ABSTRACT. Costs from volatility have largely been ignored in the optimal taxation literature because the unique characteristics of the U.S. federal government make these costs negligible. However, for other governments, especially state governments, volatility is a real and important cost. This paper demonstrates theoretically the importance of considering volatility, in both public and private consumption, for governments setting tax rates. Optimally governments tradeoff costs from volatility and deadweight loss, but this paper shows that, of these two considerations, only volatility is of first-order importance. The magnitude of the costs from volatility is estimated using a model calibrated to the United States using data from 1970 - 2010. Although volatility is of first-order importance and deadweight loss is of second-order importance, either of these two costs could have had a larger magnitude than the other. The results from the calibrated model demonstrate that the magnitude of the cost from volatility is larger than the cost due to deadweight loss. In terms of private consumption, the magnitude of the cost from setting tax rates ignoring the costs of volatility is $600 billion. Therefore, volatility is of utmost importance for policy makers to consider when setting tax policy.

JEL Numbers: H21, H7, H68, R51
Recent increases in tax revenue volatility, especially at the state level in the United States, have led to an increased discussion of the impact of volatility on optimal taxation, optimal levels of public goods, and societal welfare. Tax policy has been shown to be an important mechanism for explaining the increase in state tax revenue volatility in the 2000s, even considering the significant increases in economic volatility and important changes in tax bases (Seegert, 2012a). This paper quantifies the importance of considering tax revenue volatility when setting tax policy in two ways. First, following Harberger’s 1954 paper which shows deadweight loss is of second-order importance, I show the cost of volatility is of first-order importance with respect to a tax rate change. Second, by calibrating a stochastic general equilibrium model the welfare cost of policy-makers ignoring volatility while setting tax policy is estimated to be $600 billion per year, which is four times greater than the cost of ignoring deadweight loss.

The government has two concerns when considering the optimal response to tax revenue volatility. First, the government must consider how to distribute the underlying production risk in the economy. The government could employ lump-sum taxes, but this concentrates risk in private consumption. By taxing different state-dependent bases, such as income or consumption, the government can instead absorb some of the production risk in the public good. Diversifying the risk between public and private consumption is welfare improving and thus, lump-sum taxes are not efficient when production risk exists.

Second, the government must consider the balance between tax bases. By taxing different bases the government can hedge some of the idiosyncratic risk associated with a given tax base. The ability of the government to hedge idiosyncratic risk depends on the variance-covariance matrix of the available tax bases.

The government must tradeoff these concerns of volatility with the cost of deadweight loss caused by imposing taxes that distort people’s behavior. In this tradeoff between volatility and

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1 The production risk is modeled as shocks to technology, which affect both the wage and profit received by the representative individual. The shock, and therefore the wage and profit, is unknown at the time the government makes its decision.
deadweight loss, deadweight loss is of second-order importance. In contrast, volatility is of first-order importance. Harberger’s 1954 paper demonstrates that deadweight loss is of second-order importance by taking a Taylor expansion of the difference between expenditure functions before and after a tax rate change. Volatility is demonstrated to be of first-order importance by taking a Taylor expansion of the difference between expected utility functions before and after a tax rate change in a manner similar to Harberger (1954).

The order of importance characterizes the likelihood a small deviation from the optimum will cause a welfare loss. There will be no loss in welfare due to a sufficiently small deviation for costs that are second-order importance. In contrast, for costs that are of first-order importance even small deviations from the optimum will cause welfare losses.

Although volatility is of first-order importance and deadweight loss is of second-order importance, first-order costs are not always larger in magnitude than second-order costs. Second-order costs can become large in magnitude if the deviation is large and the objective function is relatively nonlinear. For this reason, a simple model is calibrated to quantify the costs of volatility and deadweight loss. The model, calibrated to the United States from the years 1970-2010, demonstrates that the cost of volatility is larger than the costs due to deadweight loss.

