A bridge from extensive form games to spatial models.

- Assignments. Past & present.
  - Changes.
  - The importance of the next three weeks.
- Romer-Rosenthal.
  - The implication of trembling hands
- Median Voter Theorem.
- Two-dimensional agenda games.

Your Writing Assignment

- Take your paper, change three premises.
- Consider three criteria when making the changes:
  - Service to other scientists.
  - Service to society.
  - Testability and/or generalizability.
  - *Explain your modeling choice using these criteria.*
- To the greatest extent possible, use formal logic to demonstrate whether and how your revisions change the model's substantive implications.

Paper-related Assignment

- In class
  - A 15-20 minute presentation.
    - Up to five minutes. Overview.
    - Up to five minutes. Model.
    - At least five minutes. Connecting the model to the conclusions.
    - Up to five minutes. The changes you are thinking of making.

Overview Format

- M. Motivation
- NH. Null Hypotheses
- P. Premises
  - KEY. What choices did they make?
  - Would you make the same ones?
- C. Conclusions

Extensive Form Games

- Player moves can be treated as sequential or simultaneous.

- First Models:
  - Complete information – games in which all aspects of the structure of the game –including player payoff functions -- is common knowledge.

  - Perfect information – at each move in the game the player with the move knows the full history of the play of the game thus far.

The structure of a simple game of complete and perfect information.

1. Player 1 chooses an action a_1 from the feasible set A_1.
2. Player 2 observes a_1 and then chooses a_2 from the feasible set A_2.
3. Payoffs are $u_1(a_1, a_2)$ and $u_2(a_1, a_2)$.
    1. Moves occur in sequence, all previous moves are observed, player payoffs from each move combination are common knowledge.
    2. We solve such games by backwards induction.

The central issue is credibility

Spatial Utility in one dimension

- Utility is commonly defined by the distance between an outcome and an ideal point.
- Player i's utility is maximized when the outcome is located at her ideal point, $ideal_i$.
- In one dimension, it is common to assume:
  - $u_i(outcome, ideal_i) = -|outcome - ideal_i|$.
  - Such preferences are called "single-peaked."

A Simple Model of Delegation

- There is a status quo.
- The agent makes a proposal.
- There is complete information.
- The principal accepts or vetoes the proposal.

Romer and Rosenthal (1978)

- M. Monopoly power in public finance.
- NH. Agenda Control implies unlimited power.
- P_1. Two completely informed players: an agenda setter and a voter.
  - The setter wants to maximize his budget.
  - Voter preferences are single-peaked.
- P (technical). Single-peaked preferences. A continuum of voters. A single-dimensional policy space. Majority rule. Complete & perfect information.

R&R Premises

- There exists a status quo policy, $Q \in \Re$.
- The setter makes a proposal $X \in \Re$.
- The voter chooses a winner $Y \in \{X, Q\}$.
- Each player has an ideal point and single peaked preferences
  - $U_S = -|Y-S|$
  - $U_V = -|Y-V|$

R&R Conclusions

- Suppose $V \leq Q$ (parallel solution for $V>Q$.)
- It is better for the voter to choose X only if $X \in [V-(|V-Q|), Q]$.
- If the voter is indifferent, she flips a coin.
- The setter's best response to his anticipation of voter reactions is
  - If $S \in [V-(|V-Q|), Q]$, then $X=S=Y$.
  - If $S \in [0, V-(|V-Q|))$, then $X=max[0, V-(|V-Q|)]=Y$.
  - If $S \in (Q, 1]$, then $X=(Q, 1], Y=Q$.
    - Trembling hand perfection implies $X=S$.
- In equilibrium, the outcome need not be the median voter's ideal point.
  - Prove it.

Trembling Hand Perfect NE

- An equilibrium is perfect if it is immune to the possibility that players, with some small probability, commit errors.
- Morrow, p. 193: Trembling-hand perfection tests the robustness of an equilibrium. It verifies that each player's strategy is a best response against small deviations.

