The Problem Sets

- Last Week
  - We should set 18 as the legal age for the purchase of alcoholic beverages. If people are old enough to vote, then they are certainly old enough to drink, and 18-year-olds are indeed old enough to vote.
  - Look, I don't know about anything else, but I know for a fact that Jones could never have killed anyone. So when Smith testified that Jones was the murderer, Smith must have been lying.
- Next Week
  - This one runs ahead of the lectures in class.
  - Skill development is more important than grades.

Key Premise and Questions

- Collective choice is a necessary and useful part of social life.
- What characteristics do you want a collective choice to have?
- If there exist no decision rules that have all the characteristics you desire, which combinations of characteristics can you have?

Social Choice Theory

- Examines the relationship between individual will and collective decisions.
- Focuses on preference aggregation and its implications for political/institutional engineering.
- The foundations (cooperative game theory) are positive, the uses (how preferences should be aggregated) tend to be normative.
  - Concepts: Equity. Utilitarian. Majority rule. Anonymity. Monotonicity.

Definitions: A grammar for formal models of choice.

- $N = \{1, 2, \ldots, n\}$ A set of individuals.
- $\{x, y, z\} \in S$ A set of alternatives.
- Preferences
  - Expressed as binary relations: the precursor to utility functions.
  - x R y x is ``at least as good as'' y
  - x P y x is ``strictly preferred to'' y <===> x R y and not y R x.
  - x I y x is ``indifferent to'' y <===> x R y and y R x.
  - R preference profile
  - $R = (R_1, \ldots, R_n)$

Definitions

- An element $x \in S$ is a *maximal element* of S with respect to binary relation $R \Leftrightarrow \sim[\exists(y \in S \ \& \ y \ P \ x)]$.
  - No y is strictly preferred to x.
  - There can be a y indifferent to x.

- An element $x \in S$ is a *best element* of S with respect to binary relation $R \Leftrightarrow \forall y: y \in S \rightarrow x \ R \ y$.
  - x is "at least as good" as any other element of S.
  - Has the virtue of being non-empty in a broader range of cases.

- The set of best elements of S is called its *choice set* and is denoted C(S, R).
  - What properties do you want choice sets to have?

Social Choice Theory

- Motivation: How do social choices correspond to individual desires?

- Premises
  - Alternatives $\{x, y, z\} \in S$ and individuals $i \in N$.
  - Sincere behavior. Complete information.
  - Individual preferences
    - $x \, P_i \, y$: strong ($>$). $x \, R_i \, y$ weak ($\geq$)
- Social Choice
  - CCRs convert a set S and profile R into a social choice.

Condorcet's Paradox

- M. Is majority rule optimal?
- NH. MMD aggregates preferences clearly.
- P. At least 3 voters and 3 alternatives. Complete information. Originally, sincere voting.
- C. MMD is not sufficient to produce a stable relationship between individual preferences and collective outcomes.

Example

Voter	1	2	3
Best	A	B	C
	B	C	A
Worst	C	A	B

MR Agendas: (ABC)⇒C, (ACB)⇒B, (BCA)⇒A.

The agenda determines the outcome. There is no 1:1 relationship between individual will and collective choice.

Paper Presentation Format

- M. Motivation
- NH. Null Hypotheses
- P. Premises
  - KEY. What choices did they make?
  - Would you make the same ones?
- C. Conclusions

Arrow's Theorem

- M. How do individual desires affect collective choices?
- NH (inexact). A CCR can always resolve interest conflicts.
- P. At least 2 voters and 3 alternatives. Complete information. Sincere voting.
- C. No such CCR exists.

Arrow's General Possibility Theorem

Collective Rationality

- Complete. $\forall x, y \in S$, either $x \, R \, y$, $y \, R \, x$ or both.
- Reflexive. $\forall x, y \in S$, $x \, R \, x$.
- Transitive $\forall x, y, z \in S$, $x \, R \, y$ and $y \, R \, z \Rightarrow x \, R \, z$.

- C. A collectively rational CCR cannot satisfy the following four conditions simultaneously.
  - If you want all but one of these desirable properties, you must sacrifice the remaining one.

Arrow's General Possibility Theorem

- **U**nrestricted Domain: The CCR allows us to consider any set of preferences.

- **P**areto: If everyone prefers X to Y, then Y is not chosen when X is available.

- **I**ndependence of Irrelevant Alternatives.
  $\forall x,y \in S$, and all R, R', $x\ R_i\ y \leftrightarrow x\ R'_i\ y \Rightarrow C(S,R)=C(S,R')$

- **D** There is no dictator.
  - There is no $i \in N$, s.t. $\forall x, y \in S, x\ P_i\ y \Rightarrow x\ P\ y$.

Violations

- Completeness: simple majority rule.
- Transitivity: see Condorcet paradox.
- Pareto: Random choice.
- IID: Borda Rule.
  - $r_i(x, R, S) = |\{y \in S |\ x\ P_i\ y\}|$ # of alts to which i prefers *x*.
  - $r(x, R, S) = \sum\{i \in N |\ r_i(x, R, S)\}$ Borda votes for x.
  - $C_{Borda}(R, S) = \{x \in S |\ r(x, R, S) \geq r(y, R, S)\ \forall\ y \in S.\}$ Win set.

- Example. 1: xyzw. 2: xyzw. 3: zwxy.

- What happens after y is removed?

Borda Violates IID

B	1	2	3	Total
x	3	3	1	7
y	2	2	0	4
z	1	1	3	5
w	0	0	2	2

C(R,S)=x

B	1	2	3	Total
x	2	2	0	4
z	1	1	2	4
w	0	0	1	1

C(R,S/y)={x,z}

Sen's α

Rule: x is chosen in a large set of alternatives. If other alternatives are eliminated, x should still be chosen.

# voters	4	3	3
	z	y	x
	y	x	y
	x	z	z

Using plurality rule, single vote. Last letter wins ties.

C(R, {x, y, z}) = {z}
C(R, {x, y}) = {y}
C(R, {y, z}) = {y}
C(R, {x, z}) = {x}

Sen's β

- Rule: x and y are in the choice set of a small set of alternatives.
- As the set grows, if Y is in the choice set, X must be there as well.

Borda Violates IID

B	1	2	3	Total
x	3	3	1	7
y	2	2	0	4
z	1	1	3	5
w	0	0	2	2

C(R,S)=x

B	1	2	3	Total
x	2	2	0	4
z	1	1	2	4
w	0	0	1	1

C(R,S/y)={x,z}

Next Week

Black (1948)

- M. “When a decision is reached by voting … no part of economic theory … applies.”
- NH. Many points can beat all others by a majority.
- P. One dimension. Single-peaked preferences. N voters, M alternatives. Majority rule. Complete information.
- C. The median voter theorem.

McKelvey 1979

- M. Arrow: ∃ R that yields an intransitive social ordering for any CCR. *With what likelihood?*
- NH. Majority rule generally forces outcome towards "median" alternatives.
- P. N voters, $N > 1$ dim policy space, MMD.
- C. If conditions are right, MMD yields chaos.