The structure and function of networks

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Abstract

Many systems take the form of networks, including the Internet, distribution and transport networks, neural networks, food webs, and social networks. The characterization and modelling of these systems has proved amenable to treatment using techniques drawn from statistical and computational physics, and has as a result attracted considerable attention in the physics literature in recent years. In this paper the author reviews some of the interesting issues in this area and recounts some recent work on these issues by his group and by others.

1. Introduction

Many definitions of "complex systems" have been proposed over the years, and at present there is none which is universally accepted. Most people however would agree that a fundamental property of complex systems is that they are composed of a large number of components or "agents", interacting in some way such that their collective behaviour is not a simple combination of their individual behaviours. Classic examples of such systems include social organization in human or animal communities, financial and other markets, the Internet, and ecosystems of interacting species.

Over the years, a large body of research has been directed at understanding both the behaviour of individual agents within complex systems and the nature of the interactions between them. As we are just beginning to realize however, there is a third aspect to these systems which may be even more important and which has so far received little attention, and that is the *pattern* of interaction between agents, i.e., which agents interact with which others. This pattern forms a network or graph of connections between agents [1,2], and it is on such networks that we focus in this paper.

Recent interest in the structure of networks has been spurred partly by the Internet, which has some interesting features when viewed as a graph [3]. A related but distinct network is the World-Wide Web, a network of "pages" of information which can be accessed over the Internet and which are joined to one another by "hyperlinks"—directed edges leading from one page to another [4,5]. There are also many networked systems that occur in biology, such as neural networks [6], food webs [7,8], and metabolic networks [9,10]. In addition, studies have been made of distribution networks such as airline timetables [11] or blood vessels [12,13], river networks [14], and even networks of semantic linkage between words [15]. There is however one area in which networks have been studied longer than any of these; social networks—networks of connections between people—have been studied in the sociology literature at least since the 1940s, and possibly earlier, and it is social networks which are the primary topic of this paper, although many of the

network	source	size	separation	clustering
company directors	Fortune 1000	7 673	4.60	0.588
movie actors	IMDB	449 913	3.48	0.199
physics authors	`arxiv.org`	52 909	5.92	0.452
biomedical authors	Medline	1 520 253	4.42	0.088

Table 1
Size, mean vertex–vertex separation, and clustering coefficient for four real-world networks of connections between people, as described in the text. After Ref. [25].

ideas described here may have wider applicability.

In one of the most famous early studies of social networks Stanley Milgram performed an experiment in which letters were passed from one acquaintance to another to reach a distant target individual [16]. Milgram found that the typical number of people his letters passed through to reach a specified target was only about six, and he tentatively concluded that if one were to construct the complete network of acquaintances, there would be only a short distance through this network from any person to any other, of typical length around six. Although the exact number six is certainly debatable this result is now widely accepted as being correct. It is often referred to as the "small-world effect", or the "six degrees of separation", the latter phase being taken from a 1990 play of the same name by John Guare in which Milgram's work is mentioned [17].

Acquaintance networks are the archetype of a social network, but there are many other kinds of social networks as well. Studies have addressed networks of people connected by common attendance at social events [18], common membership of clubs [19], networks of business relations between companies [20], and many others. Two types of networks which the present author has examined extensively are networks of collaborations between scientists [21,22] and networks of boards of directors of companies [23–25]. On a lighter-hearted note, there has also been a moderate amount of work on the network of collaborations of actors in films [11,26].

This paper is a personal review of recent research on social and other networks within the physics community, using a number of examples drawn from the author's own work. The outline of the paper is as follows. In Section 2 we describe three distinctive properties of real-world networks which distinguish these networks from the standard network models, such as the regular lattice, that are common in physics. In Section 3 we discuss some recent simulation and modelling work we and others have performed in an attempt to understand these properties. In Section 4 we give our conclusions.

