Outline for Today

- Problem set conversation.

- Describing a bridge that connects Arrow's Theorem and pre-Arrovian concerns about social choice and welfare to contemporary research on the design and functioning of political institutions.
  - McKelvey (1976)
  - Lupia and McCubbins (2003) – feat. Shepsle, Shepsle/Weingast, Riker

Arrow's General Possibility Theorem

Collective Rationality

- Complete. $\forall x, y \in S$, either $x \, R \, y$, $y \, R \, x$ or both.
- Reflexive. $\forall x, y \in S$, $x \, R \, x$.
- Transitive $\forall x, y, z \in S$, $x \, R \, y$ and $y \, R \, z \Rightarrow x \, R \, z$.

- C. A collectively rational CCR cannot satisfy the following four conditions simultaneously.
  - If you want all but one of these desirable properties to hold for every conceivable preference profile, then you must sacrifice the remaining property.

Arrow's General Possibility Theorem

- **U**nrestricted Domain: The CCR allows us to consider any set of preferences.
- **P**areto: If everyone prefers X to Y, then Y is not chosen when X is available.
- **I**ndependence of Irrelevant Alternatives.
  $\forall x,y \in S$, and all R, R', $x\ R_i\ y \leftrightarrow x\ R'_i\ y \Rightarrow C(S,R)=C(S,R')$
- **D** There is no dictator.
  - There is no $i \in N$, s.t. $\forall x, y \in S$, $x\ P_i\ y \Rightarrow x\ P\ y$.

Range of Applicability

- The result is often misinterpreted.
  - Arrow's claim is stated with respect to *all* preference profiles.
  - There can exist a decision rule that satisfies all five properties for N-1 preference profiles.

- Main Implication: Institutions matter.
- But when?

Plott (1967)

- M. When does MMD yield an equilibrium?
- NH. Often.
- P. M individuals, N variables.
- C. The conditions are very hard to satisfy.
  - The equilibrium be some voter's ideal point.
  - All other ideal points must be diametrically opposed to the equilibrium.

McKelvey 1976

- M. Tullock: Arrow is irrelevant in politics because the cycle set will be a small area in the policy space.
- NH. Majority rule generally forces outcome towards "median" alternatives.
- P. N voters, *N >1* dim policy space, voters have Euclidean utility functions. MMD.
- C. "It is possible for majority rule to wander anywhere in the space of alternatives."
  - When transitivity breaks down, it completely breaks down, engulfing the whole space in a single cycle set.

Hypothesis

- Analysis of a political situation requires a specification of:
  - a list of the relevant decision makers
  - the goals of these decision makers
  - the actions/strategies available to each decision maker
  - a list of feasible outcomes
  - the relationship between actions and outcomes
  - the relationships between outcomes and individual goals
  - individual perception (information)

Components of a Game

- players
- actions
- strategies
- information
- outcomes
- payoffs
- Equilibrium concept

Preference and Utility

- Utility function. u: X $\Rightarrow \Re$. x R y $\Leftrightarrow$ u(x) $\geq$ u(y)

Increasing in restrictiveness: (how much you have to know about the person to render the description).

- binary relation
- Ordinal utility.
- Cardinal utility requires more data.

Definitions

We assume a set $N=\{1,2,\ldots,n\}$ of voters, and assume that the policy space X is Euclidean m space, i.e., $X=R^m$.

For each voter $i \in N$, we assume there is a utility function $U_i: X \rightarrow R$ which is assumed to be a monotone decreasing function of Euclidean distance, i.e., $\forall\ i \in N$, $\exists\ x_i \in R^m$ s.t., $Ui(x)=\Phi_i||x-x_i||$.

Here ||.|| represents the standard Euclidean norm, and Φi is any strictly monotone decreasing function.

Definitions

We denote $x >_i y \Leftrightarrow U_i(x) > U_i(y)$, $x \geq_i y \Leftrightarrow U_i(x) \geq U_i(y)$.

Given the nature of the utility functions it follows that $x >_i y \Leftrightarrow ||x-x_i|| < ||y-x_i||$.

$|B|$ denotes the number of elements in the set of voters $B \subseteq N$. We use the shorthand $| x \geq_i y | = | i \in n | x \geq_i y |$.

Then, we can define a majority preference relation over R^m as follows. For any $x,y \in R^m$ $x \geq y \Leftrightarrow |x \geq_i y| > n/2$.

$x^* \in X$ is a majority Condorcet point iff $x^* > y$, $\forall y \in X$.

Hyperplanes

- A plane divides a space M. It has M-1 dimensions.
- For any $y \in R^m$, $c \in R$, $H_{y,c}=\{x|x' \cdot y = c\}$ is a hyperplane.
- The hyperplane partitions R^m into three sets $H_{y,c}$, $H_{y,c}^+$, and $H_{y,c}^-$ where $H_{y,c}^+=\{x|x' \cdot y > c\}$ and $H_{y,c}^-=\{x|x' \cdot y < c\}$.
- $H_{y,c}$ is a median hyperplane $\Leftrightarrow |H_{y,c}^+| \leq n/2$ and $|H_{y,c}^-| \leq n/2$.
- **M** is the set of median hyperplanes.

