A Canonical Model of Choice with Initial Endowments*

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Abstract

We use the revealed preference method to derive a model of individual decision making when one’s endowment provides a reference point that may influence her choices. This model generalizes the classical rational choice model. While the latter views choice as a consequence of “utility maximization,” the model proposed here views choice as arising from “mentally constrained utility maximization,” where the constraints are induced by one’s initial endowment. In particular, this model allows for status quo bias. A range of economic applications are presented to identify the predictive and explanatory strength of the model.

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1 Introduction

A large number of empirical studies, both within economics and psychology, have established that decision makers settle various types of choice problems in a reference-dependent manner. In particular, there is now a widespread agreement that individuals behave in the same choice situation markedly differently, depending on what sort of a “reference” they are given in the form of an initial entitlement, endowment and/or default option. Indeed, in a myriad of experimental and field studies, the relative “value” of an alternative is found to be enhanced for agents who possess that alternative as current endowment. This effect is commonly referred to as the status quo bias phenomenon – a phrase coined by Samuelson and Zeckhauser (1988). It is also well documented that the status quo position of a decision maker affects the behavior of the agent even if the agent chooses to move away from her status quo (as in reason-based decision making, or what is called the attraction effect).\(^1\)

Despite these developments, there does not exist a canonical model of choice that applies to problems with initial endowments that may serve as reference points. Such problems are typically treated by means of the Tversky-Kahneman model of loss averse preferences. However, the scope of such models is limited, because they properly apply only to consumption choice problems, but not, for instance, job search and/or voting problems (in which relevant dimensions of the problem are not exogenously given). In addition, as we shall see, their predictions may at times be at odds with the status quo phenomenon.

While, presently, there seems to be a consensus that the canonical model of rational choice leaves much to be desired from the descriptive point of view, it is undeniable that this model has served economics well due to its universal nature. After all, this model applies to any type of a choice situation (with or without an initial endowment), and hence provides a duly unifying perspective. It thus seems desirable to extend the rational choice model in a way to incorporate the experimental findings on reference-dependent choice, while preserving its canonical, thereby unifying, nature.

In this paper, we develop an individual choice model that applies to any choice problem with a feasible initial endowment as well as to those choice problems without status quo options. This model is derived by the revealed preference approach – it is thus characterized by means of behavioral, and hence directly testable, postulates. It is canonical in the sense that it preserves the universal nature of the classical model of rational choice. In addition, it allows an individual’s decisions to be affected by her initial endowment. In particular, this model leaves room for the widely documented

\(^1\)The empirical/experimental literature on reference-dependent individual decision making is too large to be cited here. We refer the reader to Camerer (1995) and Sugden (1999) for insightful surveys on this matter. For discussions of the attraction effect and related phenomena, see Simonson (1989), Shafir, Simonson and Tversky (1993), Sen (1998), Malaviya and Sivakumar (2002), Ok, Ortoleva and Riella (2011), and references cited therein.
status quo bias phenomenon, among other types of endowment-based reference effects.

Loosely speaking, the classical model of choice maintains that an agent chooses from a feasible set $S$ by maximizing a utility function on $S$. By contrast, the model developed here says that an agent, whose initial endowment is $x$, chooses from $S$ by maximizing a utility function on $S$ subject to a (psychological) constraint induced by $x$. (In the absence of an initial endowment, our model reduces to the standard one.) Put more explicitly, where $U$ is her utility function (deduced from binary choice situations), the rational choice model maintains that the agent deems an alternative $y$ in $S$ as “choosable” whenever

$$U(y) \geq U(\omega) \quad \text{for every } \omega \in S,$$

ignoring the potential presence of an initial endowment $x$. In turn, our choice model maintains this only in the absence of an initial endowment. When there is a status quo option in the problem, say $x$, this model says instead that an alternative $y$ in $S$ is “choosable” when

“$y$ is itself appealing from the perspective of $x$,”

and

“$U(y) \geq U(\omega) \quad \text{for every } \omega \in S \text{ that is appealing from the perspective of } x.$”

Just like $U$ is deduced in revealed preference theory from the choice behavior of the agent, in our analysis we derive $U$ and the psychological constraint relation of “being more appealing from the perspective of $x$.” Put differently, while the classical rational choice model captures “rationality in decision-making” by the notion of utility maximization, the canonical model we develop here captures “bounded rationality in decision-making” by the notion of (mentally) constrained utility maximization. (Figure 1 illustrates the contrast between the classical model and the one proposed here.)

The structure of the present paper is as follows. In Section 2, we introduce three simple axioms and discuss their behavioral basis in some detail. Our first axiom derives from the classical weak axiom of revealed preference (WARP). Put precisely, this axiom imposes WARP across all problems with the same initial endowment.\(^2\) Our second axiom is simply a translation of the experimental findings on status quo bias to the language of choice theory. We call this property the weak status quo bias axiom (WSQB), for it says that the choosability of an alternative in a pairwise choice situation cannot possibly deteriorate, and is possibly enhanced, when that alternative is itself the status quo option. Finally, our third axiom, which we call the status quo irrelevance (SQI), says that if a status quo option is the least desirable option in a

\(^2\)Thus, an agent must act rationally within such problems, but across choice problems with different endowments, her behavior is not restrained.
feasible set despite being a status quo, it does not affect one’s final choice in that set. Our main result, Theorem 1, characterizes a canonical choice model by means of these axioms (and an additional continuity requirement). Several special cases of this model are also discussed in Section 2.

The model derived in Theorem 1 may be thought of as a bit too general, for it does not say anything about the structure of the psychological constraint relation we mentioned above. In particular, it imposes no discipline on the structure of this constraint across different status quo options. In Section 3, we show that replacing WSQB with a stronger status quo bias axiom would yield further structure in this regard. In particular, we introduce in this section what we call here the strong status quo bias axiom (SSQB), and show in Theorem 2 that with this axiom the mental constraint relation of the model acts transitively across endowments. As a consequence, we characterize this relation in Theorem 3 as a dominance relation with respect to an (endogenously found) collection of criteria, thereby giving the model we study here the flavor of a multi-criteria choice model.

In Section 4 we present several economic applications to illustrate the potential use of the canonical choice model(s) proposed in this paper. The first of these comes from behavioral economics: we show in Section 4.1 that the endowment effect – the phenomenon that the minimum compensation demanded by an agent for a good that she owns is more than the maximum prize she is willing to pay for the same good – may not exist even if the agent exhibits status quo bias. We thus find here that the common practice of viewing the “status quo bias” and the “endowment effect” as one and the same phenomenon is not really justified. In Sections 4.2 and 4.3, we consider two financial applications, and show that particular specifications of the choice model of Theorem 2 may be used to “explain” the equity-premium puzzle, and the tendency of investors to hold on to stocks that have lost value too long while being
prone to selling stocks that have gained value. Similarly, in Section 4.4, we show that certain specifications of the choice model of Theorem 1 matches the empirical finding that home-owners who sell their houses below their purchase prices tend to set their prices too high, and as a result, such houses remain in the market more than others. Finally, in Section 4.5, we use the model of Theorem 1 to show that the classical Law of Compensated Demand extends to the case of consumers with status quo bias. However, it is found (only on the basis of our behavioral postulates, and without any particular specification of utility functions, etc.) that the involved substitution effect is bound to be less in magnitude in the presence of status quo bias. This suggests in the context of labor markets that the backward-bending labor supply phenomenon is likely to be more pronounced in the presence of status quo biased workers.

As we have noted above, a popular way of capturing the status quo bias phenomenon in the literature is through the loss aversion model of Tversky and Kahneman (1991) and its variants. Consequently, if only to clarify the nature of the present contribution, we provide in Section 5 a brief review of that model, and contrast its structure to that of the choice model developed in the body of the text. In particular, regarding the applications we considered in Section 4, we find that the qualitative predictions of these two models are different in the case of the endowment effect and the law of (compensated) demand. Finally, Section 6 provides some concluding comments, and the Appendix presents the proofs of our main results.

2 Choice with Initial Endowments

2.1 The Basic Framework

In what follows, we designate an arbitrary compact metric space $X$ to act as the universal set of all mutually exclusive alternatives. The set $X$ is thus viewed as the grand alternative space, and is kept fixed throughout the exposition. The members of $X$ are denoted as $x, y, z, \ldots$. For reasons that will become clear shortly, we designate the symbol $\Diamond$ to denote an object that does not belong to $X$. We shall use the symbol $\sigma$ to denote a generic member of $X \cup \{\Diamond\}$.

We let $\Omega_X$ denote the set of all nonempty closed subsets of $X$. By a choice problem, we mean a list $(S, \sigma)$ where $S \in \Omega_X$ and either $\sigma \in S$ or $\sigma = \Diamond$. The set of all choice problems is denoted by $\mathcal{C}(X)$.

Given any $x \in X$ and $S \in \Omega_X$ with $x \in S$, the choice problem $(S, x)$ is called a choice problem with a status quo. (The set of all such choice problems is denoted as $\mathcal{C}_{sq}(X)$.) The interpretation is that the individual is confronted with the problem of choosing an alternative from the feasible set $S$ while either she is currently endowed with the alternative $x$ or her default option is $x$. Viewed this way, choosing an alternative $y \in S \setminus \{x\}$ means that the subject individual gives up her status quo
and switches to $y$.\(^3\)

On the other hand, many real-life choice situations do not have a natural status quo alternative. Within the formalism of this paper, the choice problems of the form $(S, \Diamond)$ model such situations. Formally, then, we define a **choice problem without a status quo** as the list $(S, \Diamond)$ for any set $S$ in $\Omega_X$. (While the use of the symbol $\Diamond$ is clearly redundant here, it will prove convenient in the foregoing analysis.)

By a **choice correspondence** on $\mathcal{C}(X)$ in the present setup, we mean a function $c : \mathcal{C}(X) \to \Omega_X$ such that

$$c(S, \sigma) \subseteq S \quad \text{for every } (S, \sigma) \in \mathcal{C}(X).$$

### 2.2 Axioms for Choice with Initial Endowments

We begin our axiomatic development by introducing a rationality property familiar from the classical theory of revealed preference. As in that theory, this property warrants that some type of "utility maximization" does take place in the decision-making procedure of an individual.

**Weak Axiom of Revealed Preference (WARP).** For any $(S, \sigma)$ and $(T, \sigma)$ in $\mathcal{C}(X)$,

$$c(S, \sigma) \cap T = c(T, \sigma)$$

provided that $T \subseteq S$ and $c(S, \sigma) \cap T \neq \emptyset$.

This property conditions the behavior of a decision maker across two choice problems whose endowment structures are identical. In this sense, it is merely a reflection of the classical weak axiom of revealed preference to the framework of individual choice in the (potential) presence of an exogenously given reference alternative.\(^5\)

We next introduce a way of modeling the status quo bias phenomenon by means of a behavioral postulate.

**Weak Axiom of Status Quo Bias (WSQB).** For any $x, y \in X$,

$$y \in c(\{x, y\}, x) \quad \text{implies} \quad y \in c(\{x, y\}, \Diamond)$$

and

$$y \in c(\{x, y\}, \Diamond) \quad \text{implies} \quad y \in c(\{x, y\}, y).$$

\(^3\)In the language of Rubinstein and Salant (2008), any $(S, x)$ in $\mathcal{C}_{\text{eq}}(X)$ is a choice problem with a frame, where initial endowment $x$ provides the "frame" for the problem. We assume throughout this paper that this frame is observable.

\(^4\)Notice that a choice correspondence on $\mathcal{C}(X)$ must be nonempty-valued by definition.

\(^5\)When $\sigma = \Diamond$, our formulation of WARP reduces to the classical formulation of this property.
Consider an agent who is vulnerable to the status quo phenomenon. Suppose also that this agent moves away from her status quo option $x$ in favor of another alternative $y$. It seems reasonable that a necessary condition for this to happen is that $y$ is at least as good as $x$ in a reference-free sense. After all, if $y$ was less appealing than $x$ absent referential considerations, then, because our agent is in general (weakly) reluctant to move away from her status quo options, she would have never chosen $y$ over $x$ when endowed with $x$ initially.

The interpretation of the second part of WSQB is similar. The idea is that if the decision maker reveals the superiority of $y$ over $x$ in a reference-free setting, then, when the endowment of the agent is $y$ itself, the position of this alternative can only be stronger relative to $x$, and thus it would surely be choosable over $x$ in that case. This property, which is referred to as "conservatism" by Munro and Sugden (2003), preconditions a choice correspondence to exhibit a bias towards the status quo alternatives in a straightforward manner.