2. Statistical properties of networks

In this paper, we focus on three distinctive properties of real-world networks including social networks. These properties are as follows:

1. **The small-world effect:** As discussed in the previous section, the small-world effect is the finding that there exist short paths through a network between most pairs of vertices. In Table 1 we show the measured average distances between vertex pairs in four networks. The networks are: the network of directors of Fortune 1000 companies for 1999 (the 1000 US companies with the highest revenues), in which two directors are considered connected if they sit on the board of the same company; the network of collaborations between movie actors in the Internet Movie Database (as of 1 May 2000), in which actors are connected if they have appeared in a movie together; and two networks of scientific collaborations, for physics (from papers in the Los Alamos E-print Archive) and biomedical research (from papers in the Medline bibliographical database), in which two scientists are connected if they have coauthored a paper together. In each case the average vertex–vertex distance is small (in the range 3 to 6), certainly much smaller than the number n of vertices in the complete network.

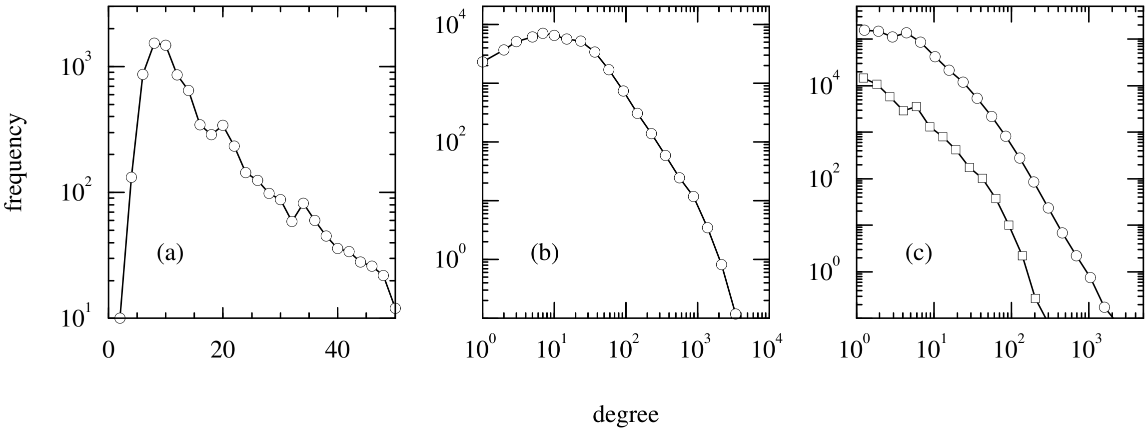

Fig. 1. Histograms of the degree distributions of the four networks of Table 1. (a) Distribution of number of directors with whom each director of a Fortune 1000 company sits on boards; (b) distribution of number of other actors whom each film actor in the Internet Movie Database as appeared in films with; (c) distributions of the total number of coauthors of authors in the Medline (top) and Los Alamos Archive (bottom) publication databases during the interval 1 January 1995 to 31 December 1999.

In recent years, following various empirical and theoretical results, the term "small-world effect" has taken on a more technical meaning, that typical distances in the network scale as $\log n$ with network size, and we will adopt this definition in this paper. In Section 3.1 we show for the particular case of the scientific collaboration networks that logarithmic scaling of this type does indeed occur in real life.

2. **Skewed degree distributions:** The degree of a vertex in a network is the number of other vertices to which it is connected. If one makes a histogram of the degrees of vertices in a real-world network, one typically finds that their distribution is highly skewed. In Fig. 1 we show such histograms for the four example networks from Table 1. As the figure shows, the network of company directors has a degree distribution which is roughly exponential after an initial rise, while the other networks have a distribution which approximately follows a power law. Networks with power-law degree distributions are sometimes called scale-free networks, although of course they may have scales of various kinds governing properties other than their degree distribution; strictly it is only the degree distribution which is scale-free. The Internet and the World-Wide Web appear to be examples of networks that are scale-free [3,4].