Medians

- What is a median? It lies halfway in between.
- $x^* \in X$ is a total median iff $\forall y \in R^m$, $\exists H_{y,c} \in \mathbf{M}$ such that $x^* \in H_y$.
- If U_i as defined, then $x^* \in X$ is a Condorcet point $\Leftrightarrow$ it is a total median.
  - If x^* is a strong total median, then the social order is transitive on X, with $x \geq y \Leftrightarrow ||x-x^*|| \leq ||y-x^*||$.
- Corners of a square example: a total median unique but not strong. Add fifth voter at total median – transitive preference order. Akin to Plott result.

Implication

- Theorem 1: If ∃ a strong total median, then the social order is transitive on X.

- “Given the severity of the restrictions needed to guarantee transitivity, it is of considerable interest to explore the nature of the intransitivities when these symmetry conditions are not met.”

The Extent of Intransitivity

- "When transitivity breaks down, it completely breaks down, engulfing the whole space in a single cycle set."

- Theorem: Assume $m \geq 2$, $n \geq 3$, and all voters have utility functions as defined above. If there is no total median, then for any x, $y \in X$, it is possible to find a sequence of alternatives, $\{\theta_0, \ldots, \theta_N\}$ with $\theta_0 = x$ and $\theta_N = y$, such that $\theta_{i+1} > \theta_i$, for $0 \leq i \leq N-1$.
  - Sketch here.

McKelvey 1976

…if any one voter, say the "Chairman," has complete control over the agenda (in the sense that he can choose, at each stage of the voting, any proposal $\theta_i \in R^n$ to be considered next) that he can construct an agenda which will arrive at any point in space, in particular at his ideal point.

- This type of manipulation is possible regardless of the preferences of the other voters and regardless of whether the "sincere" social ordering is transitive.

Key Assumptions

- The chairman must know a lot about voter preferences to cause the result.
- Voters make fine distinctions without becoming indifferent.
- Voters vote sincerely & do not collude.

Interpretations

- Arrow – Nothing will work?
- McKelvey – Chaos? Anything can happen?

- Both interpretations are overstated.

Lupia and McCubbins (2003)

- M. How do SC results affect politics & law?
- NH. Collective intent & majority will are vacuous.
- P. Standard SCT plus collective choice problems, limited energy and cognitive ability.
- C. Stable relationships between individual preferences and collective choice are likely.

A Debate About the Meaning of Choice

Riker: [P]olitics is the dismal science because we have learned from it that there are no fundamental equilibria to predict. In the absence of such equilibria, we cannot know much about the future at all.

- Disequilibrium "is the characteristic feature of politics."

Shepsle; Shepsle and Weingast

- "institutional structure … has an important independent impact on the existence of equilibrium"

- Q: "Why so much stability?"
- A: "Institutional arrangements do it."

Riker's Response

- Institutions are no more than rules and rules are themselves the product of social decisions. Consequently, the rules are also not in equilibrium."
- The claim "Institutional arrangements do it" begs, rather than answers, the question "Why so much stability?"

Our Response

- Seek N/S conditions for stability.
- The roots of stability are found in:
  - the requirements for collective action
  - systematic and universal limits on human energy, cognition, and communicative ability.
- Stability is likely for many important collective choices.

What is Stability?

- A collective choice *w* is stable if and only if, holding *S*, R, and the CCR constant, w has an empty *win set*.
- Example 1: Stability

1	2	3
y	y	z
z	z	y
x	x	x

- Example 2: Instability

1	2	3
y	z	x
z	x	y
x	y	z

The Problem

- Two assumptions “stack the deck” in favor of finding instability in SCT’s.

- There is no scarcity.
  - Scarcity makes holding another vote or implementing a new policy costly.

- There is no complexity.
  - Complexity makes people uncertain about the consequences of change.

Amendment 1

- Implementation is costly.
  - IMP_i^x is the cost to individual i of implementing alternative x as the collective choice, given q.
  - $P(IMP_i^x)$ is the preference profile after introducing these costs.

Amendment 2

- Decision making is costly, like using a machine.
- Machines require energy.
  - C_i^R is the cost to individual i of a single use of collective choice rule R.
  - $P(C_i^R)$ is the preference profile after introducing these costs.
- Maintaining the status quo, q, does not require use of the CCR.

Amendment 3

- Information affects perception.
- Information affects preferences.
- Example:

P(I-old)			P(I-new)		
1	2	3	1	2	3
x	y	z	y	y	z
y	z	x	z	z	y
z	x	y	x	x	x

- Information asymmetries make persuasion difficult.

The Stability Test

1. $\exists$ alternatives to q?
2. Given information I, is any group aware of one?
3. Given I, is any group aware of a CCR that yields an alternative from Question 2?
4. Given I and costs $M_i(x,q)$ and C_i, can it expect to benefit from the alternative?

If the answer to any of these questions is “no”, then instability is impossible.

Conclusion

- The relationship between individual preferences and collective choice is affected by information, the costs of implementation, and the costs of decision making.
- Why do we observe so much stability?
  - Collective action is not trivial.
  - Complexity and scarcity are ubiquitous.
- A collective choice can reflect “majority will.”

Implications

- Instability results identify a bounded set of universal claims that are not logically valid.
  - These results do not rule out all conditional claims about the relationship between preference and choice.

- SCT does not clarify institutional dynamics in the presence of potentially adaptive actors with resource and information limits.
  - A different formal modeling approach – non-cooperative game theory -- is needed.

Key Concepts for next PS

- Dominance
- Best Response
- Iterated Dominance
- Rationalizable Strategies