1. Model

This section presents the model in this paper using the parameterized functions from the calibrated model. However, for the analysis of first and second order importance the functions are left general. Each period begins with the government’s choice of tax rates and the provision of the public good. Second, nature chooses the production state of the world. Third, the representative individual chooses her labor supply and consumption. Finally, production and utility are realized. The model is discussed using backwards induction, starting with the realization of production and utility and ending with the government’s optimal choice of tax rates and public good provision.
A. Production

An intermediate good is transformed without cost into public and private consumption. The intermediate good is produced with labor, $L$, and a production technology, $\theta$, according to the production function in equation (1). The production function exhibits decreasing returns to scale with respect to labor, $\gamma < 1$, and constant returns to scale with respect to production technology. Production technology is subject to transitory and persistent shocks according to equation (2).

\begin{equation}
  x = f(\theta_t, L_t) = \theta_t L_t^\gamma
\end{equation}

\begin{equation}
  \theta_t = \mu_t + \varepsilon_t \quad \mu_t = \phi \mu_{t-1} + (1 - \phi) \bar{\mu} + v_t \quad \varepsilon \sim \text{Log}\mathcal{N}(0, \sigma^2_\varepsilon), \quad v \sim \text{Log}\mathcal{N}(0, \sigma^2_v), \quad \sigma_{\varepsilon,v}
\end{equation}

These shocks affect the wage and profit according to equation (3). The $\omega$ parameter determines the extent to which wages are subject to transitory shocks.\(^2\) In this way the variance of the wage is allowed to differ from the variance of profits. In addition, the correlation between wages and profits are determined by $\omega$. Wages and profits are perfectly correlated when $\omega$ is equal to one and can be positively or negatively correlated when $\omega$ differs from one.\(^3\) The calibrated model estimates $\omega$ using data on the correlation and variances of wage and profit income in the United States.

B. Individual Behavior

The representative individual has log utility over the amount of labor to supply, $L$, public consumption, $g$, and private consumption, $c$. Private consumption is divided into goods that are taxed, $\beta c$, and untaxed, $(1 - \beta)c$.\(^4\) Individuals maximize utility by choosing

\(^2\)Note that $\chi$ is determined mechanically from $f(L) = wL + \pi$.

\(^3\)The $\omega$ is a reduced-form parameter encompassing bargaining and other frictions in the labor market.

\(^4\)The budget constraint can be written as $(1 - \tau_c)wL + \pi = (1 + t_c)c_1 + c_2$. First, make the consumption good substitutions; $c_1 = \beta c$ and $c_2 = (1 - \beta)c$. Second, rearrange the budget constraint such that the right hand side equals $c(1 - t_c \beta)$. Third, define $\tau_c = t_c/(1 + t_c \beta)$ and substitute into the budget constraint. Finally, rearrange to get the budget constraint in the text.
their labor supply and division of private consumption captured by $\beta$. Individuals use their wage income, taxed at the rate $\tau_w$, and untaxed profit income, $\pi$, to pay for private consumption. The wage they receive is subject to production shocks known to the individual before she makes her labor supply decision.

\begin{equation}
U(c, \beta, L; g) = \alpha_1 \log(\beta c) + \alpha_2 \log((1 - \beta)c) + \alpha_3 \log(g) + \alpha_4 \log(1 - L)
\end{equation}

\begin{equation}
c = (1 - \tau_c \beta)[(1 - \tau_w)wL + \pi] = (1 - \tau_c \beta)y
\end{equation}

The individual optimization produces equations for labor and $\beta$ from the first-order conditions in equations (5) and (6). In equation (5) $U_2$, the derivative of utility with respect to $\beta$, is equal to zero when the consumption tax rate is zero. In this case there is no distortion between consumption goods because there is no consumption tax. When the consumption tax rate is not zero, the ratio of the marginal benefits of private consumption and $\beta$ is equal to $\tau_c y$, which is the additional tax revenue the government collects due to a marginal change in $\beta$. In the parameterized model, labor is a function of the income tax rate, the wage, profit, and utility parameters and $\beta$ is a function of the consumption tax rate and utility parameters.\(^5\)

\begin{equation}
\frac{U_2}{U_1} = \tau_c y
\end{equation}

\begin{equation}
\frac{-U_3}{U_2} = (1 - \tau_c \beta)(1 - \tau_w)w
\end{equation}

