Optimal Infeed Control for Accelerated Spark-Out in Plunge Grinding

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An optimal infeed control policy is proposed to minimize the cycle time in cylindrical plunge grinding. As compared with conventional infeed control consisting of roughing followed by spark-out, the proposed infeed control policy accelerates the spark-out by reducing the time required to recover the accumulated elastic deflection in the system and to reduce the infeed velocity to its final required value. This optimal infeed control policy is particularly advantageous for grinding systems having a long characteristic time constant. A practial method is described for implementing the optimal infeed control policy based upon direct measurement of the radial allowance remaining on the workpiece.

Introduction

The objective in external and internal cylindrical plunge grinding operations is to produce parts of revolution satisfying specified roundness, diametral, and surface finish requirements. By its very nature, however, the removal process in cylindrical grinding generates a spiral shape rather than a round shape as can be seen in Fig. 1. If the wheel is instantaneously retracted from the workpiece, a step is left whose height is equal to the infeed per revolution $a = v/n_w$ at the point where the wheel disengaged from the workpiece. A more nearly round part is obtained with a smaller infeed per revolution a, and hence a smaller radial infeed velocity v for a given workspeed, at the end of the grinding cycle. A smaller radial infeed velocity will also give a smoother surface finish. In this sense, the need to satisfy roundness and surface finish requirements can be considered as equivalent to specifying an upper limit on the radial infeed velocity at the end of the grinding cycle.

Cylindrical plunge grinding infeed control cycles generally consist of three or more stages. The commonest type of cycle illustrated in Fig. 2 consists of a roughing stage with a controlled infeed velocity u_1, a spark-out stage with $u_2 = 0$, and a rapid retraction stage. Due mainly to elastic deflection of the grinding system, the actual accumulated infeed follows the curve $q(t)$ and lags behind the controlled infeed. (Grinding wheel wear also contributes to the lag, but this effect is neglected for now.) The elastic deflection is accumulated during the transient at the beginning of the roughing stage (spark-in), and this deflection is recovered in the spark-out stage during which the part is rounded up and the surface roughness decreases as the actual radial infeed velocity decreases [1-3]. While almost all the actual material removal occurs during the roughing stage, the spark-out stage may constitute a significant portion of the total cycle time. Accordingly, the time for spark-out can significantly affect the production rate.

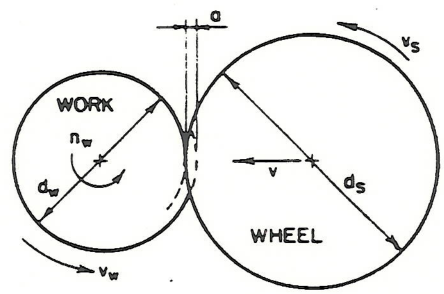

Fig. 1 Illustration of external cylindrical plunge grinding

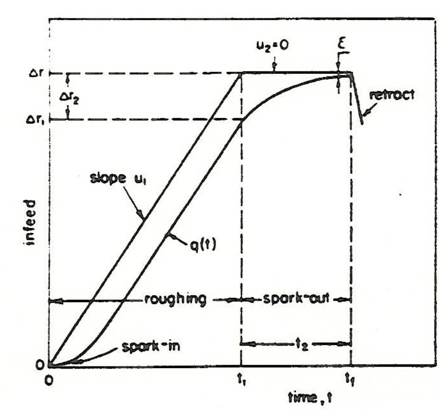

Fig. 2 Illustration of conventional grinding cycle

In the present paper, a practical approach is developed for implementing an optimal infeed control policy designed to accelerate the spark-out by reducing the infeed velocity to its final limiting value such that the total cycle time is minimized.