WSQB property seems quite appealing for a rational choice theory that aims at modeling the status quo bias phenomenon. Indeed, versions of this property is recently adopted in some other studies.\footnote{See, for instance, Sugden (2003), Munro and Sugden (2003), Masatlioglu and Ok (2005), Sagi (2006), Apesteguia and Ballester (2009), Ortoleva (2010) and Riella and Teper (2010).} Furthermore, the experimental literature on individual choice provides direct verifications of WSQB.\footnote{For instance, Knetsch (1989) has conducted two comparable treatments of a choice experiment involving two goods. In one treatment, participants were simply offered the same choice without initial entitlement. Here, almost half of the participants selected the first good. In the other treatment, the first good was given as an initial entitlement and subjects had the opportunity to exchange their endowment for the other good. Now, the vast majority of students kept their endowments (around 90 percent versus initial 50 percent). (For a critical experimental evaluation, see Plott and Zeiler (2007).) There is also plenty of (indirect) field evidence that support the (weak) status quo bias hypothesis. Particularly striking in this regard are the bias found toward default 401(k) plans, various other savings plans, and asset allocation contributions; see, for instance, Madrian and Shea (2001), Choi, et al. (2004) and Beshears, et al. (2008).}

On the other hand, while the status quo bias phenomenon seems to be a fact of life, it would be unreasonable to presume that its effects would be felt regardless of the nature of one’s status quo position. In particular, when one’s initial endowment is so bad that it is really irrelevant for one’s one final choices, it is likely that it would not exert much influence in how one makes her choices. This leads us to our next postulate which identifies those situations in which $c$ disregards the presence of a status quo alternative.

### Status Quo Irrelevance (SQI)

For any $(S, x) \in C_{sq}(X)$ with $|S| > 2$, suppose that $\{x\} \neq c(T, x)$ for every non-singleton subset $T$ of $S$ with $x \in T$. Then, $c(S, x) = c(S, \Diamond)$.\footnote{The conceptual standing of the SQI axiom remains the same in the case of choice problems $(S, x)$ with $|S| = 2$. However, this axiom implies WSQB in that case, so its full power becomes redundant. We impose the requirement of $|S| > 2$ in the statement of SQI only to ensure its independence from...}
Consider a feasible set $S$ of alternatives with at least three elements and an alternative $x$ in $S$ that is unambiguously “undesirable” in $S$ in the sense that all other alternatives in $S$ are deemed at least as good as $x$, even when $x$ acts as the status quo option. Insofar as the desirability of a status quo option is the only arbiter of its reference effects, it seems reasonable to expect the decision maker to view the presence of $x$ as a status quo as irrelevant for her final choice. SQI thus says that, in such a situation, she would settle her problem by comparing the alternatives in $S$ in a reference-free manner (that is, as if there is no status quo in the problem).

Let us illustrate the content of SQI by means of a purposely extreme example. Consider the following alternatives: $z := “accepting the job offer of a Boston company,” y := “accepting the job offer of a San Diego company,” and $x := “declining all offers and becoming homeless.” In this case, it seems indeed irrelevant that $x$ is given as a default option to the agent. The question is simply choosing between which of the two job offers to accept. Put differently, it seems quite reasonable that $c(\{x, y, z\}, x) = c(\{x, y, z\}, \diamond)$ in this instance, and this is exactly what is posited by SQI.

On the critical side, however, we should note this example overstates the appeal of the SQI property. The only way the choice data at hand can qualify an alternative $x$ as being “very bad” in a given feasible set is by verifying that this alternative would never be chosen in any feasible subset even if it is the status quo. But, in reality, this may be an inadequate method of “irrelevance” of a status quo option. To wit, consider the example above, and set $w := “being employed in a New York company.” In this case, if $w$ is the status quo option of the agent, but for whatever reason, one that she would surely opt out in favor of either (inclusive) of her job offers, it is less clear that we would have $c(\{x, y, w\}, w) = c(\{x, y, w\}, \diamond)$. After all, because it is in a big city in the East Coast of US, her current job in New York may influence the relative desirability of $x$ and $y$, and acting as some sort of a reference option, it may lead the decision maker to choose $x$ over $y$ (or otherwise) even though she would act differently had she not have $w$ as an option.

To summarize, let us note that one can think of three basic effects a person’s initial endowment can have on her choice behavior. First, due to status quo bias, it this endowment may create an attraction toward itself. Second, it may force the agent search for better alternatives than itself, thereby altering the ranking of alternatives, albeit only through how they compare against the status quo. Third, it may act as a reference point in a more subtle way, and change the relative ranking of the options directly. In our axiomatic development, WSQB is meant to capture the first effect, and SQI the second. But, we note that SQI does this at the expense of the third effect. In the next section we will show that this property still leaves some room for utilizing one’s initial endowment as a “reference” option, but allows one retain the spirit of the standard paradigm of utility maximization, thereby retaining a good amount of predictive power. However, depending on the context, the limits

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WSQB.
imposed by SQI may well be descriptively untenable, and it will surely be of interest to investigate ways of relaxing this property in future work.\footnote{We are not aware of any empirical evidence against the SQI property. There is some indirect evidence for it in the context of “attraction effect” experiments, where it is found that a decoy option that is undesirable relative to all other alternatives in a feasible set does not have referential effects; cf. Wedell (1991) and Wedell and Pettibone (1996). (This evidence is indirect because the decoy option is not assigned as the initial endowment in these experiments.) On the other hand, Masatlioglu and Uler (2008) provide some direct evidence for this property by showing that, in a three different treatments, the percentage of the subjects that satisfy SQI vary between 75% and 85%.

The final property that we shall use in this paper is a straightforward reflection of the standard continuity property for choice correspondences. This condition ensures that the choices of an individual for “similar” choice problems are “similar.”

**Continuity (C).** The map $c(\cdot, \emptyset)$ is upper hemicontinuous on $\Omega_X$. Moreover, for each $(S, x)$ in $\mathcal{C}_{sq}(X)$, and convergent sequence $(y_m)$ in $X$ with $y_m \in c(S \cup \{y_m\}, x)$ for each $m$, we have $\lim y_m \in c(S \cup \{\lim y_m\}, x)$.

Needless to say, this property is automatically satisfied by a choice correspondence $c$ when the set $X$ of alternatives is finite. Thus, when $X$ is finite, any one of the axioms introduced in this section has a behavioral basis, and hence, it is falsifiable by direct experimentation.

### 2.3 A Canonical Model of Choice

Due to the richness of the present choice domain – we owe this to initial endowments being a part of the description of the choice problems – the class of choice correspondences that satisfy WARP is quite large, and in fact, include choice models that can be viewed as *boundedly rational* at best. In fact, even the collection of all choice correspondences on $\mathcal{C}(X)$ that satisfy WARP and the additional three properties we considered in the previous subsection may correspond to somewhat unorthodox choice behavior. Fortunately, there is an easy way of thinking about all such correspondences in a unified manner. In particular, the following theorem shows that any such choice correspondence can in fact be rationalized by means of a fairly simple two-stage choice “procedure.”

**Theorem 1.** Let $X$ be a compact metric space and $c$ a choice correspondence on $\mathcal{C}(X)$. Then, $c$ satisfies WARP, WSQB, SQI and C if, and only if, there exist a continuous (utility) function $U : X \to \mathbb{R}$ and a closed-valued self-correspondence $Q$ on $X$ such that

\begin{align}
  c(S, \emptyset) &= \arg \max U(S), \\
  c(S, x) &= \arg \max U(S \cap Q(x))
\end{align}

\footnote{We are not aware of any empirical evidence against the SQI property. There is some indirect evidence for it in the context of “attraction effect” experiments, where it is found that a decoy option that is undesirable relative to all other alternatives in a feasible set does not have referential effects; cf. Wedell (1991) and Wedell and Pettibone (1996). (This evidence is indirect because the decoy option is not assigned as the initial endowment in these experiments.) On the other hand, Masatlioglu and Uler (2008) provide some direct evidence for this property by showing that, in a three different treatments, the percentage of the subjects that satisfy SQI vary between 75% and 85%.

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for every \((S, x) \in C_{sq}(X)\).

This result identifies a canonical choice model for choice problems with exogenously given endowments. To understand its basic nature, let \(c\) be a choice correspondence on \(C(X)\), \(U\) a continuous real function on \(X\), and \(Q\) a closed-valued self-correspondence on \(X\), and suppose that (1) and (2) hold for any \((S, \sigma) \in C(X)\). When dealing with a choice problem without an initial endowment, an agent whose choice behavior is modeled through \(c\) makes her decisions by maximizing the (ordinal) utility function \(U\). That is, in this case, her final choice is realized by solving the problem:

\[
\text{Maximize } U(\omega) \text{ subject to } \omega \in S.
\]

This is, of course, in complete accordance with the weak axiom of revealed preference.

In turn, this agent deals with a choice problem with an initial endowment, say, \((S, x)\), by means of a two-stage procedure. In the first stage, she uses a mental “constraint set” \(Q(x)\) and eliminates all feasible alternatives that do not belong to this constraint set. That is, in this stage, the agent identifies the set \(S \cap Q(x)\). (This set is nonempty, because (2) and the fact that \(c(\{x\}, x) = \{x\}\) imply \(x \in Q(x)\), for every \(x\) in \(X\).)

Intuitively, this set consists of all feasible alternatives that are superior to the initial entitlement of the decision maker in an unambiguous sense. Put differently, if \(y \in Q(x)\), then even her status quo bias would not stop this agent to “move” from \(x\) to \(y\). If \(y \in S \cap Q(x)\), then \(y\) not only satisfies the physical constraint of the problem (i.e. \(y \in S\)) but it also satisfies the psychological constraint entailed by the status quo position of the agent (i.e. \(y \in Q(x)\))\(^{10}\).

Once \(S \cap Q(x)\) is determined, the agent moves to the second stage of her choice procedure. At this stage, she puts her “rational” hat back on, and among the feasible alternatives that meet her psychological constraint, chooses those that maximize her utility function. If \(x\) is the only element in both \(Q(x)\), and \(S\), this stage leads her to stay with her endowment \(x\) trivially, which is a realization of her status quo bias. If there are other alternatives in \(S \cap Q(x)\) as well, then the decision maker may or may not stay with her status quo at the end of the day. Her final choice is realized by solving the problem:

\[
\text{Maximize } U(\omega) \text{ subject to } \omega \in S \cap Q(x).
\]

Notice that we can impose the property \(U(y) \geq U(x)\) for every \(y \in Q(x)\) and \(x \in X\) in this model (and in the statement of Theorem 1) without loss of generality.

\(^{10}\)Normatively speaking, one may think of the elimination of those alternatives in \(S\setminus Q(x)\) (other than \(x\)) from consideration as a “simplification” the agent uses to settle her possibly complex choice problem (which is similar to the approach used by Manzini and Mariotti (2007)).
The “bite” of the second stage of the model arises from the fact that the converse of this need not be true, that is, there may be alternatives \( y \) outside \( Q(x) \) that may have strictly higher utility value than \( x \). These alternatives are dropped from consideration when the status quo choice of the agent is \( x \) – this is how our canonical model accommodates the status quo bias phenomenon. (See Figure 1)

In summary, the choice model given by Theorem 1 combines the elements of rationality, the phenomena of status quo bias and reference dependence. An agent whose choice behavior abides by this model is indistinguishable from a standard utility maximizer in the context of a choice problem without a status quo option. Moreover, even in a choice problem with a given initial entitlement \( x \), among the alternatives that pass the tests imposed by the presence of \( x \), the final choice is determined upon maximizing a reference-independent utility function. Thus, while the standard choice model is a “utility maximization model,” the canonical choice model we derived in Theorem 1 is a “constrained utility maximization model,” where the constraint is induced by one’s default option.

In passing, we note that the choice model of Theorem 1 provides a natural way of tracking the “magnitude” of one’s status quo bias, which may be useful for performing certain types of comparative statics exercises. Intuitively, the larger the mental constraint set one has, the less status quo bias she exhibits. More precisely, if \( c_1 \) and \( c_2 \) are two choice correspondences on \( C(X) \) that are represented by \((U, Q_1)\) and \((U, Q_2)\), respectively, as in Theorem 1, then we would say that \( c_1 \) is more status quo biased than \( c_2 \) if

\[
Q_1(x) \cap \{ \omega \in X : U(\omega) \geq U(x) \} \subseteq Q_2(x) \cap \{ \omega \in X : U(\omega) \geq U(x) \}
\]

for every \( x \in X \). We will make use of this comparative notion in some of the applications presented below.

**Remark.** The characterization of our comparative notion in terms of choice data as follows: Let \( c_1 \) and \( c_2 \) satisfy WARP, WSQB, SQI and C. Then, \( c_1 \) is more status quo biased than \( c_2 \), if and only if, i) \( c_1(S, \emptyset) = c_2(S, \emptyset) \) for all \( S \), and ii) \( y \in c_1(\{x, y\}, x) \) implies \( y \in c_2(\{x, y\}, x) \) for all \( x \).

**Remark.** The uniqueness structure of the pair \((U, Q)\) found in Theorem 1 is as follows: Let \( U \) and \( V \) be two continuous real functions on \( X \), and \( Q \) and \( P \) two closed-valued self-correspondences on \( X \) such that (1) and (2) hold for every \((S, x) \in C_{sq}(X)\) and for each \((U, Q)\) and \((V, P)\). Then, and only then, there is a strictly increasing map \( f : U(X) \to \mathbb{R} \) such that \( V = f \circ U \) and \( Q(x) \cap x^\uparrow = P(x) \cap x^\uparrow \), where \( x^\uparrow \) is the set of all \( y \in X \) with \( y \in c(\{x, y\}, \emptyset) \), for every \( x \in X \).

**Remark.** The four axioms used in Theorem 1 are logically independent; we provide a proof for this in the Appendix.
2.4 Examples

This section presents some examples that illustrate the sort of behavior that is allowed by the canonical choice model outlined above.