C. Government The government maximizes the expected value of the indirect utility function, found by substituting the equations for labor, $\beta$, and consumption into the representative individual’s utility function. Two assumptions are made for expository convenience: i) the supply of

\begin{align*}
L &= \frac{(\alpha_1 + \alpha_2)(1 - \tau_w)w - \alpha_4 \pi}{(\alpha_4 + \alpha_1 + \alpha_2)(1 - \tau_w)w} \\
\beta &= \frac{(\alpha_1 + \alpha_2)(1 + \tau_c) + \alpha_1 \tau_c - (-8\alpha_1(\alpha_1 + \alpha_2)\tau_c + ((\alpha_1 + \alpha_2)(1 + \tau_c) + \alpha_1 \tau_c)^2)^{1/2}}{4(\alpha_1 + \alpha_2)\tau_c}
\end{align*}
the public good is set equal to the tax revenue and ii) the utility function is additively separable such that \( U_{1.4} = 0 \). These assumptions allow the social welfare function to be written as in equation (7), where \( \bar{c} \) and \( \bar{R} \) are the mean levels of private and public consumption and \( \sigma^2_c \) and \( \sigma^2_R \) are the variances of private and public consumption respectively. The function \( M(\cdot) \) represents the expected utility from private consumption, including leisure, and the function \( G(\cdot) \) represents the expected utility from public consumption. The shape of these functions quantifies the costs from volatility, implicitly defining the risk attitudes of public and private consumption.

\[
E[U(\beta, c, L; g)] = \int U(c, \beta, L; g) f(\theta) \equiv M(\bar{c}, \sigma^2_c, \beta, L, \sigma^2_L) + G(\bar{R}, \sigma^2_R)
\]

\[
= \alpha_1 \log(\beta \bar{c}) + \alpha_2 \log((1 - \beta) \bar{c}) + \alpha_3 \log(\bar{g}) + \alpha_4 \log(1 - \bar{L}) - \frac{(\alpha_1 + \alpha_2) \sigma^2_{\log(c)}}{2} - \frac{\alpha_3 \sigma^2_{\log(g)}}{2} - \frac{\alpha_4 \sigma^2_{\log(L)}}{2}
\]

\[
\bar{R} = g = \tau_c \beta y + \tau_w w L
\]

\[
\sigma^2 = \tau_c^2 \beta^2 \sigma^2_y + \tau_w^2 \sigma^2_{wL} + 2(\tau_c \beta \tau_w \sigma_{wL,y})
\]

In general, the expected utility function is characterized by a function of the moments of the variables in the utility function (e.g. mean, variance, skewness). In the parameterized example the expected utility function \( M(\cdot) + G(\cdot) \) is reduced to a function of the first two moments because the shocks in the model are assumed to be log-normal. Therefore, the following section assumes the expected utility function is a function of the first two moments only, however the results are robust to including higher moments.

2. **Order of Importance of Volatility and Deadweight Loss**

This section considers the welfare costs due to tax rate changes on consumption goods. The producer price for good \( i \), \( p_i \), is assumed to be fixed and the consumer price for good \( i \) is assumed to equal \( q_i = p_i + t_i \), where \( t_i \) is the tax rate. The welfare costs can be broken into three

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6 Assuming that the government must have a balanced budget abstracts away from debt issues which are not the focus of this paper. This assumption may be less of an abstraction for state governments, forty-nine of which have balanced budget requirements. In practice these balanced budget requirements do not preclude state debt but they do add additional costs. In this model the ability of the government to smooth revenue is modeled in its risk attitude.

7 \( M_1 \geq 0, G_1 \geq 0, M_2 \leq 0, G_2 \leq 0 \)
parts; deadweight loss, income effects, and volatility. Deadweight loss is defined as the costs from individuals’ behavioral responses or the substitution effect. The income effects capture the change in utility due to shifting consumption between private and public consumption.\(^8\) Finally, the volatility costs captures the loss to risk-averse individuals of volatile private and public consumption.\(^9\)

The total loss function due to tax rate changes can be constructed as the difference between an individual’s expenditure functions, utility functions, or expected utility functions before and after the tax rate changes. The literature has focused on expenditure functions because it produces an approximation which can be empirically estimated however, this measure ignores costs to volatility which are captured by the expected utility function. Constructing the loss function as the difference in utility functions provides intuition, based on the envelope theorem, for the result that deadweight loss is of second-order importance.

The difference in expenditure functions before and after the tax rate changes is the additional income needed to compensate an individual for the tax rate changes (Harberger, 1964; Diamond and McFadden, 1974; and Green and Sheshinski, 1979). To isolate the deadweight loss the literature sets the income effects to zero by assuming the tax revenues collected are rebated lump-sum back to the individual. This is done by subtracting the tax revenue collected from the difference in the expenditure functions.