3. **Clustering:** A further characteristic feature of real-world networks has been highlighted by Watts and Strogatz [26], who pointed out that most networks are highly "clustered". In the context of social networks, this means that there is a heightened probability that two people will be acquainted if they have a third acquaintance in common. Put another way, there is a heightened density of "triangles" of acquaintance in the network in which three people all know one another. One can define a clustering coefficient C which measures this effect thus [25,26]:

$$C = \frac{3\times \text{ number of triangles on the graph}}{\text{number of connected triples of vertices}}. \quad (1)$$

Here "triangles" are trios of vertices each of which is connected to both of the others, and "connected triples" are ordered trios in which at least one is connected to both the others. The factor of 3 in the numerator accounts for the fact that each triangle contributes to three connected triples of vertices, one for each of its three vertices. With this factor of 3, the value of C lies strictly in the range from zero to one, and is equal to the average probability that two of one's friends are also friends of one another. In Table 1 we show the values of this coefficient for the same four networks as before. In each case the number is reasonably high: the probabil-

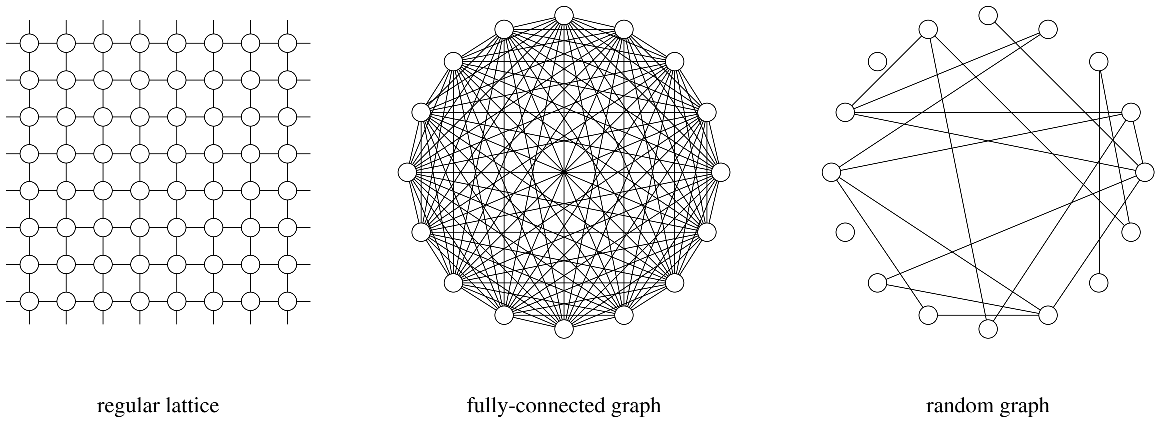

Fig. 2. Three standard network configurations which have been studied in detail in the past: the regular lattice (the square lattice, in this case, with nearest-neighbour interactions), the fully-connected graph, and the random graph.

ity of one's friends knowing each other is usually on the order of a few tens of percent.

3. Models of networks

Interactions between the components of a system are of course nothing new in physics, and a number of standard patterns of such interactions have been studied in detail in the physics literature. The three most common such prototypical networks are shown in Fig. 2. They are:

The regular lattice: This is certainly the most widely studied network in physics, but it is a very poor representation indeed of most real-world networks, particularly social networks. It does not show the small-world effect, having typical vertex–vertex distances which scale with system volume n as $n^{1/d}$, where d is the dimensionality of the lattice. Nor does it have a skewed degree distribution, although some regular lattices (e.g., the triangular lattice) do have high clustering coefficients.

The fully-connected graph: This graph, in which every vertex is connected directly to every other, is used in physics as the basis for mean-field theory, as well as for some simplified models of physical systems such as the Sherrington–Kirkpatrick spin-glass model. It shows the small-world effect after a fashion—every vertex is exactly distance 1 from every other—but not the logarithmic scaling of distance with system volume that has become the accepted definition of a small world. It also does not have a skewed degree distribution. It has clustering coefficient $C = 1$.

The random graph: Less studied in physics than in mathematics, the random graph probably comes closest to mimicking a real network. It does show the small world effect, with the typical vertex–vertex distance being $\ell = \log n / \log z$, where z is the average degree of a vertex. However, it has a Poissonian (not skewed) degree distribution, and its clustering coefficient is $C = z/n$, which is small compared to the value of C for most real-world networks.

So if none of these standard networks does a good job of representing real-world situations, what does? In the remainder of this paper we describe a number of models, introduced by various authors, which aim to capture at least some of the characteristic network features that we have discussed. All of these models in fact capture the small-world effect, and all of them also mimic either skewed degree distributions or the clustering effect as well.