Example 1. (Choice without Status Quo Bias) A decision maker who is not vulnerable to status quo bias and/or reference effects that may be induced by her initial holdings, is captured by the choice model of Theorem 1. Indeed, by setting $Q(x) := X$ for every $x \in X$, the model becomes $c(S, \sigma) = \arg \max \{U(\omega) : \omega \in S\}$ for every $(S, \sigma) \in \mathcal{C}(X)$. In the language introduced above, any choice correspondence that is represented as in Theorem 1 with the utility function $U$ is more status quo biased than this $c$. □

Example 2. (Extreme Status Quo Bias) Consider a decision maker whose choice behavior is vulnerable to status quo bias at the highest level. Such an agent is captured by the model derived in Theorem 1 by setting $Q(x) := \{x\}$ for every $x \in X$. With this specification, the canonical model entails $c(S, x) = \{x\}$ for every $(S, x) \in \mathcal{C}_q(X)$. An agent whose choice behavior is modeled by this $c$ never “moves” away from her initial endowment, no matter how good her other options are. In the language introduced above, $c$ is more status quo biased than any choice correspondence that is represented as in Theorem 1, regardless of the involved utility functions. □

Example 3. (No-Cycles) The choice model of Theorem 1 does not allow behavior that exhibits cycles. For instance, for any distinct alternatives $x, y$ and $z$, the following situation is incompatible with this model: $\{y\} = c(\{x, y, z\}, z)$, $z \in c(\{y, z\}, y)$ and $x \in c(\{x, z\}, z)$. For, by the representation of $c$ derived in Theorem 1, these statements would entail $U(y) > U(x)$, $U(z) \geq U(y)$ and $U(x) \geq U(z)$, yielding a contradiction. This shows that, for better or worse, the model at hand still embodies a considerable degree of rationality. □

Example 4. (The Attraction Effect) The model derived in Theorem 1 is suitable for modeling behavior that exhibits the “attraction effect” relative to one’s initial endowment. That is, it allows an agent to choose $x$ over $y$ when $z$ is the status quo, and choose $y$ over $x$ when $x$ is the status quo. To see this, note that “choosing $x$ over $y$ when $z$ is the status quo” is captured in our setup by the statement $\{x\} = c(\{x, y, z\}, z)$, and “choosing $y$ over $x$ when $x$ is the status quo” by $\{y\} = c(\{x, y\}, x)$. But, if $c$ is represented as in Theorem 1, these two statements would hold provided that $U(y) > U(x) > U(z)$, $x \in Q(z)$ and $y \in Q(x) \setminus Q(z)$. As Figure 4 illustrates, these conditions are duly compatible. □

In passing, we note that Example 4 points to two essential traits of the choice model of Theorem 1. First, we see that this model allows for using a status quo option
as an alternative whose “value” is somewhat enhanced for the agent, but also, it allows for it to be used as a reference point. (In Example 4, we have \( \{ y \} = c(\{ x, y \}, x) \), which means that \( y \) is superior to \( x \) unambiguously. Yet we have \( \{ x \} = c(\{ x, y, z \}, z) \), which means that “from the viewpoint of \( z \)” the option \( x \) is deemed better than \( y \), a telltale sign of reference-dependent decision making.)

Second, Example 4 points to the fact that there is a limit to the status quo bias allowed by the choice model of Theorem 1. In particular, if \( x \in c(S, z) \), then we do not have to have \( x \in c(S, x) \) as well. (Notice that WSQB requires this only for the case \( S = \{ x, z \} \).) After all, for \( S = \{ x, y, z \} \) in Example 4, we have \( \{ x \} = c(S, z) \) and \( \{ y \} = c(S, x) \).

In fact, these two points are closely related. It turns out that a property like “\( x \in c(S, z) \) implies \( x \in c(S, x) \)” — this is even weaker than what is dubbed the “Status Quo Bias Axiom” in Masatlioglu and Ok (2005) — robs the involved choice model from attributing a referential status to one’s initial endowment. Allowing for reference dependence (on the initial endowment) and status-quo bias phenomenon jointly, then, one needs an axiom that is weaker than the “Status Quo Bias Axiom.” This is achieved here by means of WSQB.

The final two examples we shall consider in this section illustrate parametric specializations of the choice model of Theorem 1.

**Example 5.** Let \( X \) stand for the commodity space \( \mathbb{R}_+^n \) and take any continuous and strictly increasing utility function \( U : X \rightarrow \mathbb{R} \). Let \( B \) be a nonempty finite set of \( n \)-vectors, and \( \alpha \) a nonnegative \( n \)-vector. We define the self-correspondence \( Q_{B,\alpha} \) on \( X \) as follows:

\[
Q_{B,\alpha}(x) := \{ \omega \in X : U(\omega) \geq U(x) \text{ and } \beta \omega \geq \beta(x - \alpha) \text{ for all } \beta \in B \}.
\]

(See Figure 2a) With this specification, the choice model of Theorem 1 becomes

\[
c_{B,\alpha}(S, x) = \text{arg max}\{ U(\omega) : \beta \omega \geq \beta(x - \alpha) \text{ for all } \beta \in B \}
\]

for every nonempty compact subset \( S \) of \( X \) and every initial endowment \( x \) in \( S \). (For instance, suppose \( B \) consists exactly of the unit \( n \)-vectors. Then, as illustrated in Figure 2b, the consumption choice behavior implied by \( c_{B,\alpha} \) does not tolerate a loss of more than \( \alpha_i \) units of good \( i = 1, ..., n \). In the extreme case where \( \alpha_i = 0 \) for each \( i \), the agent does not tolerate any loss relative to any of the components of her initial holdings.)

This parametric formulation facilitates making comparative statics by means of the “more status quo biased” relation introduced in Section 2.3. For instance, we see here that \( c_{B,\alpha} \) is more status quo biased than \( c_{B,\alpha'} \) iff \( \alpha \leq \alpha' \) (component-wise). As each component of \( \alpha \) converges to \( \infty \), \( c_{B,\alpha} \) converges to the choice correspondence of a person who maximizes \( U \) in a reference-free manner, so we recover the standard rational choice model in the limit.
Example 6. Let $X$ stand for the commodity space $\mathbb{R}^2_{++}$, and assume that the decision maker has a Cobb-Douglas utility function $U$, defined by $U(x) := \sqrt{x_1 x_2}$, on $X$. Let $\rho$ be a non-positive function on $X$, and define the self-correspondence $Q_\rho$ on $X$ as

$$Q_\rho(x) := \{ y \in X : v_x(y) \geq v_x(x) \},$$

where $v_x : X \to \mathbb{R}$ is defined by $v_x(y) := (a_x y_1^\rho(x) + b_x y_2^\rho(x))^{1/\rho(x)}$ with $a_x := x_2^\rho(x)/2$ and $b_x := x_1^\rho(x)/2$. (The elasticity of substitution of $v_x$ is equal to $1/(1 - \rho(x))$.)

With these specifications, the choice model of Theorem 1 becomes

$$c_\rho(S, x) = \arg \max \{ \sqrt{\omega_1 \omega_2} : \omega \in S \text{ and } v_x(\omega) \geq v_x(x) \}.$$ 

It is easy to check that if $\rho_1$ and $\rho_2$ are such that $\rho_1 \leq \rho_2$ (pointwise), then $c_{\rho_1}$ is more status quo biased than $c_{\rho_2}$. Furthermore, in the special case of constant elasticity of substitution, that is, when $\rho_1$ and $\rho_2$ are constant functions – Figure 3 depicts this situation – $c_{\rho_1}$ is more status quo biased than $c_{\rho_2}$ iff $\rho_1 \leq \rho_2$. □

3 Multi-criteria Decision Making with Status Quo Bias

The choice model characterized in Theorem 1 does not impose any structure on how the mental constraints induced by various status quo options may relate to each other. According to this model, regardless of the relation between two choice prospects $x$ and $y$, one may seem extremely status quo biased when $x$ is the status quo option (the case of small $Q(x)$) while she may even act reference-free when her status quo
is \( y \) (the case of large \( Q(y) \)). While this provides strong explanatory power, it limits the predictive power of this model.\(^{11}\)

In applications, one can of course impose a suitable structure on \( Q \) to avoid these sorts of issues. However, it is not clear if, and how, one can sharpen the behavioral/axiomatic basis of the present choice model in order to obtain such structural restrictions on \( Q \). In this section, our goal is to show that one can in fact do this on the basis of alternative formulations of the status quo bias phenomenon. Clearly, this requires using a behavioral postulate that is stronger than WSQB. We will focus on the following strengthening of this property.

**Strong Axiom of Status Quo Bias (SSQB).** For any \( x, y \in X \),

\[
y \in c(\{x, y\}, x) \quad \text{implies} \quad \{y\} = c(\{x, y\}, \diamond)
\]

and, for any \( S \in \Omega_X \) with \( x, y \in S \),

\[
y \in c(S, \diamond) \quad \text{implies} \quad \{y\} = c(S, y).
\]

The first part of this postulate is only mildly stronger than the corresponding part of WSQB. It says simply that if an agent were to move away from her status quo

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\(^{11}\)To illustrate, consider an agent whose choice correspondence is represented by \( U \) and \( Q \) as in Theorem 1. If \( U(y) > U(x) \) and \( Q(x) = \{x, y\} \), then this agent would leave her status quo \( x \) only for \( y \). It then stands to reason that it would be even harder for this agent to leave \( y \) when \( y \) is itself the status quo, thereby exhibiting at least as much status quo bias as she does when \( x \) is her endowment. Yet, the model allows even for \( Q(y) = X \).
option $x$ in favor of an alternative $y$, then this must be because she deems $y$ strictly better than $x$ in a reference-free sense. Clearly, what this brings over WSQB is only that ties be broken in favor of the status quo options.

The second part of SSQB is substantially stronger than the corresponding part of WSQB. In words, it says that if the decision maker reveals that $y$ is a best alternative in a (possibly non-binary) choice situation, then the agent would opt for $y$, had $y$ been the status quo option to begin with. While this may seem at first like an innocent extension of the second part of WSQB, we shall see shortly that it has unexpectedly strong implications. Let us first give a concrete example of a choice correspondence that satisfies WSQB but not SSQB.

**Example 6** [Continued]. Consider the choice correspondence $c_\rho$ of Example 6 which is sure to satisfy WSQB. If the elasticity of substitution $\rho$ is constant here, then $c_\rho$ satisfies SSQB as well. If $\rho$ is not a constant function, however, $c_\rho$ may well fail to satisfy SSQB, as illustrated in Figure 4.

![Figure 4: A General Non-linear Constraint Model](image)

Figure 4: A General Non-linear Constraint Model where \( \{x\} = c(\{x, y, z\}, z) \) but \( \{y\} = c(\{x, y\}, x) \) and \( \{y\} = c(\{x, y, z\}, x) \)

The following result shows how Theorem 1 would modify if WSQB were replaced by SSQB.

**Theorem 2.** Let $X$ be a compact metric space and $c$ a choice correspondence on $\mathcal{C}(X)$. Then, $c$ satisfies WARP, SSQB, SQI and C if, and only if, there exist a continuous (utility) function $U : X \to \mathbb{R}$ and a closed-valued self-correspondence $Q$ on $X$ such that $Q \circ Q \subseteq Q$ and

\[
U(y) > U(x) \quad \text{for every } x \in X \text{ and } y \in Q(x) \setminus \{x\}
\]

while (1) and (2) hold for every $(S, x) \in \mathcal{C}_{sq}(X)$.  

16
As expected, replacing WSQB with SSQB in Theorem 1 amounts to imposing further structure on the psychological constraint correspondence \( \mathcal{Q} \). In particular, any alternative in \( \mathcal{Q}(x) \) other than \( x \) itself must now be strictly better than \( x \) in the reference-free sense (according to \( U \)). While this goes along with the interpretation that \( \mathcal{Q}(x) \) consists of “unambiguously better options than \( x \),” it implies also that an initial endowment can never be declared indifferent to any other alternative. In particular, the rational choice model (Example 1) is not a special case of the choice model of Theorem 2.

A more important restriction on \( \mathcal{Q} \) that is guaranteed by imposing SSQB on top of the postulates of Theorem 1 is the transitivity of \( \mathcal{Q} \), that is, \( \mathcal{Q} \circ \mathcal{Q} \subseteq \mathcal{Q} \). This property says that if \( y \in \mathcal{Q}(x) \) and \( z \in \mathcal{Q}(y) \) then we must have \( z \in \mathcal{Q}(x) \), for any \( x, y, z \in X \). In other words, if \( z \) is not eliminated from the viewpoint of the status quo \( y \), and \( y \) itself is not eliminated from the perspective of \( x \), then \( z \) is not be eliminated from the perspective of \( x \). This adds quite a bit of structure to the choice model of Theorem 1. In particular, it entails that this model is incompatible with the “attraction effect” type behavior we have discussed in Example 4.

On the positive side, Theorem 2 builds a bridge between the “mentally constrained choice” model of Theorem 1 and the “multi-criteria choice” models that are studied elsewhere. This is because, due to the transitivity of \( \mathcal{Q} \), the graph of this correspondence act as a preorder. (This preorder is, in fact, a partial order in the present setup, that is, it is antisymmetric.) As any preorder equals the intersection of all of its completions, we find that an expression like \( y \in \mathcal{Q}(x) \) can in this case be expressed as saying that \( y \) “dominates” \( x \) with respect to all of these completions, and we arrive at a choice model that is based on multi-criteria decision-making. This argument becomes particularly strong when \( X \) is finite. For, in that case, we can represent each of the completions by means of utility functions, and furthermore, using (3) we may express the reference-free utility function of the model as an aggregation of these criteria. Our final representation result provides a precise description of this situation.