Harberger approximates this loss function using a Taylor series approximation and demonstrates deadweight loss can be estimated by the second-order term in the Taylor series expansion. This approximation is useful because the utility function would have to be known to estimate the loss with expenditure functions but, the second-order approximation depends only on the slope

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\(^8\)With lump-sum taxes the efficient split of consumption sets the marginal benefits of public and private consumption equal. If the consumption split is not efficient a shift of consumption toward the efficient split increases welfare and a shift away decreases welfare.

\(^9\)For example, with lump-sum taxes consumption volatility is concentrated in private consumption, however the government can distribute some of the volatility to public consumption by taxing state-dependent tax bases.
of the demand function. This result also implies deadweight loss is of second-order importance because there are no first-order terms in the Taylor series expansion due to deadweight loss.\footnote{There are first-order terms in the Taylor series expansion of the loss function corresponding to the income effects however, the literature assumes these effects cancel.} Hence, to a linear approximation of a small change in the tax rate there is no welfare loss due to deadweight loss.

The difference in expected utility functions before and after the tax rate changes is the additional expected utility needed to compensate an individual for the tax rate changes. For a risk-averse individual these costs include changes in the volatility of public and private consumption induced by the tax rate changes. The expected utility function captures this cost because it is a function of the variance of public and private consumption, as shown in the previous section.

As an intermediary step the welfare costs are constructed as the difference in utility functions. An application of the envelope theorem demonstrates deadweight loss is of second-order importance and that this result does not depend on the absence of other distortions or taxes in the economy. This contrasts with the fact that costs from volatility are of first-order importance, which is shown using the expected utility formula for loss.

2.1. **Expenditure Function Loss Function.** Deadweight loss can be written as the difference in expenditure functions for prices that exist before and after a tax change, $E(p + t, u) - E(p, u)$, minus the change in tax revenue collected, $T(p + t, p, u)$, where $p$ and $t$ are price and tax vectors. The second line in equation (8) approximates the loss function, $L(p + t, p, u)$, using a second-order Taylor series expansion.\footnote{This approximation is done in Harberger (1964), Diamond and McFadden (1974), and Green and Sheshinski (1979).} The first term in the second line of equation (8) is the first-order term of the Taylor series expansion. Using Shepard’s lemma the first-order term reduces to the quantity demanded multiplied by the tax rate which cancels with the tax revenue collected. Hence, the first-order term captures the income effect which is not part of deadweight loss. Deadweight loss is captured by the substitution effect which is approximated as the second-order
term of the Taylor series expansion given in line 3 of equation (8) and hence is of second-order importance. This derivation demonstrates deadweight loss can be approximated by a function of only the slopes of the compensated demand functions, suggested by Hotelling (1938), Hicks (1939), and Harberger (1964).

\[
L(p + t, p, u) = E(p + t, u) - E(p, u) - T(p + t, p, u)
\]

\[
(8)
\approx \frac{\partial E(p, u)}{\partial t}((p + t) - p) + \frac{1}{2} \frac{\partial^2 E(p, u)}{\partial t^2}((p + t) - p)^2 - T(p + t, p, u)
\]

\[
= -\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} s_{i,j} t_i t_j
\]

The cancelation of the first-order terms depends on the assumption that there were no previous tax distortions. In their 1979 paper Green and Sheshinski show that if there are taxes previous to a change in tax revenue there would be a first-order term representing the change in the previous tax revenue. Stiglitz and Dasgupta (1970) demonstrate the first-order term is negative if the tax rate change is on goods complementary to other taxed goods and positive if the taxed goods are substitutes.\(^{12}\) In these cases the first-order term does not represent deadweight loss, defined as the utility loss due to behavioral responses, but represents changes in tax revenue. To provide additional intuition for why the first-order effects are not due to behavioral responses the loss function is constructed in terms of utility functions.

2.2. **Utility Function Loss Function.** Deadweight loss can be constructed as the difference in utility before and after a tax change as in the first line in equation (18). The utility change from the change in public consumption, \(g\), due to the change in tax rates is given by \(U_3 \frac{\partial g}{\partial t}\). The

\[^{12}\text{The first-order term may also have different signs due to horizontal and vertical externalities. For example, the first-order term could be negative due to a negative vertical externality on federal income tax revenue caused by a change in the state level income tax rate.}\]
first-order Taylor series approximation is given in line 2 of equation (18).

\[
\hat{L} = U(x_2, y_2, g_2) - U(x_1, y_1, g_1)
\]

\[
\approx \left( U_1 \frac{\partial x_1}{\partial p} + U_2 \frac{\partial y_1}{\partial t} + U_3 \frac{\partial g_1}{\partial t} \right) \left( p + t_1 - p - t_2 \right)
\]