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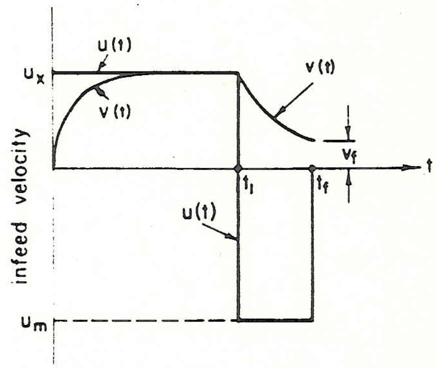

Fig. 3 Optimal control infeed velocity and actual velocity

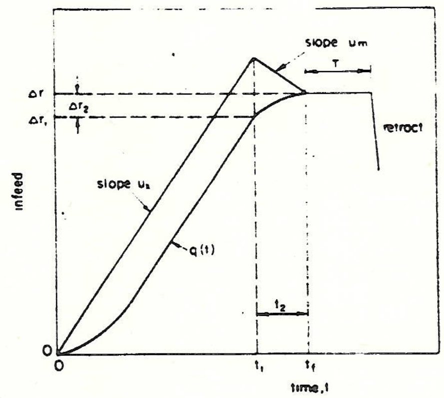

Fig. 4 Optimal grinding cycle with additional spark-out

A similar optimal control problem was previously analyzed [4], but the solution and its practical utilization differ from what is presented here.

Model

As a basis for developing the optimal infeed control policy, it is first necessary to derive the differential equations relating the control infeed velocity $u(t)$ to the actual infeed velocity $v(t)$ of the grinding system. Neglecting wheel wear, the difference between the control and actual infeed velocities can be attributed to the changing radial elastic deflection δ_e in the grinding system:

$$u(t)-v(t)=\frac{d(\delta_e)}{dt} \tag{1}$$

Let us assume that the normal grinding force is proportional to the infeed velocity:

$$F_n=Cbv(t) \tag{2}$$

and that the elastic deflection is, in turn, proportional to the normal force F_n:

$$F_n=k\delta_e \tag{3}$$

where b is the grinding width, C is a constant, and k is the overall grinding system stiffness. Combining equations (1-3) leads to

$$\dot{v}=\frac{1}{\tau}[u(t)-v(t)] \tag{4}$$

where τ is a time constant defined as:

$$\tau\equiv\frac{Cb}{k} \tag{5}$$

The rate at which the workpiece radius actually decreases is also equal to the actual infeed velocity:

$$\dot{q}=v(t) \tag{6}$$

Equations (4) and (6) comprise a set of two differential equations which describe the grinding process together with the initial conditions

$$v(0)=0 \tag{7a}$$

$$q(0)=0 \tag{7b}$$

and the final conditions

$$v(t_f)=v_f \tag{8a}$$

$$q(t_f)=\Delta r \tag{8b}$$

where v_f is the final infeed velocity ($v_f\geq 0$) based upon surface finish and roundness requirements, and Δr is the total amount of material to be removed from the workpiece radius.

Conventional Control Policy

Consider first the conventional grinding cycle shown in Fig. 2. In order to minimize the grinding cycle time, the infeed velocity control in the first stage should be the maximum allowable one $u_1 = u_x$, which will usually be limited by surface integrity or wheel breakdown. Solving equations (4) and (6) for the first stage gives:

$$v(t)=(1-E)u_x \tag{9}$$

$$q(t)=(t+\tau E-\tau)u_x \tag{10}$$

where

$$E\equiv\exp(-t/\tau) \tag{11}$$

In a typical grinding $t_1>>\tau$, so that $E=0$ at the end of the first stage and

$$v(t_1)=u_x \tag{12}$$

$$q(t_1)=u_x t_1-u_x\tau \tag{13}$$

The quantity $u_x\tau$ is the steady-state lag of the actual infeed behind the control infeed.

Equations (12) and (13) provide the initial condition for the spark-out stage with $u(t)=0$. Solving equations (4) and (6) at the final time t_f gives:

$$v(t_f)=E_2u_x \tag{14}$$

$$q(t_f)=q(t_1)+(1-E_2)\tau u_x=t_1u_x-E_2\tau u_x \tag{15}$$

where E_2 is the value of E at $t=t_2$, and t_2 is the spark-out duration ($t_2 = t_f - t_1$). Since $\Delta r = t_1u_x$, equation (15) can also be written:

$$q(t_f)=\Delta r-\epsilon \tag{16}$$

This result contradicts the requirement of equation (8(*b*)), since there is always an oversize error $\epsilon = E_2\tau u_x$ which can only be practically eliminated if $t_2>>\tau$.

