Outline

- Problem Sets
- Nash Equilibrium in Extensive Form Games.
- Backwards Induction
- Subgame Perfection

Problem Set

- Generally good
- Waiting for Cournot
  - I did not grade this problem.
  - Basics of how to do it.

Extensive Form Games

- Player moves can be treated as sequential or simultaneous.

- First Models:
  - Complete information – games in which all aspects of the structure of the game –including player payoff functions -- is common knowledge.

  - Perfect information – at each move in the game the player with the move knows the full history of the play of the game thus far.

The structure of a simple game of complete and perfect information.

1. Player 1 chooses an action a_1 from the feasible set A_1.
2. Player 2 observes a_1 and then chooses a_2 from the feasible set A_2.
3. Payoffs are $u_1(a_1, a_2)$ and $u_2(a_1, a_2)$.
   1. Moves occur in sequence, all previous moves are observed, player payoffs from each move combination are common knowledge.
   2. We solve such games by backwards induction.

The central issue is credibility.

Example 1 Here

3 legislators

Choices: Yes, No

Outcomes: Pass, Not.

Backwards Induction

- When player 2 gets the move at the second stage of the game, he or she faces the following problem, given the previously chosen action a_1, $max_{a2 \in A2}\ u_2(a_1, a_2)$.
- Assume, for the moment, that for each $a_1 \in A_1$, player 2's optimization problem has a unique solution denoted by $R_2(a_1)$.
- Since player 1 can solve player 2's problem as well as 2 can, player 1 should anticipate player 2's reaction to each action a_1 that 1 might take.
- So 1's problem at the first stage amounts to $max_{a1 \in A1}\ u_1(a_1, R_2(a_1))$.
- $(a^*_1, R_2(a^*_1))$ is the backward induction outcome of this game.

- Implies sophisticated rather than sincere behavior.
- Implies that the sequence of action can affect equilibrium strategies.

Example 2

- Morrow, p. 124.
- Even though backwards induction predicts that the game will end at a particular stage, an important part of the argument concerns what would happen if the game did not end in the first stage.

Requirements for BI

- o Thinking through strategic behavior requires us to assume that decision makes are interested in, and capable of, counterfactual reasoning.
  - o In some cases, the amount of counterfactual reasoning required is quite substantial.

- o If people reason “as if” they undertake such calculations, then the theory’s validity is not imperiled.

- *When can we assume that people are, or act as if they are, capable of thinking through counterfactuals?*

Example 3

- Gibbons, p. 60.
- Even though backwards induction predicts that the game will end at a particular stage, an important part of the argument concerns what would happen if the game did not end in the first stage.

Subgame Perfect NE

A NE is subgame perfect if players' strategies constitute a Nash Equilibrium in every subgame.

- Player 1 chooses action a_1 from feasible set $A_{1.}$
- Player 2 observes a_1 and then chooses action a_2 from feasible set A_2.
- Player 3 observes a_1 and a_2 and then chooses action a_3 from feasible set A_3.
- Payoffs are $u_i(a_1,a_2,a_3)$ for i=1,….,3.
- $(a_1, a_2{}^*(a_1), a_3{}^*(a_1, a_2))$ is the subgame-perfect outcome of this two-stage game.

Backwards Induction & Subgame Perfection

- The BI outcome involves only credible threats: player 1 anticipates that player 2 will respond optimally to any action a_1 that 1 might choose, by playing $R_2(a_1)$; player 1 gives no credence to threats by player 2 to respond in ways that will not be in 2's self-interest when the second stage arrives.
- A NE is subgame perfect if it does not involve a noncredible threat.
- A dynamic game may have many NE, but the only subgame-perfect NE is the one associated with the backwards-induction outcome.

Repeated Games

Consider a two-stage Prisoners' Dilemma.

	Defect	**Cooperate**
Defect	1, 1	5, 0
Cooperate	0, 5	4, 4

In the second stage, the equilibrium will be defect, defect. Therefore, the first period of the two stage game is equivalent to the following one-stage game.