**Theorem 3.** Let \( X \) be a nonempty finite set and \( c \) a choice correspondence on \( \mathcal{C}(X) \). Then, \( c \) satisfies WARP, SSQB and SQI if, and only if, there exist a positive integer \( k \) and an injection \( u : X \to \mathbb{R}^k \) (whose \( i \)-th component map is denoted by \( u_i \)) such that

\[
c(S, \diamond) = \arg \max_{\omega \in S} \sum_{i=1}^k u_i(\omega),
\]

and

\[
c(S, x) = \arg \max_{\omega \in S \text{ and } u(\omega) \geq u(x)} \sum_{i=1}^k u_i(\omega)
\]

for every \( S \in \Omega_X \) and \( x \in S \).
Consider a decision maker whose choice correspondence \( c \) on \( C(X) \) is represented by finitely many real maps \( u_1, \ldots, u_k \) on \( X \) as in Theorem 3. We may think of each \( u_i \) as measuring how a given alternative fares with respect to a criterion that the agent deems relevant for her choices. (Thus, the agent makes her evaluations on the basis of \( k \), subjectively determined, criteria.) When dealing with a choice problem without an initial endowment, the agent makes her decisions by maximizing a utility function that is obtained by the additive aggregation of these criteria. If, on the other hand, the decision maker has a status quo choice in a problem \( S \), say, \( x \), then she proceeds to settle that problem by comparing every feasible alternative to \( x \) with respect to all criteria. If none of the feasible alternatives \( y \) in \( S \) dominates \( x \) with respect to all objectives – that is, for any feasible \( y \) there is a rationale \( i \) such that \( u_i(x) > u_i(y) \) – then the agent chooses to stay with her status quo. If, however, at least one feasible alternative “beats” \( x \) with respect to all criteria, then she concentrates (only) on such alternatives, and chooses among them the one(s) that yield the highest satisfaction in terms of the additive aggregation of her choice criteria.

This choice model relates closely to those heuristically suggested by Simon (1955) and Bewley (1986). There are also more recent contributions in the literature on rational choice theory where related models are axiomatically characterized.\(^{12}\)

**Remark.** The finiteness of \( X \) plays an essential role in Theorem 3. First, without this assumption, we cannot ensure that the completions of the preorder induced by \( Q \) of Theorem 2 have utility representations. Even more pressing is that, when \( X \) is infinite, this partial order need not be the intersection of finitely many of its completions. (In the jargon of order theory, the problem is that the dimension of this partial order need not be finite.) This would make it impossible to obtain finitely many attribute functions \( u_1, \ldots, u_k \) in general. Finally, even in those cases where finitely may such objectives can be found, we may not be able to guarantee the linear aggregation of these objectives as in Theorem 3 when \( X \) is not finite.\(^{13}\)

**Remark.** The axioms used in Theorem 2 (and hence in Theorem 3) are logically independent; we provide a proof for this in the Appendix.

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\(^{12}\) In particular, there is a close connection between this result and Theorem 1 of Masatlioglu and Ok (2005). Given the finiteness of \( X \), the systems of axioms used in these results are in fact equivalent. However, the present system of axioms is significantly simpler. Furthermore, Theorem 3 shows also that the multiple objectives found in that paper can be aggregated *additively* into a utility function that governs the behavior of the agent in choice problems without initial endowments. For other multi-criteria choice models that have a similar flavor to that of Theorem 3, see Bossert and Sprumont (2001), Sugden (2003), Munro and Sugden (2003), Giraud (2004), Mandler (2005), Sagi (2006), Houy (2008), Apesteguia and Ballester (2009), Ortoleva (2010), and Riella and Teper (2010).

\(^{13}\) One can easily check that we could instead use \( F \circ u \) instead of \( u_1 + \cdots + u_k \) in Theorem 2, provided that \( F \) is a strictly increasing real function on \( u(X) \).
4 Economic Applications

In this section we present several applications of the canonical choice models we introduced in Sections 2 and 3 in various economic environments. On the one hand, these applications demonstrate the “applicability” of these models, and on the other, they provide a comparison of some of the concrete implications of our model with those of the loss aversion models (to be reviewed in Section 5).

4.1 Application to Behavioral Economics: The Endowment Effect

There are numerous experimental studies that provide evidence to the effect that the minimum compensation demanded by an agent for a good that she owns may be more than the maximum price she is willing to pay for the same good. This phenomenon, that is, the (negative) gap between one’s willingness to pay (WTP) and willingness to accept (WTA), is known as the endowment effect – see Thaler (1980) – and it is commonly argued to be a consequence of the status quo bias phenomenon. However, insofar as the notion of status quo bias is modeled through the WSQB property, it does not, in fact, entail the endowment effect. This fact, which seems to have been overlooked in the literature, sits particularly square with the recent empirical findings that in certain environments – for instance in the context of the market interactions of experienced traders – the endowment effect is likely to be absent. We next provide a brief demonstration of this point by using an instant of the canonical choice model derived in Theorem 1.

To conform with the previous experimental literature, we adopt a framework that involves only money and a single unit of a commodity, say, a mug. Consequently, we designate \( X := I \times \{0, 1\} \) as the alternative space for this environment, where \( I \) is an interval of the form \([0, \alpha]\) with \( \alpha > 0 \) being a positive number. (Here a pair like \((a, 1)\) in \( X \) is interpreted as the agent possessing \( a \) dollars and the mug, while \((b, 0)\) in \( X \) corresponds to the bundle that contains \( b \) dollars and no mug.)

Consider a decision maker whose initial monetary endowment is \( w_0 \) dollars, where \( 0 \leq w_0 \leq \alpha/2 \). We define the willingness to accept of this agent (for the mug) as

\[
\text{WTA}(c) := \inf\{a \geq 0 : (w_0 + a, 0) \in c((w_0 + a, 0), (w_0, 1)), (w_0, 1))\},
\]

where \( c \) corresponds to her choice correspondence on \( C(X) \). That is, \( \text{WTA}(c) \) is the smallest amount of money that this agent, if endowed with the mug, would charge to sell the mug. The formulation of WTP is less straightforward. We follow Tversky and Kahneman (1991) in postulating that the act of giving up money to buy goods

\[^{14}\text{See Kahneman, Knetsch and Thaler (1991) and Camerer (1995, pp. 665-670) for detailed surveys on this matter.}\]

\[^{15}\text{See Shogren et al. (1994), List (2003, 2004), and Plott and Zeiler (2005).}\]
is viewed by the agent not as a loss, but as a foregone gain of money. This leads to defining the willingness to pay of this agent (for the mug) as

$$WTP(c) := \sup \{w_o \geq a \geq 0 : (w_o, 1) \in c\{(w_o, 1), (w_o + a, 0)\}\}.$$ 

That is, $WTP(c)$ is the largest amount of monetary increment the agent would be willing to give up to be able to obtain the mug. We assume $w_o > WTP(c)$, that is, the agent is not willing to give up all of her wealth to receive the mug.

Despite the commonly held contention that “status quo bias implies the endowment effect” – this is indeed a formal implication of various models in the literature; see Example 9 below – certain special cases of the canonical choice model of Theorem 1 (but not of Theorem 2) allow for $WTA(c) = WTP(c)$. This is, in fact, trivially true, because the rational choice model is a special case of our canonical model (Example 1). Less trivially, this equality may still hold for choice correspondences that exhibit status quo bias in a strict sense. In particular, for a choice correspondence $c$ on $C(X)$ that is represented by a utility function $U$ on $X$ and a closed self-correspondence $Q$ on $X$ as in Theorem 1, the absence of the endowment effect is characterized as follows: Let $p$ stand for the unique positive number with $U(w_0 + p, 0) = U(w_0, 1)$; this is the reference-free fair price of the mug. Then, $WTA(c) = WTP(c)$ iff $(w_0 + p, 0) \in Q(w_0, 1)$ and $(w_0, 1) \in Q(w_0 + p, 0)$.

To illustrate, we exhibit in Figure 5 two instances, one in which $WTA(c) = WTP(c)$ and one in which $WTA(c) > WTP(c)$. The choice model of Theorem 1 is thus general enough to allow for the presence and absence of the endowment effect while remaining status quo biased (in the sense of WSQB) at all times. It is even possible for such a model that the endowment effect is present for some trades and not for others. For instance, it is easy to modify the graphical illustrations in Figure 5 so that no gap is present in a unit sale of the mug, but a gap is present in the sale of two or more mugs.

### 4.2 Application to Finance: The Equity Premium Puzzle

One of the well-known dilemmas in finance is based on the empirical observation that equity returns are, on average, significantly better than those of bonds. The simultaneous presence of bonds and equities in the market can thus be justified by the utility maximization model only if one presumes unrealistically high risk aversion on the part of the investors. This is dubbed the equity premium puzzle (Mehra and Prescott (1985)). It is well-known that this puzzle can be explained by using a (constant) loss aversion model (cf. Barberis, Huang and Santos (2001)). We now illustrate how one may utilize the canonical choice model of Section 4 to do the same.  

16Alternatively, one could choose to model the act of giving up money in exchange of goods as a “loss” as well – see Bateman, et al. (1997, 2005) – but the conclusions drawn in this section would remain unaltered within that formulation as well.
Consider a financial market in which there is a bond with price \( p_b > 0 \) and a stock with price \( p_s > 0 \). Annually, the bond yields \( B \) dollars with certainty, and the stock yields \( M \) dollars with probability \( \alpha \) and \( m \) dollars with probability \( 1 - \alpha \). Naturally, we assume that \( B > p_b \) and \( M > B > m \). To ensure comparability, we also posit that the expected per period value of the stock is equal to the return on the bond, that is, \( \alpha M + (1 - \alpha)m = B \). Now consider a representative agent in this economy who is risk neutral. If this agent is an expected utility maximizer, an equilibrium in which both the stock and bond are purchased can be justified only if this agent is indifferent between buying the stock and the bond, that is, when

\[
\alpha(M - p_s) + (1 - \alpha)(m - p_s) = B - p_b.
\]

As \( \alpha M + (1 - \alpha)m = B \), therefore, this happens only if \( p_s = p_b \), that is, in the absence of any equity premium.

Let us now model the decisions of our agent through the choice model of Theorem 3. Let \( X \) be the set of all probability distributions on \( \mathbb{R} \). In what follows, we denote the expectation of a lottery \( \mu \) in \( X \) by \( \mathbb{E}_\mu \), and by \( \delta_a \) we mean the degenerate lottery that pays \( a \) dollars with probability one. Consequently, the stock option corresponds to the lottery \( \mu_s := \alpha \delta_{M - p_s} + (1 - \alpha) \delta_{m - p_s} \) and the bond option is the degenerate lottery \( \mu_b := \delta_{B - p_b} \). Now take any nonempty collection \( W \) of continuous and stochastically strictly increasing real functions on \( X \), and assume that there is a \( V \in \mathcal{W} \) such that \( V(\delta_{\mathbb{E}_\mu}) > V(\mu) \) for every non-degenerate \( \mu \in X \). (To wit, \( V \) represents the preferences of a risk averse individual who may not be an expected utility maximizer.) We define \( U(\mu) := \mathbb{E}_\mu \) and \( Q(\mu) := \{ \nu \in X : W(\nu) \geq W(\mu) \text{ for all } W \in \mathcal{W} \} \) for each \( \mu \in X \). Consequently, when her default option is \( \mu \), the choice of the agent between stocks and bonds is found as:

\[
c(S, \mu) = \arg \max \{ \mathbb{E}_\nu : \nu \in S \text{ and } W(\nu) \geq W(\mu) \text{ for all } W \in \mathcal{W} \},
\]

where \( S := \{ \mu_s, \mu_b, \mu \} \). But then, if the default option of this individual is to buy

---

**Figure 5:** WTA versus WTP
the bonds, that is, $\mu = \mu_b$, the stock belongs to the mentally constrained set if $V(\mu_s) \geq V(\mu_b)$. By the risk aversion property of $V$, therefore, $V(\delta_0 \delta_{M-p_s} + (1-\alpha) \delta_{m-p_s}) > V(B-p_b)$, so, as $V$ is stochastically strictly increasing, $\alpha(M-p_s) + (1-\alpha)(m-p_s) > B - p_b$, that is, $p_b > p_s$. In words, the positive demand to the stock in the market is possible only if the bond sells at a higher price than the stock, even though the representative agent acts as a risk neutral expected utility maximizer in the absence of any status quo effects and the expected values of the bond and the stock are the same.\footnote{As $p_b > p_s$, we have $((E_{\mu_s} - p_s)/p_s) > (B - p_b)/p_b$, that is, the expected per-dollar return on the stock exceeds that of the bond; there is positive risk premium in the economy.}

It is also readily checked that the same conclusion holds when the default option of the agent is not to participate in the market, that is, $\mu = \delta_0$, provided that $V$ is risk averse enough that $V(\delta_0) > V(\alpha \delta_{M-p_b} + (1-\alpha) \delta_{m-p_b})$.\footnote{The stock should yield higher $V$-value than the default option, that is, $V(\mu_s) \geq V(\delta_0) > V(\alpha \delta_{M-p_b} + (1-\alpha) \delta_{m-p_b})$, which implies $p_b > p_s$.}

The intuition behind this illustration is not unlike how loss aversion theory “explains” the equity premium puzzle. In the present formulation, the agent behaves as a risk neutral expected utility maximizer, and hence her choices are not compatible with the presence of positive risk premium (and hence with the equity premium puzzle), when she acts in the absence of a status quo option. If, on the other hand, the agent has a default option, and she is reluctant to move away from this alternative unless a dominant investment option is available, the situation is different. In particular, if she views buying bonds as her default option, and compares alternatives to this option through several ranking criteria in her mind – this is captured by the set $W$ above – and at least one of these criteria is that of a mildly risk averse person – this is captured by $V$ above – then her behavior may well entail positive risk premium.