(9)

\[
= \Delta_t \left( U_1 s_x - U_1 \frac{\partial x_1}{\partial m} x_1 + U_2 s_y - U_2 \frac{\partial y_1}{\partial m} y_1 + U_3 \frac{\partial g_1}{\partial t} \right)
\]

Slutsky Decomposition

\[
= \Delta_t \left( U_1 s_x + U_2 s_y \right) + \Delta_t \left( U_3 \frac{\partial g_1}{\partial t} - U_1 \frac{\partial x_1}{\partial m} x_1 - U_2 \frac{\partial y_1}{\partial m} y_1 \right)
\]

Deadweight Loss

Income Effect = 0

\[
= \Delta_t U_1 \left( s_x + \frac{U_2}{U_1} - \frac{p x s_x}{p_x} \right) = \Delta_t U_1 (s_x - s_x) = 0
\]

The Slutsky decomposition given in line 3 of equation (18) separates the approximation into the income and substitution effects where \( s_x \) represents the derivative of the compensated demand for good \( x \). The income effect, given in line 4 of equation (18), compares the utility from public consumption, the first term in the income effect, and private consumption, the second and third terms in the income effect. In this case without volatility the income effect equals the FOC from the government’s optimization, therefore if the Taylor series approximation is taken around the optimum the income effect is zero. In this case without volatility if the Taylor series approximation is taken around the optimum the income effect is zero because the income effect equals the FOC from the government’s optimization.\(^{13}\)

The first-order approximation of deadweight loss (substitution effect) is zero for any set of tax rates not only the optimal set. Line 5 of equation (18) demonstrates the individual’s optimization, specifically the FOC \( \frac{U_2}{U_1} = 1/p_x \), causes the deadweight loss first-order term to be

\(^{13}\)The income effect is also zero if, following the literature, there is no public good and the tax revenues are rebated lump-sum back to the individual.
Therefore, deadweight loss being of second-order importance does not depend on the government’s optimization or on the absence of other distortions in the economy but only on the individual’s optimization.

An application of the envelope theorem in this case confirms the first-order term is zero. For example, plot the utility of the individual against the relative price. If the individual always consumes the old bundle regardless of the relative price, assuming she has enough income to buy the old bundle, the utility would be represented as a horizontal line in utility relative-price space. Separately plot the maximized utility for each relative price. This curve lies weakly above the horizontal line, touching at the old relative price which made the old bundle the optimal bundle. Therefore, by the envelope theorem there is no first-order effect from the relative price change.

2.3. Expected Utility Function Loss Function. Finally, the appropriate measure of the total loss resulting from tax rate changes include deadweight loss, income effects, and volatility. Constructing the total loss function as the difference in expected utility functions before and after the change in tax rates, given in equation (10), captures all three of these costs. As shown in the previous section, expected utility can be written as 

\[ M(\bar{c}_2, \sigma^2_{c,2}, \beta_2) + G(\bar{R}_2, \sigma^2_{R,2}) \]

where \( M(\cdot) \) is the expected utility in private consumption and \( G(\cdot) \) is the expected utility in public consumption. For risk-averse individuals these functions include higher moments. For simplicity, the expected utility functions have been restricted to the cases in which they can be fully characterized by their first two moments (mean and variance) but the results are robust to allowing for additional moments. The loss function is estimated using a first-order Taylor series approximation given in line 2 of equation (10) to demonstrate the first-order approximation of the cost from volatility.

\[ (p_2 - p_1)(M_3 \sigma^3_{c,2} + M_2 \sigma^2_{c,2} + G_2 \sigma^2_{R,2} + M_4 \sigma^4_{c,2} + G_4 \sigma^4_{R,2}). \]

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14 Line 5 of equation (18) totally differentiates the budget constraint to get \( s_y = -p_x s_x \) and uses the individual’s first order condition to get \( U_2/U_1 = 1/p_x \).

15 The individual does have enough income to always buy the old bundle if the tax revenue is returned to her with a lump-sum transfer.