3.1. *Random graphs with skewed degree distributions*

The random graph of Erdős and Rényi [27] mentioned above, in which a certain number of edges are placed randomly between the vertices of an initially empty network, captures the small-world effect nicely but has a degree distribution which is Poissonian rather than exponential or power-law in form. As pointed out by Molloy and Reed [28], however, the model can be modified to correct this problem as follows. Suppose we wish to construct a network with a given degree distribution in which the probability of a randomly-chosen vertex having degree k is p_k. We can do this by taking n vertices and giving each of them a degree k drawn at random from p_k. We can think of the degrees as being represented by the "stumps" of edges emerging from the vertices. Then we choose pairs of these stumps at random and join them together. This procedure produces a network which has the desired degree distribution, but which is otherwise random. It turns out that many average properties of networks of this kind can be calculated exactly, using combinatoric methods [25,28]. For example, it can be shown that the typical vertex–vertex distance in such a network is given by

$$\ell = \frac{\log n/z_1}{\log z_2/z_1} + 1, \qquad (2)$$

where z_1 is the average degree of a vertex (previously called z) and z_2 is the average number of second-nearest neighbours of a vertex. These numbers can easily be measured for any given network, and in Fig. 3 we show a comparison of the vertex–vertex distance on a number of networks of scientific collaborations, plotted against the predicted value of the same quantity from Eq. (2). As the figure shows, the agreement between theory and empirical results is good, giving us some hope that models of this kind might be a useful representation of real-world networks.

3.2. *Growth models for skewed degree distributions*

While the generalized random graph model above can represent a network with any degree distribution we desire, it offers no explanation for where degree distributions come from. Another model which does this for the case of a power-law degree distribution has been proposed by Barabási and Albert [29]. In their model, they propose that power-law degree distributions arise as a result of "preferential attachment", a process whereby ver-

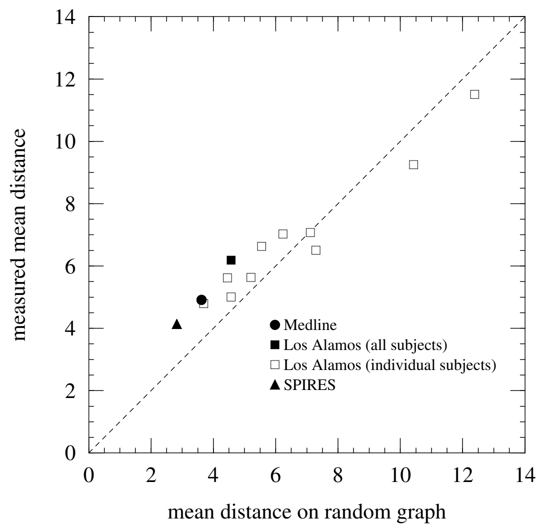

Fig. 3. Measured vertex–vertex distances for 13 scientific collaboration networks drawn from bibliographical databases in biology (Medline), physics (Los Alamos E-print Archive), and high-energy physics (SPIRES), plotted against theoretical predictions for the same distances from Eq. (2). After Ref. [21].

tices and edges are continually added to a network and edges attach with greater likelihood to existing vertices which already have high degree. Herb Simon showed in 1955 [30] that a process of this type can give rise to a power-law distribution of the wealth of individuals in an economy. Employing similar mathematical techniques, Barabási and Albert showed that the same is true for degree distributions of networks. In particular, if the probability of an edge attaching to a vertex of given degree is simply proportional to that degree, then the resulting degree distribution follows a power law with exponent -3. In real-world networks such as the Internet and the World-Wide Web, the exponent of the degree distribution is usually between 2 and 3, which agrees reasonably with the theory. Growth models such as these have some other interesting properties as well. For example, some of them seem to exhibit an infinite-order phase transition at the point where a giant component of connected vertices first appears [31,32]. They do not, however, appear to give high clustering coefficients [2].