	Defect	**Cooperate**
Defect	2, 2	6, 1
Cooperate	1, 6	5, 5

A general result.

- Definition: Given a stage game G, let *G(T)* denote the *finitely repeated* game in which G is played T times, with the outcomes of all preceding plays observed before the next play begins. The payoff for *G(T)* are simply the sum of the payoffs from the T stage games.

- If the stage game G has *a unique NE* then, for any finite T, the repeated game *G(T)* has a unique subgame perfect outcome: the NE of G is played in every stage.

Cooperation from Repetition?

- Proposition: If $G=\{A_1,\ldots A_n;u_1,\ldots u_n\}$ is a static game of complete information with multiple NE then there may be subgame perfect outcomes of the repeated game $G(T)$ in which, for any $t<T$, the outcome in stage T is not a Nash equilibrium of G.

The prisoners' dilemma with one action added for each player.

	Defect	**Cooperate**	**Right**
Defect	1, 1	5, 0	0, 0
Cooperate	0, 5	4, 4	0, 0
Bottom	0, 0	0, 0	3, 3

	Defect	Cooperate	Right
Defect	1, 1	5, 0	0, 0
Cooperate	0, 5	4, 4	0, 0
Bottom	0, 0	0, 0	3, 3

- o Suppose that the players anticipate that (Bottom, Right) will be the second stage outcome if the first stage outcome is (Cooperate, Cooperate), but that (Defect, Left) will be the second-stage outcome if any of the eight other first stage outcomes occurs.

- o The players, first stage interaction then amounts to the following one-shot game:

	Defect	Cooperate	Right
Defect	2, 2	6, 1	1, 1
Cooperate	1, 6	7, 7	1, 1
Bottom	1, 1	1, 1	4, 4

Example 6

- Morrow, p. 131. Figure 5.16.

Example 7

- S-PNE on a Voting Tree. (Agenda: abcde)
  - TYPE 1 D A B C E
  - TYPE 2 A B C E D
  - TYPE 3 C B E D A
  - TYPE E e D A C B

The Folk Theorem

- Let G be a finite, static game of complete information. Let $(e_1, \ldots e_n)$ denote the payoffs from a NE of G, and let $(x_1, \ldots x_n)$ denote any other feasible payoffs from G. If $x_i > e_i$ for every player i and if δ is sufficiently close to one, then there exists a subgame-perfect NE of the infinitely repeated game $G(\infty, \delta)$ that achieves $(x_1, \ldots x_n)$ as the average payoff.
  - Insights from one-shot games do not automatically transfer to repeated interactions.
  - Repeated games require special assumptions about time.
  - Credible threats or promises about future behavior can influence current behavior.

Rubenstein (1982)

Premises

- The following sequence repeats until an offer is accepted.
  - Player 1 proposes a split.
  - Player 2 accepts immediately or, after delay, makes a counteroffer.
  - Player 1 accepts immediately or, after delay, makes a counteroffer….
- Players prefer money now. Discount rate: δ - present value of a next period $.

Results

- The unique subgame perfect equilibrium is for Player 1 to take $100/(1+\delta)$ and leave $100\delta/(1+\delta)$ for Player 2, and for Player 2 to accept this offer and spurn any offer that is worse.
- If $\delta=1$, player 1 takes & leaves 50.
- If $\delta=.5$, player 1 takes 67, leaves 33.
- If $\delta=0$, player 1 takes 100.

- Higher discount rates are sufficient to imply lower walk-away values in the current period.

Rubenstein Implications

- The amount of the offer reflects the net present value to player 2 of playing the game.

- The less 2 likes waiting for payoffs – the higher their discount rates – the more that player 2 will sacrifice for a payoff now.
  - At δ=1, s=1/2. No one fears the future. No one has an advantage.
  - At δ=.5, s=2/3.
  - At δ=0, s=1. Also true if a one-shot game where if player 2 rejects player 1's offer, all payoffs are zero.

- In the comparative statics result, no variable other than discount rates shift, yet the results change dramatically.