16 Let the expected utility be given by 

\[ M(\bar{c}_2, \sigma^2_{c,2}, \beta_2, \Omega_{c,2}) + G(\bar{R}_2, \sigma^2_{R,2}, \Omega_{g,2}) \]

where \( \Omega_{c,2} \) and \( \Omega_{g,2} \) are vectors of higher moments of private and public consumption. The last line in equation (10) would then be \( (p_2 - p_1)(M_3 \sigma^3_{c,2} + M_2 \sigma^2_{c,2} + G_2 \sigma^2_{R,2} + M_4 \sigma^4_{c,2} + G_4 \sigma^4_{R,2}). \)
\[ L = M(\bar{c}_2, \sigma_{c,2}^2, \beta_2) + G(\bar{R}_2, \sigma_{R,2}^2) - M(\bar{c}_1, \sigma_{c,1}^2, \beta_1) - G(\bar{R}_1, \sigma_{R,1}^2) \]

\approx (p + t_1 - p - t_2)(M_2 \frac{\partial \sigma_{c,1}^2}{\partial p} + G_2 \frac{\partial \sigma_{R,1}^2}{\partial p} + M_3 \frac{\partial \beta_1}{\partial p} + M_1 \frac{\partial \bar{c}_1}{\partial p} + G_1 \frac{\partial \bar{R}_1}{\partial p})

The first-order term representing deadweight loss is given by the utility cost of the individual shifting between taxed and untaxed goods. In equation (10) this term is given by \( M_3 \frac{\partial \beta_1}{\partial p} \) which by the FOC of the individual given in equation (5) equals zero if \( \tau_c = 0 \). If \( \tau_c \neq 0 \) then \( M_3 \frac{\partial \beta_1}{\partial p} = \tau_c y \) which is a first-order term accounting for the change in tax revenue but not a cost from behavioral changes. Therefore, once again the first-order approximation of deadweight loss is zero because of the individual’s optimization.

In contrast, the first-order approximation of the cost from volatility given by the changes in the variances of private and public consumption is not zero. In equation (10) the costs from volatility are given by \( M_2 \frac{\partial \sigma_{c,2}^2}{\partial p} + G_2 \frac{\partial \sigma_{R,2}^2}{\partial p} \). At the optimum the government trades off costs from deadweight loss and volatility, hence the cost from volatility is not minimized because the government accepts a higher cost of volatility in order to lower the cost of deadweight loss. If deadweight loss did not exist the government would minimize the cost of volatility and the first-order term would be zero at the optimum. Even though the cost of deadweight loss is also not minimized at the optimum the first-order term for deadweight loss is still zero because of the individual’s optimization. In contrast, the individual’s optimization does not minimize the cost of volatility. Therefore, the cost from volatility is of first-order importance around the optimum and at any point at which the cost from volatility is not minimized.

Figures (1) and (2) demonstrate the intuition graphically for why volatility is of first-order importance and deadweight loss is of second-order importance. Figure (1) depicts utility with respect to \( \beta \). The Taylor series expansion is taken around the optimum values, represented by the peak of the concave function due to the individual’s optimization. Even if the Taylor series
expansion were taken around tax rates that were not optimal, the individual’s optimization would still cause the Taylor series expansion to be taken at values at the peak of the concave function. At the peak, the linear approximation to a change in $\beta$ is zero.

Figure (2) depicts the utility cost of volatility (captured by $M(\cdot, \sigma^2(\tau), \cdot, \cdot) + G(\cdot, \sigma_g^2(\tau))$) with respect to a tax rate. The cost is U-shaped meaning an increase in the tax rate decreases the cost of volatility for small values of the tax rate and increases the cost for large values. The Taylor expansion is taken around the optimum values, away from the nadir because the government trades off some additional cost to volatility for less deadweight loss. In this example, the linear approximation to a change in the tax rate is positive.

The linear approximation of deadweight loss and volatility depend on the slope at which the Taylor series approximation is taken. If the approximation is taken at the peak and nadir of the two curves the linear approximations of deadweight loss and volatility are both zero. In contrast, if the approximation is taken away from the peak and nadir of the two curves the linear
approximations of deadweight loss and volatility are both not zero. The individual’s optimization causes the approximation to be taken at the peak of the curve for deadweight loss, for any tax rates the approximation is taken around. In contrast, the individual’s optimization does not cause the approximation to be taken around the nadir of the cost of volatility. In addition, if the Taylor series approximation is taken around the optimal tax rates then by the government’s FOCs the approximation will not be around the nadir and hence, the cost of volatility is of first-order importance.