The same holds if the default option of the agent is not to invest at all, provided that one of the criteria she uses to check for dominance is sufficiently risk averse. It is also true that if an agent is more status quo biased compared to another one, her willingness to pay for the stock is lower.

4.3 Application to Finance: The Disposition Effect

Another famous behavioral puzzle in the context of finance concerns the tendency of investors to hold on to stocks that have lost value too long while being prone to selling stocks that have gained value. This behavior is called the disposition effect; it was predicted by Shefrin and Statman (1985), and it was later corroborated empirically by several studies (cf. Weber and Camerer (1998), Odean (1998) and Ivkovich, Poterba and Weisbenner (2005)). It is known that the disposition effect is can be “explained” through loss aversion models. We next illustrate how one may utilize the canonical choice model of Section 3 to account for this effect as well.

Consider a stock that yields, per period, $200 with probability $\alpha$ and $-100 with probability $1-\alpha$, where $0 < \alpha < 1$. In this simplified context, the disposition effect is
captured by the selling price of this stock after it has produced a gain being strictly less than that of the stock after it has yielded a loss. We wish to show that this may well be the case if the owner of the stock is a rational individual with a status quo bias whose behavior is represented by an instance of the choice model of Section 3.

Let $X$ denote the collection of all probability distributions on $\mathbb{R}$, and define $U : X \to \mathbb{R}$ by $U(\mu) := \mathbb{E}_\mu(u)$, where $u$ is a strictly increasing and strictly concave von Neumann-Morgenstern utility function. (Here $\mathbb{E}_\mu(u)$ is the expected value of $u$ with respect to the lottery $\mu$.) Next, take any continuous and strictly increasing $f : \mathbb{R} \to \mathbb{R}$ that is affine (or concave) on the interval $(-\infty, 100]$ and strictly concave on the interval $[100, \infty)$, and define the self-correspondence $Q$ on $X$ by

$$Q(\mu) := \{ \nu \in X : \mathbb{E}_\nu(f) \geq \mathbb{E}_\mu(f) \}.$$  

Then, according to the choice model of Theorem 2, the choices from a given nonempty (compact) subset $S$ of $X$ when a lottery $\mu$ is the status quo are found as:

$$c(S, \mu) = \arg \max \{ \mathbb{E}_\nu(u) : \nu \in S \text{ and } \mathbb{E}_\nu(f) \geq \mathbb{E}_\mu(f) \}. \quad (19)$$

Now, consider two lotteries that yield

$$\begin{cases} 
100 & \text{with probability } \alpha \\
-200 & \text{with probability } 1 - \alpha
\end{cases} \quad \text{and} \quad \begin{cases} 
400 & \text{with probability } \alpha \\
100 & \text{with probability } 1 - \alpha,
\end{cases}$$

respectively. The first lottery here – we denote it by $\mu_L$ – is what our stock looks like after it has yielded a loss in its first period. (Given that a loss of $100$ is incurred in the first period, at the start of the second period, this stock yields $-100 + 200$ with probability $\alpha$, and $-100 - 100$ with probability $1 - \alpha$.) Given the loss in the first period, if the agent sells this stock in the second period for $p$, the gain is then $p - 100$ (disregarding discounting effects). Thus, the minimum selling price of the stock in this case is the least $p \geq 0$ such that $\delta_{p-100} \in c(\{\delta_{p-100}, \mu_L\}, \mu_L)$. \footnote{For instance, in the case of lotteries whose supports are included in $[100, \infty)$, this individual may not leave her status quo for a riskier lottery even if the latter yields a higher expected utility in terms of $u$.}

Similarly, the second lottery above – we denote it by $\mu_G$ – corresponds to our stock in the second period after this stock has produced a gain of $200$. The minimum selling price of the stock in this case is the least $p \geq 0$ such that $\delta_{p+200} \in c(\{\delta_{p+200}, \mu_G\}, \mu_G)$. Let us denote this selling price by $p_*$. Then, by definition of $c$, $p_* + 200$ must be equal to the larger of the certainty equivalents of $\mu_G$ with respect to the expectation of $u$ and $f$. But, as $u$ and $f$ are strictly concave on an interval that contains the support of $\mu_G$, both of these certainty equivalents are strictly less than the expected value of $\mu_G$, which is $300\alpha + 100$. It follows that $p_* < 300\alpha - 100$. On the other hand, the certainty equivalent of $\mu_L$ with respect to $f$ is simply the expected value of $\mu_L$, that is, $300\alpha - 200$. Thus $p_* - 100$ is strictly less than the certainty equivalent of $\mu_L$ with respect to $f$, that is, $\delta_{p_*-100}$ does not belong to $Q(\mu_L)$, and hence, $\delta_{p_*-100} \notin Q(\mu_L)$.

\footnote{Here, $\delta_a$ stands for the degenerate lottery that gives $a$ with probability one.}
\(c(\{\delta_{p-100}, \mu_L\}, \mu_L)\). It follows that the minimum selling price for \(\mu_L\) must exceed \(p^*\) in accordance with the disposition effect. Furthermore, the gap between the minimum selling price of \(\mu_L\) and that of \(\mu_G\) increases as the agent gets more status quo biased.

The intuition behind this illustration lies in the structure of \(Q\) that we have chosen above. This constraint correspondence utilizes the strict concavity of \(f\) to posit risk averse behavior for “winner” stocks – in this example, a “winner” stock is one that yields a lottery whose support is contained in \([100, \infty)\) – in addition to the one entailed by the concavity of \(u\). It, however, does not do this for “loser” stocks, because \(f\) is affine on the interval \((-\infty, 100]\). In this sense, the channel through which the present model “explains” the disposition effect is analogous to that of prospect theory (cf. DellaVigna (2009)).

4.4 Application to Price Determination in Housing Markets

An analogue of the disposition effect has been observed in the context of housing markets. Genovese and Meyer (2001) have shown empirically that home-owners who sell their houses below their purchase prices tend to set their prices too high, and as a result, such houses remain in the market more than others. One can use a loss aversion model to “explain” this phenomenon; see, for instance, DellaVigna (2009). The same can be done, in this case in an extremely simple fashion, by means of the canonical choice model of Section 3. We illustrate this next.

Consider an agent who has bought her house for \(p_0\) dollars. Following the illustration given in DellaVigna (2009), we imagine that setting sales price for this house has a direct effect through one’s utility, and an indirect effect through the probability of being able to sell the house. To be specific, let us denote the contingency of “not selling the house,” by \(h\), and fix two functions, \(u : \mathbb{R}_+ \cup \{h\} \to \mathbb{R}\) and \(\alpha : \mathbb{R}_+ \to [0, 1]\). For any number \(p \geq 0\), we interpret \(u(p)\) as the utility of the sale of the house at price \(p\), and \(\alpha(p)\) as the probability that the house would actually sell at price \(p\). In addition, \(u(h)\) is the reservation utility of the agent (from keeping the house). It is thus reasonable to assume that \(u\) is strictly increasing and strictly concave on \(\mathbb{R}_+\) and that \(\alpha\) is decreasing and concave. We also assume that \(0\) belongs to the range of \(\alpha\) so that \(\bar{p} := \inf\{p \geq 0 : \alpha(p) = 0\}\) exists. Notice that \(\bar{p} := u^{-1}(u(h))\) is the largest price at which selling the house yields less utility than the reservation utility of the owner. To avoid trivialities, we assume henceforth that \(\bar{p} > p\). The expected utility of the owner in this setting is thus given by the function \(U : \mathbb{R}_+ \to \mathbb{R}\) defined by \(U(p) := \alpha(p)u(p) + (1 - \alpha(p))u(h)\) if \(\bar{p} > p > p_0\), and \(U(p) := u(h)\) otherwise. Our assumptions guarantee that \(U\) is strictly concave on \([p, \bar{p}]\), and hence there is a unique maximizer of this function, say, \(p^*\), and this price satisfies \(\bar{p} > p^* > p_0\).

Suppose the home-owner is in the mind-set that she would never sell her house at more than a \(\theta\) percent loss. It is straightforward to capture this person through the canonical choice model of Theorem 1 by setting \(X := \mathbb{R}_+ \cup \{h\}\) and extending
U to X by defining $U(h) := u(h)$. In addition, we choose any closed-valued self-correspondence $\mathcal{Q}$ on $X$ such that $\mathcal{Q}(h) = [\theta p_0, \infty) \cup \{h\}$. Then, the choice model of Theorem 1 maintains that

$$c(X, h) = \begin{cases} 
\{p^*\}, & \text{if } p^* \geq \theta p_0 \\
\{\theta p_0\}, & \bar{p} > \theta p_0 > p^* \\
[\theta p_0, \infty) \cup \{h\}, & \text{if } \theta p_0 > \bar{p}.
\end{cases}$$

In particular, if the home-owner is expected to make a nominal loss, that is, when $p_0 > p^*$, then she may well charge a price higher than the optimal price $p^*$; this actually happens when the loss is substantial enough so that $\theta p_0 > p^*$. Here, $\theta$ provides a measure of the size of the status quo bias of a home-owner. Higher $\theta$ corresponds to being more status quo-biased, and asking for higher selling prices.

### 4.5 Application to Consumer Theory: The Law of Demand

Consider an economy in which an individual may consume two goods, the second one being a composite (numeraire) good (standing for everything else the consumer may wish to consume other than the first good). The utility function of this individual is given by a continuous, strictly increasing and strictly quasiconcave map $U : \mathbb{R}_{++}^2 \rightarrow \mathbb{R}$, and her income is denoted by $w > 0$. When the prices in the economy is given by $p \in \mathbb{R}_{++}^2$ with $p_2 = 1$, therefore, the optimal consumption choice of this individual is found upon maximizing $U(x)$ subject to the budget constraint $px \leq w$. (At this stage in the analysis, we treat the individual as acting without a status quo option; more on this shortly.) Let us denote this choice by $x(p, w)$, the demand of the individual at prices $p$ and income $w$.

Assume now that, perhaps even before the agent exercised her choice, the price of the first good decreases, so that the new price in the economy is $q \in \mathbb{R}^2_{++}$ where $p_1 > q_1$ and $p_2 = q_2 = 1$. In this case, it seems reasonable to model the consumer’s problem as making a choice from the new budget set $\{x \in \mathbb{R}^2_{++} : qx \leq w\}$ with $x(p, w)$ acting as her status quo option. Consequently, if we posit that the agent behaves in accordance with the choice model given in Theorem 1, her problem is:

$$\text{Maximize } U(x) \text{ subject to } qx \leq w \text{ and } x \in \mathcal{Q}(x(p, w))$$

where $\mathcal{Q}$ is a closed-valued self-correspondence on $\mathbb{R}^2_{++}$. Here, of course, $\mathcal{Q}(x(p, w))$ corresponds to the consumption bundles that look particularly appealing “relative to the status quo choice $x(p, w)$,” that is, this set consists of all feasible alternatives that are superior to $x(p, w)$ in an unambiguous sense. Given that the (reference-free) preferences of the agent are monotonic, therefore, it is natural to assume that $\mathcal{Q}$ has increasing values, that is, $x + \mathbb{R}^2_{++} \subseteq \mathcal{Q}(x)$. Similarly, the convexity of the preferences of the agent makes it reasonable to posit that $\mathcal{Q}$ is convex-valued.

---

21Formally, we can think of $X$ as a metric space by first metrizing $\{0\} \times \mathbb{R}_+$ by a bounded metric, and then adjoining $h$ (an object that does not belong to $\mathbb{R}_+$) to this set as an isolated point.
We now perform a classical Slutsky analysis to understand whether the law of compensated demand holds for this agent, and if so, how the substitution effect in this case relates to that in the standard case where the agent acts without status quo considerations. To this end, let us set \( w' := qx(p, w) \), the income level that makes the earlier consumption \( x(p, w) \) exhaust one’s budget in the new prices \( q \). (Here \( w' - w < 0 \) is the Slutsky compensation.) The (hypothetical) choice problem of the agent (in which the income effect of the change of prices from \( p \) to \( q \) is eliminated) is:

\[
\text{Maximize } U(x) \text{ subject to } qx \leq w' \text{ and } x \in \mathcal{Q}(x(p, w)).
\] (4)

We denote the solution of this problem as \( x(q, w') \).