If preferential attachment provides a possible ex-

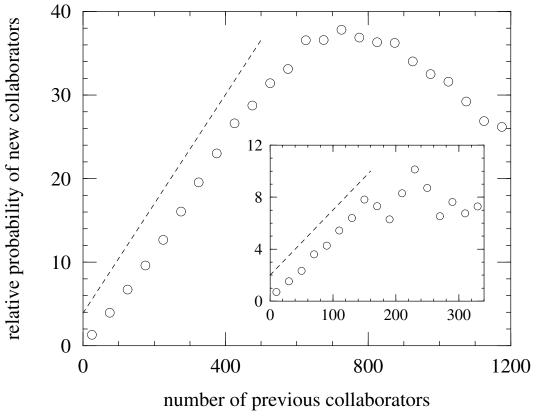

Fig. 4. The probability that a new edge in a collaboration network will attach to a vertex of given degree as a function of that degree, measured relative to the probability of the same occurrence on a network with the same topology but on which preferential attachment does not occur. The roughly linear increase in the relative probability indicates that linear preferential attachment occurs in these networks, at least for low degree. Main figure: data from the Medline biomedical database. Inset: data from the Los Alamos physics e-print archive. After Ref. [33].

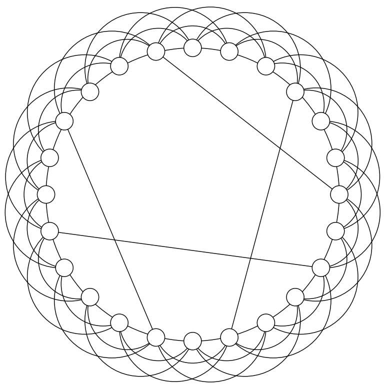

Fig. 5. The Watts-Strogatz "small-world" model in one dimension with near-neighbour links out to distance $k = 3$ and shortcut density $\phi = 0.05$.

planation for the skewed degree distributions we see in real-world networks, the next step is to try to confirm if preferential attachment does actually take place in those networks. In order to do this, we need network data with good time resolution; we need to know the order in which edges are added to a network, so that we can observe whether preferential attachment is indeed taking place. One class of networks for which this is possible is the scientific collaboration networks discussed earlier. In these networks, two scientists are considered connected if they have coauthored one or more papers, and the time of addition of each link in the network is the time at which the corresponding paper was submitted or published. Using this idea, Newman [33] has calculated the relative probability of edges attaching to different vertices in various collaboration networks as a function of the current degree of those vertices. Figure 4 shows this relative probability for collaborations in biology and in physics, and it appears from the figure that indeed linear preferential attachment does take place in these networks. There is an upper limit in the degree above which preferential attachment no longer takes place, but interestingly this upper limit corresponds roughly to the point at which power-law behaviour in the degree distribution ceases for these networks, which only lends further support to the preferential attachment hypothesis. Jeong *et al.* [34] have performed similar measurements of preferential attachment for citation networks, the Internet, and the network of collaborations of film actors, with similarly encouraging results.

3.3. *The Watts–Strogatz model for clustering*

Turning now to the question of clustering in networks, we introduce a number of models which may explain clustering in some situations.

One of the first models to offer an explanation for the simultaneous appearance of the small-world effect and clustering in the same network was the "small-world model" of Watts and Strogatz [26]. This model proposes that networks are constructed from a regular lattice, representing local connections between individual people, plus a low density of "shortcut links" which join randomly chosen pairs of individuals at arbitrarily great distances. These shortcut links might represent the acquaintances one has in other countries or in other walks of life. The most widely studied case of the model is built on a one-dimensional regular lattice with periodic boundary conditions and links between near neighbours out to some maximum range k, as illus-

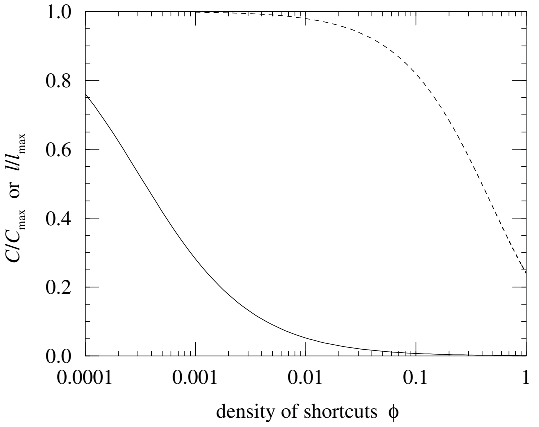

Fig. 6. Average distance between vertices (solid line) and clustering coefficient (dotted line) for the one-dimensional Watts–Strogatz model with $k = 1$ as a function of the density ϕ of shortcuts. The cluster coefficient is measured relative to its value $C_{\max}$ on a fully connected graph, and ℓ relative to its value $\ell_{\max}$ on a random graph. The value of ℓ is taken from the mean-field treatment of Ref. [35] and the value of C from Eq. (3).

trated in Fig. 5. The number of shortcut bonds is usually expressed as a fraction ϕ of the number of bonds on the underlying one-dimensional lattice.