Optimal Control Policy

The problem of specifying an opitmal control infeed policy can be stated as the following optimization problem:

Transfer the system

$$\begin{bmatrix}\dot{q}\\ \dot{v}\end{bmatrix}=\begin{bmatrix}0 & 1\\ 0 & -1/\tau\end{bmatrix}\begin{bmatrix}q(t)\\ v(t)\end{bmatrix}+\begin{bmatrix}0\\ 1/\tau\end{bmatrix}u(t) \tag{17}$$

with a control variable constraint $u_m \leq u(t) \leq u_x$ from its initial condition given by equation (7) to the final condition given by equation (8) in minimum time. Equation (17) corresponds to equations (4) and (6).

This problem belongs to a class of so-called "time optimal problems'' which can be solved on the basis of Pontryagin's Maximum Principle. The general solution has the following features [5]:

1 If the system equations are linear with constant coefficients (the case of equation (17)) and the admissible control is limited between given constants ($u_m \le u(t) \le u_x$), the solution is bang-bang, which means that the control switches from u_x to u_m, or vice-versa.

2 If all eigenvalues of the state matrix (the first matrix on the right-hand side of equation (17)) are real and the control $u(t)$ is bang-bang, then there can be at most $(n-1)$ switches of $u(t)$, where n is the order of the state matrix.

Since the state matrix in equation (17) is second order, there should be at most one switch of $u(t)$ at time t_1. Therefore, the optimal control interval is divided into two periods during which the extremal controls are provided:

$$u(t) = u_x \quad \text{for} \quad 0 \le t \le t_1$$

$$u(t) = u_m \quad \text{for} \quad t_1 \le t \le t_f$$

This control policy is indicated in Fig. 3. The optimal switching time t_1 and cycle time t_f can be found by solving equation (17) forward in time within the two periods and comparing the final state with the required one in equation (8). At the end of the first period, the state (v_1, q_1) is given by equations (12) and (13). At this point, the minimum infeed control velocity u_m is applied, which is negative. The final state at time t_f is:

$$v(t_f) = E_2(1-E_1)u_x - (E_2 - 1)u_m \tag{18a}$$

$$q(t_f) = q(t_1) + \tau(1-E_2)(1-E_1)u_x + (t_2 + \tau E_2 - \tau)u_m \tag{18b}$$

where E_1 and E_2 are the values of E at t_1 and t_2, respectively. By substituting equations (8) and (13) into equation (18) and assuming that $E_1 \approx 0$ as before, the two time periods are obtained:

$$t_1 = \frac{\Delta r}{u_x} + \frac{\tau}{u_x}(v_f - u_m \ln\beta) \tag{19}$$

$$t_2 = \tau \ln\beta \tag{20}$$

where

$$\beta \equiv \frac{u_x - u_m}{v_f - u_m} \tag{21}$$

The optimal grinding cycle time t_f^* obtained by adding t_1 and t_2 becomes

$$t_f^* = \frac{\Delta r}{u_x} + \tau\left[\frac{v_f}{u_x} + \left(1 - \frac{u_m}{u_x}\right)\ln\beta\right] \tag{22}$$

Practical Considerations

It is apparent that with the infeed control policy in Fig. 3, the final infeed velocity requirement of $v(t_f) = v_f$ is reached at only one point on the part periphery. The foregoing analysis does not explicitly take account of the need to satisfy the surface quality requirement around the whole circumference. In the ideal case of $v_f = 0$, this condition is readily obtained by adding a third stage with $u = 0$ for a period of at least $T = 1/n_w$, corresponding to one revolution of the workpiece, followed by a rapid retraction. The whole grinding cycle is illustrated in Fig. 4.

In order to implement this control policy, it is necessary to know at which point to switch from the first to the second stage. As a practical matter, the time t_1 cannot be directly measured accurately enough due to such factors as initial part-to-part variation. A more feasible approach is to switch when the meaured remaining allowance to be removed after the first stage reaches Δr_2. By combining equations (13) and (19):