The Slutsky substitution effect, or the compensated own-price effect, is defined as the magnitude of \( x_1(p, w) - x_1(q, w') \). In turn, the classical Law of Compensated Demand maintains that this effect is always negative, provided that \( \mathcal{Q} = \mathbb{R}_+^2 \), that is, when the consumer is a standard utility-maximizer. As we show by means of a revealed preference analysis, this law remains intact in the present framework in which the consumer acts in a status quo biased manner.

**Fact 1.** (The Law of Compensated Demand) \( x_1(q, w') \geq x_1(p, w) \).

**Proof.** Since \( x(p, w) \in \mathcal{Q}(x(p, w)) \) and \( qx(p, w) = w' \), we have \( U(x(q, w')) \geq U(x(p, w)) \). So, if \( px(p, w) > px(q, w') \), then, as \( U \) is strictly increasing, \( x(p, w) \) would not have been the optimal choice for the initial problem of the agent. Thus:

\[
px(p, w) \leq px(q, w').
\] (5)

On the other hand, \( qx(q, w') = w' \) (by the monotonicity of \( U \)). In view of the choice of \( w' \), therefore,

\[
qx(p, w) = qx(q, w').
\] (6)

As \( p_2 = q_2 \), combining (5) and (6) yields

\[
p_1(x_1(p, w) - x_1(q, w')) \leq q_1(x_1(p, w) - x_1(q, w')).
\]

As \( p_1 > q_1 \), this proves our assertion.

Let us now pose the following problem. How does the magnitude of the substitution effect we found above compare with the substitution effect the consumer would exhibit if she did not have a status quo bias? To formulate this question formally, we denote the optimal solution of the problem

\[
\text{Maximize } U(x) \text{ subject to } qx \leq w' \text{ and } x \in \mathbb{R}_+^2
\]
by \(x^*(q, w')\), that is, \(x^*(q, w')\) is the demand of the consumer at prices \(q\) and income \(w'\), provided that she acts according to the standard consumer choice model. The classical Law of Compensated Demand can then be stated as saying that \(x^*_1(q, w') \geq x_1(p, w)\). In view of Fact 1, then, our question concerns the comparison of \(x^*_1(q, w')\) and \(x^*_1(q, w')\). The answer is given in the next result.

**Fact 2.** (The Comparative Law of Compensated Demand) \(x^*_1(q, w') \geq x_1(q, w')\).

**Proof.** Suppose, to derive a contradiction, we have \(x^*_1(q, w') < x_1(q, w')\). By the classical Law of Compensated Demand, therefore, 
\[
x_1(p, w) \leq x^*_1(q, w') < x_1(q, w').
\]
As \(x(p, w), x^*(q, w')\) and \(x(q, w')\) all lie on the budget line \(\{x \in \mathbb{R}^2_+ : qx = w'\}\), this means that \(x^*(q, w')\) can be written as a convex combination of the 2-vectors \(x(p, w)\) and \(x(q, w')\). As both of these 2-vectors belong to \(Q(x(p, w))\), and \(Q\) is convex-valued, therefore, we have \(x^*(q, w') \in Q(x(p, w))\) as well. It follows that \(U(x(q, w')) \geq U(x^*(q, w'))\), while the converse of this inequality is obviously true by the choice of \(x^*(q, w')\). But then \(x(q, w')\) and \(x^*(q, w')\) are two distinct optimal solutions for the problem (4), which contradicts the strict quasiconcavity of \(U\).

The gist of the two facts we proved above can be summarized as follows:

<table>
<thead>
<tr>
<th>Substitution Effect</th>
<th>Substitution Effect</th>
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<tbody>
<tr>
<td>without</td>
<td>≥</td>
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<tr>
<td>Status Quo Bias</td>
<td>Status Quo Bias</td>
</tr>
</tbody>
</table>

Thus, insofar it is modeled by the canonical choice model of Theorem 1, we see here that the compensated own-price effect is less pronounced for consumers that are subject to status quo bias. This suggests that “backward-bending labor supply” type phenomena may be more prevalent than one may at first suspect, and this, due to the status quo bias phenomenon.

**Remark.** The analysis above presumes that the initial demand \(x(p, w)\) of the consumer is obtained in the absence of a status quo option. If we relax this hypothesis, Fact 1 ceases to be correct, unless we put further structure on the mental constraint correspondence \(Q\). In particular, if we assume that the agent is strongly status quo-biased, that is, her choice behavior is consistent with SSQB, then, as we have seen in Theorem 2, \(Q \circ Q \subseteq Q\) holds in the context of the choice model of Theorem 1, and this property ensures the validity of both Facts 1 and 2 even when the initial problem is solved relative to any (fixed) status quo option.
5 Loss Aversion versus Status Quo Bias

In behavioral economics, *loss aversion* refers to a decision maker’s tendency to put more emphasis on avoiding losses than on acquiring gains. While its underlying intuition is unambiguous, it appears difficult to define this concept formally in a context-free fashion. This draws a stark contrast with the status quo bias phenomenon which can be formulated as WSQB (or a variant of it) independently of the context of choice. Nevertheless, loss aversion phenomenon appears very much related to that of status quo bias, so much that it is often argued in behavioral economics that status quo bias is but a manifestation of one’s loss aversion. In concert with this, the most common way of modeling the decision-making procedure of an individual with status quo bias is by using a loss aversion model. In this section, we aim to point to some difficulties with this practice, and provide a comparison of the choice model we introduced above to those that would be rationalized by loss averse preferences.\footnote{We concentrate here exclusively on works in which one’s reference is modeled exogenously as her (observable) initial holdings and/or default option. At least as interesting as this is, of course, the case where one’s reference is not observable, such as one’s aspirations, as in the work of K"{o}szegi and Rabin (2006), or such as endogenously formed reference points, as in Ok, Ortoleva and Riella (2012). We have little to say about such forms of reference dependence in the present paper, however.}

The prototypical example of loss aversion models is the one introduced by Tversky and Kahneman (1991) in the context of riskless choice; we shall use this model to couch our comparison of loss aversion and status quo bias phenomena. This model is viewed within the behavioral economics folklore as the “standard” model of reference-dependent decision making. It is defined in a framework where the objects of choice have multiple, say $n \geq 2$, attributes, all of which are observable. It is thus particularly suitable to study choice decisions of individuals over feasible bundles of $n$ goods. Indeed, this is primarily how the model is used in practice.

The basic premise of the Tversky-Kahneman model, which we shall henceforth refer to as the TK-model, is that an agent, whose initial entitlement is some non-negative $n$-vector $x$, chooses those alternatives from a given feasible subset of $\mathbb{R}_+^n$ by maximizing a utility function $U_x : \mathbb{R}_+^n \to \mathbb{R}$ of the following form:

$$U_x(y) := \sum_{i=1}^{n} u_i(y_i - x_i).$$

Here, for each $i$, $u_i : \mathbb{R} \to \mathbb{R}$ is a continuous and strictly increasing function that satisfies the following two properties:

1. $u_i(0) = 0$ and $u_i(t) < -u_i(-t)$ for each $t \neq 0$;
2. $u_i$ is concave on $\mathbb{R}_+$ and $u_i$ is convex on $\mathbb{R}_-$.

Property (1) captures the phenomenon of *loss aversion*, which says that, in terms of the $i$th commodity, a gain is less important to the agent than a loss of equal
In turn, Property (2) says that the marginal value of such gains and losses are decreasing, and hence corresponds to the so-called *diminishing sensitivity* effect.

A number of implications of this model is found to be consistent with experiments in which the presence of an “initial entitlement” affects one’s choice behavior. There are also a good number of market-related “anomalies” that are found to be duly consistent with the TK-model. For instance, in the context of all three of the applications we have considered in Sections 4.2-4.4, it is known that the predictions of the TK-model match the empirical regularities quite well. Moreover, the TK-model has a particularly simple mathematical structure that makes it amenable to applications, which is surely an appealing property. Notwithstanding its incipient popularity, however, there are several difficulties that surround this model. First of all, it is not a “canonical” choice model in that it does not readily apply to individual choice problems in which the objects of choice are not consumption bundles. Indeed, it is difficult to see how to make use of this model, say, in the context of voting over political candidates, choosing between insurance or retirement policies, comparing job offers, etc. The TK-model thus seems best situated within the context of consumption decisions alone. Considering, in addition, the fact that this model is based on an “additive aggregation” hypothesis, it becomes transparent that it cannot serve as a canonical model of reference-dependent choice.

Even in the context of choosing among consumption bundles, however, there are some issues with the TK-model, at least insofar as one is interested in capturing the choice behavior of a decision-maker with status quo bias. A major reason for this is that this model does not leave room for studying the decisions of an agent who does not have a status quo position. This, in turn, hampers the ability of the model to deal with the *status quo bias phenomenon* properly, contrary to the commonly held view in behavioral economics.

To clarify this point, let us recall that a basic premise of the status quo bias phenomenon is this:

![A person who exhibits status quo bias may choose an alternative $x$ over another alternative $y$ when she is originally endowed with $x$, even though she would in fact prefer $y$ over $x$ absent any reference effects.]

---

23 This definition was suggested by Wakker and Tversky (1993).

24 A typical response to this, say, in the context of job offers, is that every “job contract” is a multidimensional object — the dimensions being, for instance, work location, salary, job quality, etc. — and hence, once the dimensions are specified, the TK-model becomes applicable to this context. The difficulty here is that these dimensions are not observable, simply because which dimensions are rendered relevant to the problem are known only to the decision maker. Furthermore, even when they are prespecified, these dimensions need not be quantifiable, so one has to view $x$ in this context as the “utility profile” that the agent derives from her current job. In turn, $y_i - x_i$ corresponds to the gain/loss of utility (of switching from $x$ to $y$) with respect to the $i$th dimension. This leads one to subscribe to disconcertingly strong cardinality-of-utility assumptions just to be able to view $y_i - x_i$ as a meaningful expression, and surely makes the quantity $\sum_i^n u_i(y_i - x_i)$ rather difficult to interpret.
Put differently, the status quo bias phenomenon maintains that, being a status quo may not decrease, and possibly increase, the value of an alternative (in the eyes of a decision-maker) relative to the reference-free value of this alternative. This is precisely the second part of the WSQB axiom, and it is indeed the nature of the status quo bias phenomenon discovered in the experimental studies. And yet, the TK-model is ill-prepared to model this premise, precisely because this model does not apply to problems without status quo points, and hence does not specify when an agent prefers a bundle over another “absent any reference effects.”

In applications, this difficulty is either ignored (by not considering those choice problems without reference points), or it is dealt with by designating a particular commodity bundle to act as the “no reference” point. (More often than not, this point is chosen to be the least preferable bundle, namely, the one that has 0 units of all goods.) Unfortunately, this does not solve the problem. For, no matter which bundle is chosen as the “no reference” point, the formalism of the TK-model sees that point as a reference, and hence, it predicts behavior that may well be inconsistent with the status quo bias phenomenon. We illustrate this next.

Example 7. (The TK–Model and the Status Quo Bias Phenomenon) Consider an environment in which an agent needs to choose among pairs \((M, m)\), where \(M\) and \(m\) stand for the units of mugs and money, respectively. We shall use a particular specification of the TK-model in which the utility of the bundle \((M, m)\) for an agent whose initial (status quo) endowment is \((M^*, m^*)\), is

\[
U_{(M^*, m^*)}(M, m) = u(M - M^*) + v(m - m^*),
\]

where

\[
u(t) := \begin{cases} 
t, & \text{if } t \geq 0 \\
2t, & \text{if } t < 0
\end{cases}
\quad \text{and} \quad
v(t) := \begin{cases} 
t^{0.8}, & \text{if } t \geq 0 \\
-2|t|^{0.8}, & \text{if } t < 0
\end{cases}.
\]

This specification, while robust, is the same with that estimated by Tversky and Kahneman (1992).\(^{25}\)

Now suppose the agent has initially no holdings of either mugs or money, and hence designate \((0, 0)\) as the “no reference” point (but note that the present example can be adapted to the case where any 2-vector is designated to serve as the “no reference” point). Consequently, when offered to choose between “one mug and $100” and “no mug and $103,” this agent would choose the first bundle because

\[
U_{(0, 0)}(1, 100) = 1 + 100^{0.8} > 103^{0.8} = U_{(0, 0)}(0, 103).
\]

Now, right after the agent made her choice, in which she declared that \((1, 100)\) is strictly better than \((0, 103)\) in a reference-free manner, let us offer her the option of choosing “no mug and $103” again. Given that her current status quo is \((1, 100)\), and

\(^{25}\)Here \(u\), which measures the significance of gain/loss of mugs, exhibits constant loss aversion, while \(v\), which measures that of gain/loss of money, exhibits strictly diminishing sensitivity.
we know already that this bundle is better than \((0,103)\) for her even when \((1,100)\) is not a status quo, the status quo bias phenomenon (in the form considered above) would maintain that the decision maker would remain with her current endowment. Surprisingly, the model at hand would predict quite the opposite of this:

\[
U_{(1,100)}(1,100) = 0 < 2(-1) + 3^{0.8} = U_{(1,100)}(0,103).
\]

The TK-model thus predicts here that being a status quo decreases the value of the bundle \((1,100)\) in the eyes of our agent relative to the “reference-free” value of this alternative. Clearly, this prediction goes against the basic premise of the status quo bias phenomenon.  