One of the nice things about this model is that many of its properties can be calculated analytically [35–37]. In the limit of large system size, for example, the clustering coefficient is given by

$$C = \frac{3k(k-1)}{2k(2k-1) + 8\phi k^2 + 4\phi^2 k^2}. \tag{3}$$

In Fig. 6 we show results for the typical vertex–vertex distance ℓ in the model and the clustering coefficient C as a function of ϕ. The crucial point to notice about the figure is that there is a substantial regime of intermediate values of ϕ in which ℓ is small and C is simultaneously large, so that the network shows both the small-world effect and clustering.

The model of Watts and Strogatz however is clearly not a realistic model for most networks (a possible exception is the use of the two-dimensional version of the model to represent the propagation of plant diseases [38]), and so researchers have turned to the development of more realistic models.

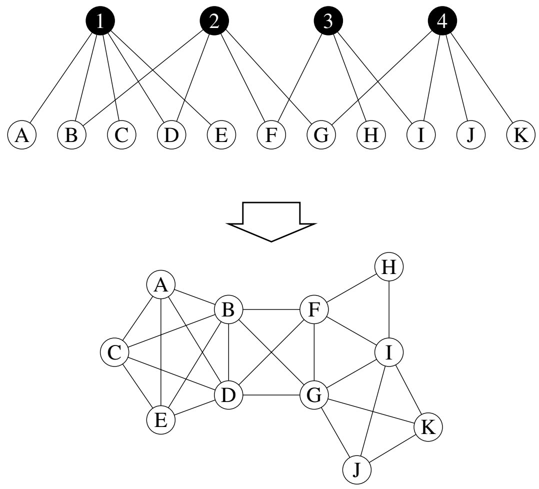

Fig. 7. Top: A bipartite graph of, for example, scientists (A to K) and scientific papers (1 to 4) with lines linking each scientist to the paper on which their name appeared as a coauthor. Bottom: the projection of the same network onto just the scientists.

3.4. *Bipartite graph models and clustering*

A better and very simple explanation for clustering arises in bipartite graphs, or "affiliation networks". These are networks in which vertices are joined together via common membership in groups. All of the example networks from Table 1 and Fig. 1 have this form: the scientific collaboration networks, the film actor collaboration network, and the network of company directors. Each can be represented as a bipartite graph, as in Fig. 7, in which there are two types of vertices representing scientists (actors, directors) and papers (films, boards), with edges running between each scientist and the papers on which their name appeared as a coauthor. Typically one does not represent such networks in their full bipartite form however, but rather one projects the network down onto just the authors, so that there is an edge running between any two authors who have coauthored a paper (see Fig. 7 again). This projection produces a network which contains a high density of triangles; any paper with three or more authors contributes at least one such triangle. These triangles can then produce a finite value for the clustering coefficient, following Eq. (1).

In fact, it turns out to be fairly straightforward

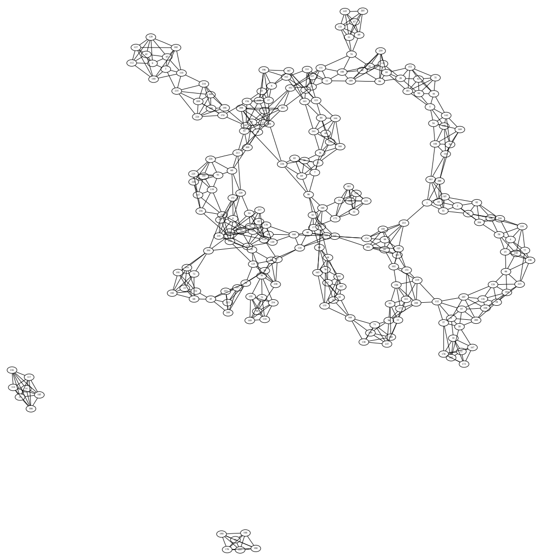

Fig. 8. An example of a simulated social network generated using one of the algorithms proposed by Jin *et al.* [39], in which links are preferentially formed between pairs of individuals who have one or more mutual acquaintances.