R&R Example

S	V	Q
1	5	9
3	7	1
3	9	5
7	1	5
6	8	1
1	2	3
9	5	4
3	7	9

R&R Example 1

S	V	Q	Range	Outcome
1	5	9	1-9	1
3	7	1	1-13	3
3	9	5	5-13	**5**
7	1	5	0-5	**5**
6	8	1	1-15	6
1	2	3	1-3	1
9	5	4	4-6	**6**
3	7	9	5-9	**5**

Lessons from Romer and Rosenthal

- Causal factors:
  - **Interest Proximity**
    - closer interests, better outcome
  - **Reversion Point**
    - if bad for principal, agent gains
  - **Proposal & Amendment Rights**
    - if proposal restricts future actions, proposer benefits
- Missing: information problems.

Black (1948)

- M. "When a decision is reached by voting or is arrived at by a group all of whose members are not in complete accord, there is no part of economic theory which applies."
- NH. Is there more than one point that can beat all others by a simple majority?
- P. One dimension. Single-peaked preferences. A continuum of voters. A single-dimensional policy space. Majority rule. Complete & perfect information.
- C. Voters choosing the alternative closes to them and both candidates choosing the median voter's ideal point is the unique Nash equilibrium.

Variations of the Median Voter Theorem

- Two candidates.
  - Prove it.
- Four candidates.
  - Prove it.

MVT Premises

- Each voter has an ideal point $V_j \in \Re, j \in \{1, \ldots, N\}$
  - N *is* large, finite, and odd.
- Each voter has single peaked preferences, $U_i = -|Y - V_j|$
  - Indifferent voters flip a coin.

- The voters choose a winner $Y \in \{c_1, c_2\}$.
  - The probability of c_1 winning =1 if the number of voters preferring it to c_2 is greater than $N/2$.
  - The probability of c_1 winning =.5 if the number of voters preferring it to c_2 is $N/2$.
  - Otherwise, the probability is zero.

- Each candidate makes a proposal $c_i \in \Re, i \in \{1, 2\}$.
  - Each candidate wants to maximize the number of votes.

Spatial Utility in 2 dimensions

- Utility is commonly defined by the distance between an outcome and an ideal point.
- Player i's utility is maximized when the outcome is located at her ideal point, $ideal_i$.
- Recall that $a^2 + b^2 = c^2$ implies $c = \sqrt{a^2 + b^2}$
- In two dimensions, it is common to assume:

$$u_i(outcome, ideal_i) = -\sqrt{|outcome_x - ideal_x|^2 + |outcome_y - ideal_y|^2}$$

Example

Ordeshook 1992: 82-85

- Five committee members have standard preferences over a two-dimensional Euclidean policy space
  - $x_1=(5,0)$
  - $x_2=(10,0)$
  - $x_3=(5,10)$
  - $x_4=(0,10)$
  - $x_5=(5,5)$
- The status quo, Q=(30,30).
- Motions are made under an open rule and voted on sequentially in a pairwise fashion.
- Which outcome prevails?

Example’

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- Five committee members have standard preferences over a two-dimensional Euclidean policy space
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  - $x_3=(5,10)$
  - $x_4=(0,10)$
  - $x_5=(5,5)$
- The status quo, Q=(30,30).
- Motions are made under an open rule and voted on sequentially in a pairwise fashion.
- Voter 5 is not allowed to make a motion.
- Which outcome prevails?

Example’’

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  - $x_3=(5,10)$
  - $x_4=(0,10)$
  - $x_5=(5,5)$
- The status quo, Q=(30,30).
- Motions are voted on sequentially in a pairwise fashion.
- Voter 1 and 2 can make one motion each.
- Voter 1 first decides whether to make the first motion or to let voter 2 do so.
- Which outcome prevails?

If Voter 1 makes the first motion

- If voter 1 proposes his ideal point, voter 2 will respond by making a motion on the line connecting x_2 and x_5 that is to her own ideal point and least as close to x_5 as x_1.

- If voter 1 proposes the point that lies midway between x_2 and x_5, voter 2 cannot make a motion that leaves both her and voter 5r better off.

- Therefore, the outcome is (7.5, 7.5).

If Voter 1 passes

- There are 2 cases to consider
  - Voter 2's motion is further from x_5 than is Voter 1's ideal point.
    - Voter 1 will offer his ideal point as a motion.
  - Voter 2's motion is closer to x_5 than is Voter 1's ideal point.
    - Voter 1 will offer a motion that is slightly closer to x_5.
    - Of these points, Voter 2 most prefers voter 1's ideal point.
- Therefore, the outcome is voter 1's ideal point.
- Thus, voter 1 prefers to pass on the opportunity to make the first motion.