Example 7 shows that the TK-model is, in general, not consistent with the basic premise of status quo bias, and certain specifications of it may even exhibit status quo aversive behavior. While this model may provide a good way of capturing the choice behavior of loss averse individuals, the same does not appear to be true for the choice behavior of decision-makers with status quo bias. More generally, it seems like the commonly held contention that “loss aversion implies status quo bias” is suspect, at least insofar as loss aversion is modeled through the classical TK-model.

**Remark.** The observation above stems from the fact the diminishing sensitivity (for losses) may at times act counter to loss aversion. It is, however, generic in the sense that for any TK-model such that \(u_i|_{\mathbb{R}^+}\) is strictly concave on some interval (for some \(i\)) we can find such a violation of the status quo bias phenomenon. In other words, the only special case of the TK-model that is globally consistent with the basic premise of the status quo bias phenomenon is the one of constant loss aversion (in which each \(u_i|_{\mathbb{R}^+}\) and \(u_i|_{\mathbb{R}^-}\) are assumed to be linear). But, clearly, this version of the TK-model is simply too restrictive to serve as a canonical model of choice with status quo bias. In particular, it fails the law of diminishing marginal utility (for every good involved). Indeed, all parametric estimations of this model (that we are aware of) exhibit some degree of diminishing sensitivity.

This observation suggests that the behavioral implications of the phenomena of loss aversion and status quo bias may well be distinct. This is indeed the case when loss aversion is modeled through the TK-model and status quo bias through the choice

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26 While the content of Example 7 seems novel, we should note that it was Sagi (2006) who has first found that the loss aversion models cannot avoid such adverse consequences (due to the kinks they involve in the associated indifference curves). A related difficulty, namely, the possibility of cyclical choices under the TK-model, was pointed out by Munro and Sugden (2003). (That is, an agent that behaves according to this model may strictly prefer \(y\) to \(x\) when endowed with \(x\), and strictly prefer \(z\) to \(y\) when endowed with \(y\), and finally, \(x\) to \(z\) when endowed with \(z\).)

27 In recent experimental work, Masatlioglu and Uler (2008) have directly compared the explanatory power of the constant loss aversion model with that of the general T-K model. The discrepancy that they have found is substantial.
model of Theorem 1. These two models may have similar qualitative implications in some economic contexts, to be sure. For instance, in the context of the equity premium puzzle and the disposition effect (as well as the pricing anomaly in housing markets considered in Section 4.4), these two models appear in concert, suggesting that these anomalies may be due either to loss aversion or status quo bias. However, there are economic instances where the predictions of these two models diverge, as we illustrate next.

Example 8. (The TK–Model and the Endowment Effect) We consider the setting introduced in Section 4.1 when comparing the status quo bias phenomenon with the endowment effect. Let \( c \) be the choice correspondence in this context that arises through the maximization of an endowment-dependent utility function as in the TK-model. Then, labelling \( u_1 \) and \( u_2 \) as \( u \) and \( v \), respectively, we have

\[
WTA(c) = \inf\{a \geq 0 : u(w_o + a - w_o) + v(-1) \geq 0\}
\]

(since \( u(0) = 0 = v(0) \)). As \( u \) is continuous and strictly increasing, therefore, \( u(WTA(c)) = -v(-1) \). Similarly,

\[
WTP(c) = \sup\{w_o \geq a \geq 0 : u(w_o - (w_o + a)) + v(1) \geq 0\},
\]

and it follows that \( u(-WTP(c)) = -v(1) \). Consequently, \( u(WTA(c)) = -v(-1) > v(1) = -u(-WTP(c)) > u(WTP(c)) \), where the strict inequalities follow from the hypothesis of loss aversion. Since \( u \) is strictly increasing, we find that \( WTA(c) > WTP(c) \). Conclusion: Every choice correspondence induced by the TK-model is subject to the endowment effect (with respect to every trade).

We have seen in Section 4.1 that status quo bias phenomenon (as formulated by WSQB) does not necessarily entail the endowment effect. This seems like an important observation from the behavioral point of view, suggesting again that one should regard these traits as different phenomena. It appears that loss aversion, but not the status quo bias phenomenon, provides sound behavioral foundations for the endowment effect. In particular, the finding that the endowment effect disappears in environments with repeated trades – see, for instance, List (2003, 2004) – entails the same for loss aversion but not for status quo bias.

Example 9. (The TK–Model and the Law of Demand) We consider the framework of Section 4.5, and as in Example 7, work with a particular specification of the TK-model in which the utility of the bundle \( y \in \mathbb{R}^2_+ \) for an agent whose initial endowment

\[28\]The type of behavior we demonstrated in Example 7 cannot occur according to the model derived in Theorem 1. To see this, note that “choosing \( x \) over \( y \) in the absence of an initial entitlement” is captured in our setup by the statement \( \{x\} = c(\{x, y\}, \emptyset) \), and “choosing \( y \) over \( x \) when \( x \) is the status quo” is even stronger than saying \( y \in c(\{x, y\}, x) \). But, if \( c \) were represented as in Theorem 1, the first of these statements would entail \( U(x) > U(y) \), and the second \( U(y) \geq U(x) \).
is \( x \), is \( U_x(y) = u(y_1 - x_1) + v(y_2 - x_2) \), where

\[
u(t) := \begin{cases} t, & \text{if } t \geq 0 \\ 2t, & \text{if } t < 0 \end{cases}
\quad \text{and} \quad v(t) := \begin{cases} t^\alpha, & \text{if } t \geq 0 \\ -2|t|^\alpha, & \text{if } t < 0 \end{cases}
\]

with \( 0 < \alpha < 1 \). (We consider here a parametric model to ensure that the following analysis does not pertain to a knife-edge scenario.) When the prices in the economy are given by \( p \in \mathbb{R}^2_+ \) with \( p_2 = 1 \), we assume that the optimal consumption choice of this individual is found upon maximizing \( U((0,0))(x) \) over the budget set \( \{ x \in \mathbb{R}^2_+: px \leq w \} \). In what follows, we shall adopt the following parametric restriction: \( p_1 < 2 < w^{1-\alpha} \). Under this condition, the solution to this problem, denoted by \( x(p,w) \), is found as \( x(p,w) = \left( \frac{1}{p_1}(w - (\alpha p_1)^\theta), (\alpha p_1)^\theta \right) \), where \( \theta := 1/(1-\alpha) \).

Consider next a decline in the price of good 1; the new price in the economy is \( q \in \mathbb{R}^2_+ \) where \( p_1 > q_1 \) and \( p_2 = q_2 = 1 \). As in Section 4.5, then, we set \( x(p,w) \) as the reference point of the individual. In the present context, this means that the consumer’s new choice problem is maximizing \( U_{x(p,w)}(x) \) over the budget set \( \{ x \in \mathbb{R}^2_+: qx \leq w \} \). We denote this solution by \( \hat{x}(q,w) \). Next, to perform the Slutsky analysis, we set \( w' := qx(p,w) \) so that the (hypothetical) choice problem of the agent (in which the income effect of the change of prices from \( p \) to \( q \) is eliminated) becomes maximizing \( U_{x(p,w)}(x) \) over all \( x \in \mathbb{R}^2_+ \) with \( qx \leq w' \). We denote the solution of this problem as \( \hat{x}(q,w') \). Clearly, what is required by the Law of Compensated Demand in this context is: \( \hat{x}_1(q,w') \geq x_1(p,w) \). That is, in the absence of the income effect, a decrease in the price of good 1 should result in increase of the consumption of that good. Yet, under a wide specification of the parameters at hand this does not happen. Indeed, whenever \( 1 < q_1 < p_1 < 2 < w^{1-\alpha} \), it can be shown by calculus that \( \hat{x}_1(q,w') < x_1(p,w) \), that is, the Law of Compensated Demand fails.

We conclude from Example 9 that the TK-model is not compatible with the Law of Compensated Demand in general, and it may well entail a positive substitution effect. Comparing this finding with Fact 1 shows that the implications of the status quo bias phenomenon (as captured by the choice model of Theorem 1) and that of loss aversion (as modeled by the TK-model) may well be quite distinct in certain economic environments.

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29As in Example 7, we treat here the individual as acting without a status quo option at this point, which we try to capture within the TK-model upon setting the reference point as \( (0,0) \). The substance of the following demonstration would, however, remain intact (albeit, with a different parametric restriction) with any other choice of a reference point for the consumer.

30Straightforward calculations show that \( \hat{x}_1(q,w') < \hat{x}_1(q,w) < x_1(p,w) \) for some \( \epsilon > 0 \) where \( p_1 - \epsilon < q_1 < p_1 < 2 < w^{1-\alpha} \). Thus, the Law of Demand as well fails under this specification, that is, \( \hat{x}_1(q,w) < x_1(p,w) \). By consuming less of good 1 at the lower price \( q_1 \), the consumer treats this commodity as a Giffen good in this price range. Yet, the income effect is positive here, that is, \( \hat{x}_1(q,w') < \hat{x}_1(q,w) \), so good 1 is not inferior for the consumer. Thus, the TK-model maintains that the Law of Demand fails in this instance because of the substitution effect, not despite it, contrary to the intuition behind the “Giffen good phenomenon.”
6 Conclusion

In this paper, we adopted the revealed preference method to derive an individual decision making model that allows for an agent to make use of her status quo option as a reference point. This model is “canonical” in the sense that it applies to any choice problem independently of the context. The classical canonical choice model, which posits that choices arise due to utility maximization, is realized as a special case of our model. In fact, the choice model proposed here extends the “utility maximization” paradigm to one of “constrained utility maximization,” where constraints are induced, psychologically, by one’s initial endowments.

An axiomatic characterization of a choice model provides a complete list of behavioral postulates that allows one to test that model directly. As we have noted earlier, our main behavioral axiom, namely, WSQB, has already been tested in the literature. This is, however, a rather conservative formulation of the status quo bias phenomenon. It is of interest to see if stronger formulations of this phenomenon, such as the SSQB property we used in Section 3, would be supported or falsified in related experiments. Similarly, our model is based on the SQI property, which, loosely speaking, says that status quo bias would not exist with undesirable endowments. This property should be tested in future experimental work.

The ultimate arbiter of the success of a choice model is, of course, its performance in the context of economic applications. In Section 4 we have gone through several such applications to illustrate the potential of the model developed here. However, much is left for future empirical research. One thing that stands out about the Tversky-Kahneman model of loss aversion, or more generally, the prospect theory, is its impressive ability to explain, and thus unify, seemingly distinct behavioral anomalies in a variety of economic contexts. It is too early to tell if the model of Theorem 1 will perform as well in this regard.

Finally, we note that there are many directions in which the models introduced in this paper can be extended. We have assumed here the feasibility of the status quo options at all times, so our models cannot be applied to, say, job search problems of an individual who has lost her current (status quo) job. Similarly, we have not considered here how choice under risk and ambiguity can be modeled in the presence of initial endowments, nor we have anything to say currently on how to extend our models to dynamic environments. Much work remains to be done in future research.

Appendix

Proof of Theorem 1

We first prove the “if” part of the theorem. Let $U : X \to \mathbb{R}$ be a continuous function and $Q$ be a closed-valued self-correspondence on $X$, and take any choice correspondence $c$ on $C(X)$ that satisfies (1) and (2) for any $(S, x) \in C_{sq}(X)$. It is obvious that $c$ satisfies WARP. To show that $c$ also satisfies WSQB and SQI, we need to make the following simple observation:
Claim 1.1. \( x \in Q(x) \) for every \( x \in X \).

Proof of Claim 1.1. As \( c \) is a choice correspondence on \( C(X) \), we must have \( c(\{ x \}, x) = \{ x \} \) for any \( x \) in \( X \). Our claim thus follows from (2).

Now take any \( x, y \in X \), and suppose that \( y \in c(\{ x, y \}, x) \). By (2) and Claim 1.1, we must have \( U(y) \geq U(x) \), which, by (1), is equivalent to \( y \in c(\{ x, y \}, \emptyset) \) as we sought. On the other hand, if \( y \in c(\{ x, y \}, \emptyset) \), that is, \( U(y) \geq U(x) \), then, as \( y \in Q(y) \) by Claim 1.1, (2) entails \( y \in c(\{ x, y \}, y) \), as we sought. Conclusion: \( c \) satisfies WSQB.

Let us now show that \( c \) satisfies SQI. Take any \( (S, x) \in C_{sq}(X) \), and suppose that \( \{ x \} \neq c(T, x) \) for every non-singleton subset \( T \) of \( S \) with \( x \in T \). If \( S \) is itself a singleton, then \( S = \{ x \} \), and we get \( c(S, x) = \{ x \} = c(S, \emptyset) \) by virtue of \( c \) being a choice correspondence. If \( S \) is not a singleton, then, by hypothesis, \( y \in c(\{ x, y \}, x) \), that is, \( y \in Q(x) \), for every \( y \in S \setminus \{ x \} \). Combining this fact with Claim 1.1, then, we find \( S \subseteq Q(x) \), that is, \( S \cap Q(x) = S \), and it follows from (1) and (2) that \( c(S, x) = c(S, \emptyset) \), as we sought.

It remains to prove that \( c \) satisfies C. But this is a straightforward consequence of the Berge Maximum Theorem and the closed-valuedness of \( Q \).