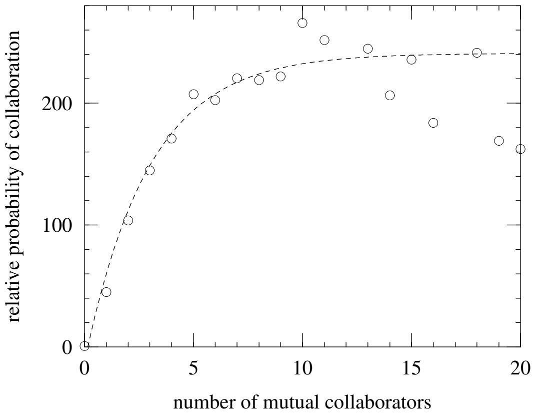

Fig. 9. The relative probability of two scientists collaborating as a function of their number of previous mutual collaborators. After Ref. [33].

to calculate the clustering coefficient for these networks analytically, given the degree distributions of the two types of vertices in the original bipartite network [25]. The resulting values are often quite high and in some cases compare favourably with the real-world values. For example, in the network of the boards of directors of the Fortune 1000 companies, theory predicts a clustering coefficient of 0.590, where the measured value is 0.588. In other cases, however, the agreement is much poorer. For the collaboration networks of scientists and film actors for example, the theory underestimates the true value by about a factor of two.

3.5. *Growth models for clustering*

In social networks the standard explanation for clustering is that people tend to introduce pairs of their friends to one another, thus completing triangles in the network and increasing the clustering coefficient. A mechanism of this sort could explain why the simple theory of the previous section often underestimates clustering coefficients. One can construct a growth model of a social network in which one preferentially makes links between pairs of individuals who have one or more common acquaintances. Models of this kind have been proposed by Watts [36], Jin *et al.* [39], and Davidsen *et al.* [40]. In Fig. 8 we show an example of a network grown with the model of Jin *et al.* The resulting network shows high clustering ($C = 0.45$ in this case), but it also has another interesting feature. It shows clear groups or "communities" of vertices which have many connections to one another and fewer to vertices outside the group. This unanticipated feature of the model may reflect a genuine mechanism for community formation in real social networks: the local mechanism of people introducing pairs of their friends to one another can produce the global phenomenon of cliquishness and social groupings.

In the same way as we did for preferential attachment models in Section 3.2, we can test the assumptions of growth models for clustering by looking at time-dependent network data. Taking the example of the scientific collaboration networks again, we show in Fig. 9 the relative probability of two authors in the Los Alamos physics archive, that have not previously appeared as coauthors on the same paper, coauthoring a paper as a function of their number m of previous mutual collaborators. As the figure shows, this probability goes up extremely fast with number of mutual collobora-tors. Initially the increase appears roughly linear, although it levels off as m becomes large. The dotted line in the figure is a least squares fit to the form $A - B\,\mathrm{e}^{-m/m_0}$, although this form should be taken with a pinch of salt: the data above about $m = 10$ in the figure have large statistical errors, since the number of pairs of people in the database who have 10 or more mutual past collaborators but have not themselves collaborated is small. Nonetheless, the overall message of the figure is clear, that having previous mutual collaborators does indeed strongly dispose a pair of scientists to work together.

4. Conclusions

In this paper the author has given a brief personal overview of some recent work within the physics community on networks of various kinds. The discussion centres on the development of models to explain three crucial features which seem to be common to most real-world networks: short vertex–vertex path lengths (the "small-world" effect), skewed degree distributions, and clustering. A number of models have been put forward which explain one or more of these features well, including generalized random graph models, preferential attachment models, and the Watts–Strogatz social network model.

So far, however, no models have been proposed which simply and convincing explain how all three of these features come to be present in a network simultaneously. This is one of the interesting open questions in the field.

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