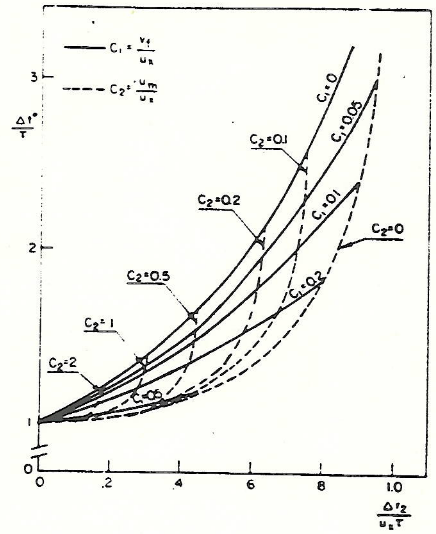

Fig. 5 Cross-plot of $\Delta t^*/\tau$ versus $\Delta r_2/u_x\tau$ for various values of C_1 and C_2

$$\Delta r_2 = \Delta r - q(t_1) = \tau(u_x - v_f + u_m \ln\beta) \tag{23}$$

In this case, the system performance may become limited by how accurately Δr_2 can be specified or measured, which leads to a trade-off between the grinding allowance Δr_2 and the grinding time. From equation (22) the optimal grinding time (excluding T in Fig. 4) can be written:

$$t_f^* = t_0 + \Delta t^* \tag{24}$$

where t_0, corresponding to the first term in equation (22), is the time to remove all the material at the maximum infeed velocity, and Δt^*, corresponding to the second term, is the additional optimal time required to recover elastic deflection and decelerate the infeed velocity to its final value. The time Δt^* can be written in dimensionless form as:

$$\frac{\Delta t^*}{\tau} = C_1 + (1 + C_2)\ln\left(\frac{1+C_2}{C_1+C_2}\right) \tag{25}$$

where

$$C_1 \equiv \frac{v_f}{u_x} \tag{26}$$

and

$$C_2 \equiv \frac{-u_m}{u_x} \tag{27}$$

Likewise, the allowance in equation (23) can also be written in dimensionless form as a fraction of the steady-state lag in the first stage:

$$\frac{\Delta r_2}{u_x \tau} = 1 - C_1 - C_2 \ln\left(\frac{1+C_2}{C_1+C_2}\right) \tag{28}$$

The dimensionless parameters C_1 and C_2 now indicate the final infeed velocity and control infeed velocity during the second stage, respectively. (Note that C_2 is defined as positive for a negative control velocity as shown in Figs. 3 and 4).

The tradeoff between $\Delta t^*/\tau$ and $\Delta r_2/u_x\tau$ is shown in Fig. 5 for various values of C_1 and C_2. The curve for $C_2 = 0$ corresponds to the conventional policy in Fig. 2 as a special case. The reduction in cycle time which is possible by adopting the optimal policy in Fig. 3 can be readily seen in Fig. 5. For example, in order to achieve a final condition $C_1 = 0.05$, the conventional control policy ($C_2 = 0$) would require $\Delta t^*/\tau = 3$ as compared with $\Delta t^*/\tau = 1.4$ for $C_2 = 1$. The optimal policy becomes increasingly advantageous as the final requirement on C_2 becomes more stringent. Furthermore the final

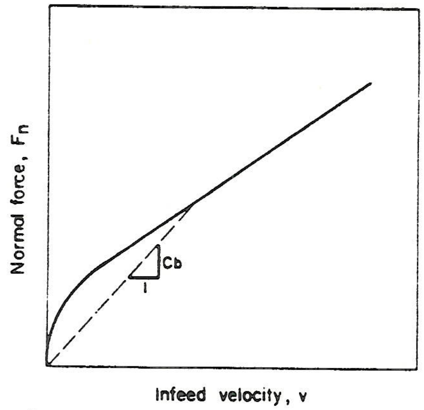

Fig 6 Normal force versus infeed velocity

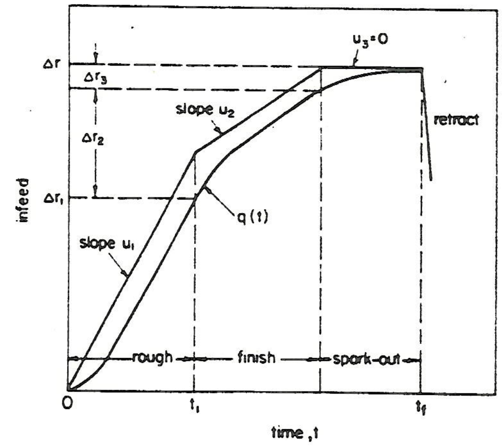

Fig. 7 Conventional grinding cycle with additional finishing stage

requirement has relatively little effect on the grinding time. For example, with $C_2 = 1$ the final condition $C_1 = 0.1$ requires $\Delta t^*/\tau = 1.3$, whereas $C_1 = 0$ is reached with $\Delta t^*/\tau \approx 1.4$. It is generally worthwhile to aim for complete spark-out ($C_1 = 0$) with this control policy.