We now move to prove the “only if” part of Theorem 1. Let \( c \) be a choice correspondence on \( C(X) \) that satisfies WARP, WSQB, SQI and C. Define the binary relation \( \succeq \) on \( X \) by

\[ y \succeq x \quad \text{if and only if} \quad y \in c(\{ x, y \}, \emptyset). \]

A standard argument, based on WARP, shows that \( \succeq \) is a complete preorder on \( X \). It is also easy to see that \( C \) ensures that \( \succeq \) is continuous. Indeed, for any \( x \in X \) and any convergent sequence \( (y_m) \) in \( X \) such that \( y_m \succeq x \) for each \( m \), we have \( \{ x, y_m \} \rightarrow \{ x, y \} \), where \( y := \lim y_m \). Thus, it follows from \( C \) that \( y \in c(\{ x, y \}, \emptyset) \), that is, \( y \in Q(x) \). Therefore:

\[ c(S, \emptyset) = \arg \max \{ U(\omega) : \omega \in S \} \quad \text{for every } S \in \Omega_X. \] (8)

We now define the self-correspondence \( Q \) on \( X \) by

\[ Q(x) := \{ y \in X : y \in c(\{ x, y \}, x) \}. \]

Claim 1.2. \( Q \) is closed-valued.

Proof of Claim 1.2. Pick any \( x \in X \), and let \( (y_m) \) be a sequence in \( Q(x) \) with \( y_m \rightarrow y \) for some \( y \in X \). Then, \( y_m \in c(\{ x, y_m \}, x) \) for each \( m \), so, by \( C \), we find \( y \in c(\{ x, y \}, x) \), that is, \( y \in Q(x) \).

Claim 1.3. For every \( (S, x) \) in \( C_{sq}(X) \), we have

\[ c(S, x) = c(S \cap Q(x), x). \]

Proof of Claim 1.3. Let \( T := S \cap Q(x) \), and pick any \( y \in c(S, x) \). By WARP, we have \( y \in c(\{ x, y \}, x) \), which means that \( y \in Q(x) \), and hence \( y \in T \). Conclusion: \( c(S, x) \subseteq T \). Therefore, \( c(S, x) \cap T \) equals \( c(S, x) \), which ensures that it is a nonempty set. We may thus apply WARP to conclude that \( c(S, x) = c(S, x) \cap T = c(T, x) \), and we are done.

Claim 1.4. For every \( (S, x) \) in \( C_{sq}(X) \), we have
Proof of Claim 1.4. Take any \((S, x)\) in \(C_{\text{sq}}(X)\), and let \(T\) be the collection of all subsets \(T\) of 
\(S \cap Q(x)\) such that \(|T| > 1\). We wish to show that \(\{x\} \neq c(T, x)\) for every \(T \in T\). Indeed, for any 
given \(T \in T\), there exists a \(w\) in \(T\), distinct from \(x\), such that \(w \in Q(x)\), that is, \(w \in c(\{x, w\}, x)\). 
Therefore, if \(c(T) = \{x\}\) were the case, WARP would imply 
\[
\{x\} = c(T) \cap \{x, w\} = c(\{x, w\}, x) \ni w,
\]
that is \(x = w\), a contradiction. Conclusion: \(c(T, x) \neq \{x\}\) for any \(T \in T\). This observation allows 
us to apply SQI to conclude that (9) is valid.

Combining Claims 1.3 and 1.4, we find \(c(S, x) = c(S \cap Q(x), \omega)\), and hence, by (8), 
\[
c(S, x) = \arg \max \{U(\omega) : \omega \in S \cap Q(x)\},
\]
for every \((S, x) \in C_{\text{sq}}(X)\). In view of Claims 1.2, the proof of Theorem 1 is complete.

Proofs of Theorems 2 and 3

Given Theorem 1, the “if” parts of Theorems 2 and 3 are fairly straightforward, so we solely 
concentrate on the “only if” parts of these result. Let \(c\) be a rational choice correspondence on 
\(C(X)\) that satisfies WARP, SSQB and SQI. Define the map \(U\) and the self-correspondence \(Q\) on \(X\) 
as in the proof of Theorem 1.

Claim 2.1. For any \(x \in X\), 
\[
U(y) > U(x) \quad \text{for every } y \in Q(x) \setminus \{x\}.
\]

Proof of Claim 2.1. Take any distinct \(x\) and \(y\) in \(X\) with \(y \in Q(x)\). Then, \(y \in c(\{x, y\}, x)\), and 
hence we have \(\{y\} = c(\{x, y\}, \omega)\) by SSQB. The definition of \(U\), therefore, entails that \(U(y) > U(x)\).

Claim 2.2. \(Q \circ Q \subseteq Q\).

Proof of Claim 2.2. Take any \(x, y, z \in X\) with \(y \in Q(x)\) and \(z \in Q(y)\). We wish to show that 
\(z \in Q(x)\). We may assume that \(x, y\) and \(z\) are distinct, otherwise \(z \in Q(x)\) obtains trivially. In that 
case, by Claim 2.1, we have 
\[
U(z) > U(y) > U(x).
\]

Now, set \(S := \{x, y, z\}\), and suppose \(x \in c(S, x)\). By (10), this implies that \(U(x) \geq U(\omega)\) for every 
\(\omega \in S \cap Q(x)\). In particular, \(U(x) \geq U(y)\), contradicting (11). On the other hand, if \(y \in c(S, x)\), then 
\(\{y\} = c(S, y)\) by SSQB. By WARP, then, \(\{y\} = c(\{y, z\}, y)\), contradicting \(z \in Q(y)\). Conclusion: 
\(\{z\} = c(S, x)\). By WARP, then, \(z \in c(\{x, y\}, x)\), that is, \(z \in Q(x)\), as we sought.

The proof of Theorem 2 is complete at this point. To proceed with the proof of Theorem 3, we 
assume henceforth that \(X\) is finite, and define the binary relation \(\succ\) on \(X\) as follows: 
\[
y \succ x \quad \text{if and only if } \quad y \in Q(x).
\]

The following claim identifies the main properties of this binary relation.

Claim 2.3. \(\succ\) is a partial order on \(X\) such that, for any \(x\) and \(y\) in \(X\), 
\[
y \succ x \neq y \quad \text{implies } U(y) > U(x).
\]

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Proof of Claim 2.3. As \( x \in Q(x) \) for every \( x \in X \), the binary relation \( \succcurlyeq \) is reflexive. By Claim 2.2, it is transitive as well. The antisymmetry of \( \succcurlyeq \) follows from (12), while (12) is a restatement of Claim 2.1.

We now recall the following result from order theory.

Lemma A. Let \( X \) be a nonempty finite set, and let \( \succeq \) be a complete preorder on \( Y \). Suppose \( \succsim \) is a reflexive binary relation on \( X \) such that

\[
y \succsim x \neq y \implies \text{not}(x \succeq y)
\]

for any \( x, y \in X \). Then, there exists a map \( \alpha : X \rightarrow \mathbb{R} \) such that \( \alpha(\omega) < 0 \) for every \( \omega \in X \), and

\[
y \succeq x \quad \text{if and only if} \quad \sum \{ \alpha(\omega) : \omega \succsim y \} \geq \sum \{ \alpha(\omega) : \omega \succsim x \}
\]

for every \( x, y \in X \).

Proof of Lemma A. This is Lemma 3 of Kreps (1979), and is easily proved by mathematical induction. \( \square \)

In view of Claim 2.3, we may apply Lemma A to the complete preorder represented by \( U \) and the partial order \( \succcurlyeq \) on \( X \) to find a map \( \alpha : X \rightarrow \mathbb{R} \) such that

\[
U(y) \geq U(x) \quad \text{if and only if} \quad \sum \{ \alpha(\omega) : \omega \succcurlyeq y \} \geq \sum \{ \alpha(\omega) : \omega \succcurlyeq x \}.
\]

Now let \( k \) be the number of elements of \( X \), and enumerate \( X \) as \( \{x_1, \ldots, x_k\} \). We define the function \( u_i : X \rightarrow \mathbb{R} \) by

\[
u_i(z) := \begin{cases} \alpha(x_i), & \text{if } x_i \succcurlyeq z \\ 0, & \text{otherwise}\end{cases}
\]

for each \( i = 1, \ldots, k \). It follows from this definition that

\[
\sum_{i=1}^{k} u_i(z) = \sum \{ \alpha(\omega) : \omega \succcurlyeq z \} \quad \text{for every } z \in X.
\]

Therefore, we have

\[
U(y) \geq U(x) \quad \text{if and only if} \quad \sum_{i=1}^{k} u_i(y) \geq \sum_{i=1}^{k} u_i(x), \quad (13)
\]

for any \( x \) and \( y \) in \( X \).

To complete the proof of Theorem 3, take any two alternatives \( x \) and \( y \) in \( X \). Suppose \( y \succcurlyeq x \) and take any \( i \in \{1, \ldots, k\} \). If \( x_i \succcurlyeq y \), then \( x_i \succcurlyeq x \) as well (by transitivity of \( \succcurlyeq \)), and hence \( u_i(y) = u_i(x) \). If \( x_i \succcurlyeq y \) does not hold, then \( u_i(y) = 0 \geq u_i(x) \). Conclusion:

\[
y \succcurlyeq x \quad \text{implies} \quad u_i(y) \geq u_i(x) \quad \text{for every } i = 1, \ldots, k.
\]

Conversely, suppose \( u_i(y) \geq u_i(x) \) holds for every \( i = 1, \ldots, k \), but, to derive a contradiction, assume that \( y \succcurlyeq x \) is false. Suppose \( y \) corresponds to the \( j \)th alternative in the enumeration \( x_1, \ldots, x_k \), that is, \( y = x_j \). Then, as \( y \succcurlyeq x \) is false, we have \( u_j(x) = 0 \), and hence, as \( x_j \succcurlyeq y \) holds by reflexivity of \( \succcurlyeq \), we find \( u_j(y) = \alpha(x_j) < 0 = u_j(x) \), a contradiction. Conclusion:

\[
y \succcurlyeq x \quad \text{if and only if} \quad u_i(y) \geq u_i(x) \quad \text{for every } i = 1, \ldots, k. \quad (14)
\]

We now define the map \( u : X \rightarrow \mathbb{R}^k \) by \( u(\omega) := (u_1(\omega), \ldots, u_k(\omega)) \). Then, by definition of \( \succcurlyeq \) and (14), we have

\[
y \in Q(x) \quad \text{if and only if} \quad u(y) \geq u(x). \quad (15)
\]
Then, if $u(y) = u(x)$ were to hold for two distinct $x$ and $y$ in $X$, Claim 2.3 would have entailed that $U(y) > U(x)$, and hence, by (13), we would have found that $u_1(y) + \cdots + u_k(y) > u_1(x) + \cdots + u_k(x)$ which contradicts $u(y)$ and $u(x)$ being the same numbers. Therefore, $u(y) = u(x)$ can hold only if $x = y$, that is, $u$ is an injection. Finally, combining (13) and (15) with (1) and (2) completes the proof of Theorem 3.

**Independence of Axioms**

Pick an arbitrary integer $k \geq 3$, and set $X := \{0, 1, \ldots, k\}^2$. First, consider the choice correspondence $c$ on $C(X)$ defined as

\[ c(S, \Diamond) := \arg \max_{(i,j) \in S} i + j \quad \text{and} \quad c(S, x) := \arg \max_{(i,j) \in S \text{ and } (i,j) \geq x} 2i + j, \]

for every nonempty subset $S$ of $X$ and $x \in S$. (Here $\geq$ is the coordinatewise ordering on $X$.) It is easily checked that this choice correspondence satisfies WARP, WSQB and C, but not SQI. On the other hand, the choice correspondence $c$ on $C(X)$, defined as

\[ c(S, \Diamond) := \begin{cases} 
\{(0,0)\}, 
& \text{if } S = X \\
\arg \max_{(i,j) \in S} i + j, 
& \text{otherwise}
\end{cases} \quad \text{and} \quad c(S, x) := \{x\}, \]

for every nonempty subset $S$ of $X$ and $x \in S$, satisfies WSQB, SQI and C, but not WARP. Next, consider the choice correspondence $c$ on $C(X)$, defined as

\[ c(S, \Diamond) := \arg \max_{(i,j) \in S} i + j \quad \text{and} \quad c(S, x) := \begin{cases} 
\{x\}, 
& \text{if } x \neq 1 \\
\{0\}, 
& \text{if } x = 1 \text{ and } 0 \in S \\
\{1\}, 
& \text{otherwise.}
\end{cases} \]

(Here $0 := (0,0)$ and $1 := (1,1)$.) This choice correspondence satisfies WARP, SQI and C, but not WSQB. Finally, set $X := [0,1]^2$ and define the map $u : X \to \{0,1\}$ by $u(x) = 1$ if $x = 1$. Then, the choice correspondence $c$ on $C(X)$, defined as $c(S, x) := \arg \max u(S)$ for every nonempty subset $S$ of $X$ and $x \in S \cup \{\Diamond\}$, satisfies WARP, WSQB and SQI, but not C. Conclusion: The axioms used in Theorem 1 are logically independent.

As WSQB can be interchanged with SSQB in the previous paragraph, these examples also establish the logical independence of the axioms used in Theorems 2 and 3.

**References**