It is apparent from Fig. 5 that the grinding time can be reduced by making C_2 bigger (faster retraction). This, in turn, will require a smaller allowance Δr_2 at the switching point. The magnitude of C_2 is likely to be constrained by a lower practical limit on Δr_2 due to such factors as the accuracy and sensitivity of the in-process gaging system, the tolerance requirements, and the response time of the machine tool and infeed control system. At the other extreme, the allowance must be less than the steady-state deflection ($\Delta r_2/u_x\tau < 1$). Once the allowance Δr_2 is specified, the required value of C_2 can be obtained from Fig. 5 or equation (28). For example, with $\Delta r_2/u_x\tau = 0.3$ as the limiting case, it can be seen in Fig. 5 that u_m should be chosen such that $C_2 \approx 1$ for a final requirement of $C_1 = 0$. Solving directly for C_2 from equation (28) is very tedious, but it can be shown for $0.2 < \Delta r_2/u_x\tau < 0.8$ and $C_1 = 0$ that C_2 can be approximated with less than 5 percent error by the relationship:

$$C_2 = \exp\left(1.65 - 5.15\frac{\Delta r_2}{u_x\tau}\right) \tag{29}$$

An example with computed values of τ, $u_x\tau$, $\Delta\gamma_2$, Δt^*, and T is given in Appendix A. Time constants on the order of $\tau = 1$ s appear to be more or less typical for external cylindrical grinding of solid components. With flexible components, the time constants would be bigger due to decreased stiffness of the grindng system. Big time constants are also typical of internal grinding operations mainly due to low system stiffness. With bigger time constants, longer spark-out times are needed with the conventional control policy, and it becomes relatively more advantageous to use the optimal infeed control policy to accelerate the spark-out.

In order to implement this optimal infeed control policy, it is necessary to specify the time constant τ of the grinding system. In general, τ cannot be directly obtained from equation (5) because the parameters C and k are not known. One practical solution is to make an on-line estimate of τ by measuring the steady-state lag at $t >> \tau$ in the first stage. The steady-state lag can be obtained as the difference between the total accumulated radial infeed $u_x t$ and the corresponding decrease in part radius as determined by in-process gaging of the part. Since the steady state lag is equal to $u_x\tau$, dividing by u_x yields the time constant τ. This method can be expected to result in a "measured" time constant which is longer than the effective one. This can be seen by noting that although the grinding model assumes a proportional relationship between the normal grinding force and the infeed velocity, the experimental measurements typically show the behavior illustrated in Fig. 6 [6]. The apparent proportionality factor Cb in Fig. 6 will decrease as the actual infeed velocity increases. For typical production grinding conditions, this overestimate of the time constant should lead to a slight overestimate of the allowance Δr_2 in equation (24), which might be compensated for by extending the time T before rapid retraction at the end of the cycle in Fig. 4.

Up to now, the effect of wheel wear has been neglected. A modified analysis taking wear into account is presented in Appendix B. With external grinding, where the wheel diameter is usually much greater than the workpiece diameter, the effect of wheel wear will be very small for grinding ratios obtained in most production operations. For a relatively smaller wheel diameter, and especially in the case of internal grinding, wheel wear will have an increasingly greater effect on the optimal control policy.

Aside from the conventional control policy in Fig. 2, another type of control policy often found in practice includes an additional finishing stage between the roughing and spark-out stages such as illustrated in Fig. 7. From the optimization analysis, it is apparent that the addition of this stage will always lengthen the grinding cycle. However, this additional stage may be necessary in order to remove subsurface damage from rough grinding, such as an overtempered layer of hardened steel. This same cycle can also be improved by adopting the accelerated spark-out control policy shown in Fig. 8, analogous to the one in Fig. 4, with the infeed velocity control u_2 now limited so as to avoid subsurface damage. The best point at which to switch from u_2 to u_3 in Fig. 7 can be found by following the same procedure as described above for the cycle in Fig. 4.

The optimal infeed control policy reduces the grinding cycle time by accelerating the spark-out. It has been proposed [7] that this accelerated spark-out policy can be advantageously incorporated into the Adaptive Control Optimization (ACO) grinding system which has been recently developed [8]. The ACO grinding system is based upon a strategy which optimizes the grinding and dressing parameters for maximum removal rate subject to constraints on workpiece burn and surface finish. In the prototype ACO system, a conventional grinding cycle was used like the one shown in Fig. 2 with a fixed spark-out time duration, and the optimization objective was essentially equivalent to maximizing u_1, thereby minimizing the roughing time. With the addition of the optimal infeed control policy, the spark-out time can also be reduced. Furthermore, the part can now be fully sparked out, which may have been impractical with the conventional cycle due to the magnitude of the time constant τ. More complete spark-out should improve the final surface finish. This, in effect, means that the surface finish constraint will be relaxed, and implementation of the ACO optimization strategy in this

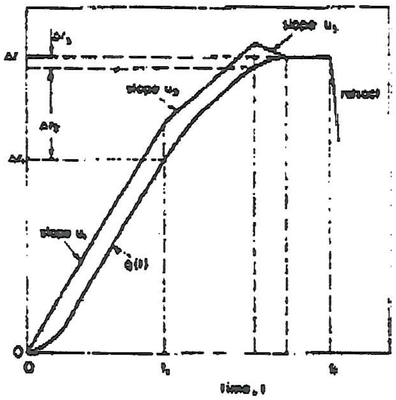

Fig. 8 Grinding cycle with finishing stage modified for accelerated spark-out

case will lead to coarser dressing and a faster allowable infeed velocity in the roughing stage with a further reduction in cycle time. Coarser dressing will also reduce the proportionality factor C in equation (2), decreasing the time constant τ and reducing still further the time for accelerated spark-out. A more advanced ACO grinding system is being developed which combines the optimal infeed control policy for accelerated spark-out together with the grinding and dressing optimization strategy of the previous ACO system.

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APPENDIX A

Consider a cylindrical external plunge grinding operation with the following parameters:

normal force (max)	$F_n = 600$ N
wheel velocity	$v_s = 45$ m/s
work velocity	$v_w = 45$ m/min
wheel diameter	$d_s = 750$ mm
work diameter	$d_w = 80$ mm
grinding width	$b = 30$ mm
system stiffness	$k = 10$ N/μm
infeed velocity (max)	$u_x = 54$ μm/s

The normal force given is the maximum value at $t >> \tau$ in the first stage when the actual infeed velocity reaches the controlled maximum value of u_x. The time constant is calculated from equations (5) and (2) as

$$\tau = \frac{Cb}{k} = \frac{F_n}{u_x k} = 1.1 \text{ s}$$

and the steady-state lag in the first stage at $t >> \tau$ is $u_x\tau = 60\mu$m. For $C_1 = 0$ and $C_2 = 1$, the required allowance from Fig. 5 is $\Delta r_2/u_x\tau = 0.3$ or $\Delta r_2 = 18\mu$m, for which $\Delta t^*/\tau = 1.4$ or $\Delta t^* = 1.5$ s. The minimum additional time T in Fig. 4 for one revolution of the workpiece is 0.34 s.

APPENDIX B

The grinding model can be readily modified to include the effect of wheel wear for a given grinding ratio G. The grinding ratio is defined as the volumetric ratio of metal removal rate of wheel wear rate:

$$G = \frac{\pi d_w b v(t)}{\pi d_s b w(t)} = \frac{d_w v(t)}{d_s w(t)} \tag{B-1}$$

where $w(t)$ is the radial wear rate of the wheel. Taking wheel wear into account, the continuity condition analogous to equation (1) becomes:

$$u(t) - v(t) - w(t) = \frac{d(\delta e)}{dt} \tag{B-2}$$

Combining equations (B-1) and (B-2) with equations (2) and (3) leads to

$$\dot{v} = \frac{1}{\tau'}[u'(t) - v(t)] \tag{B-3}$$

where

$$\tau' = \frac{\tau}{1 + \dfrac{d_w}{d_s G}}$$

and

$$u'(t) = \frac{u(t)}{1 + \dfrac{d_w}{d_s G}}$$

Equations (B-3) and (6) are now the two differential equations which describe the grinding process in place of equations (4) and (6). Therefore, the same analysis still applies provided that τ is replaced by τ' and $u(t)$ is replaced by $u'(t)$.