Figures (1) and (2) highlight the fact that the total cost of deadweight loss could be larger than volatility in magnitudes despite the fact that deadweight loss is of second-order importance and volatility is of first-order importance. In the following section a calibrated model is used to quantify and compare the magnitudes of the costs due to volatility and deadweight loss.
3. Calibrated Model

3.1. Volatility-unaware and volatility-conscious governments. This subsection calculates the first-order conditions for a government that does not take into account the costs of volatility ("volatility-unaware") and a government that does account for the costs of volatility ("volatility-conscious"). The volatility-unaware government maximizes utility of the representative individual. The government is constrained to collect an exogenously given level of expected revenue $g$. This constraint is a common constraint in the optimal taxation literature but it abstracts from the costs of volatility. In contrast, the volatility-conscious government maximizes the representative individual’s expected utility. In this case, the variances of public and private consumption enter the government’s objective function directly.

C.1 Volatility-Unaware Government. The volatility-unaware government sets the income and consumption tax rates to maximize utility subject to the constraint that government revenues, on average, equal an exogenous level of revenue $g$ used to produce the public good. The government maximizes the indirect utility function, which substitutes the equilibrium values for $\beta, L, c$ from the individual’s optimization into the utility function.

$$U(\tau_w, \tau_c; g) = \alpha_1 \log(\beta c) + \alpha_2 \log((1 - \beta)c) + \alpha_3 \log(g) + \alpha_4 \log(1 - L)$$

$$g = \tau_c \beta E[y] + \tau_w E[wL]$$

First-order conditions for the volatility-unaware government.

(11) \[ \partial \tau_c : \left[ \frac{\alpha_1}{\beta} - \frac{\alpha_2}{1 - \beta} - \frac{\alpha_1 + \alpha_2}{c} \tau_c y \right] \frac{\beta}{\partial \tau_c} - \frac{\alpha_1 + \alpha_2}{c} \beta y + \frac{\alpha_3}{g} \frac{\partial g}{\partial \tau_c} = 0 \]

The use of the envelope theorem in simplifying the first-order condition of the volatility-unaware government provides additional intuition for why deadweight loss is of second order importance. The effects of raising the consumption tax rate can be split into an income effect, transferring
income from the individual to the government, and a substitution effect due to changes in the individual’s consumption behavior, captured by $\partial \beta / \partial \tau_c$. The individual’s first-order condition with respect to $\beta$ is the term that multiplies $\partial \beta / \partial \tau_c$ causing this term to be zero, at least to a first-order approximation. Therefore, the welfare cost of raising the consumption tax rate due to behavioral changes in consumption is mitigated by the individual’s maximization and drops out of the first-order condition for the government.

$$\partial \tau_w : \frac{\alpha_3}{g} \frac{\partial g}{\partial \tau_w} = \frac{\alpha_1 + \alpha_2}{c} (1 - \tau_c \beta) wL - \frac{\alpha_1 + \alpha_2}{c} \frac{\partial L}{\partial \tau_w} \left[ \frac{\partial c}{\partial \tau_w} + \frac{\alpha_4}{L} \right] - \frac{\alpha_1 + \alpha_2}{c} \left[ \frac{\partial c}{\partial \pi} \frac{\partial \pi}{\partial \tau_w} - \frac{\partial c}{\partial w} \frac{\partial w}{\partial \tau_w} \right]$$

(12) $\frac{\alpha_3}{g} wL + \frac{\alpha_3}{g} \frac{\partial wL}{\partial \tau_w} + \frac{\alpha_3}{g} \frac{\partial y}{\partial \tau_w}$ $\quad$ Leakage $\quad$ Horizontal Externality $\quad$ GE effects

The typical considerations in optimal taxation are present in the first-order conditions for the volatility-unaware government. The first term on both sides of equation (12) represent the income effect, transferring income from the individual to the government weighted by the marginal utility of private and public consumption. The second term on the left hand side represents the leakage from the income transfer due to behavioral responses of the individual. The leakage increases the cost of providing the public good because some income is lost to both the individual and government when the government uses distortionary taxes to raise revenue. The third term on the left hand side captures the interplay between taxes, the horizontal externality. In this model, raising the income tax rate causes the individual to spend less, thus decreasing the consumption tax revenue. Finally, the second term on the right hand side captures the general equilibrium effects taxes have on wages and profits. The volatility-conscious government has these same considerations but also considers how tax rates change the volatility of public and private consumption.
C.2 Volatility-Conscious Government. The volatility-conscious government sets the income and consumption tax rates to maximize the expected utility of the representative individual. The government is constrained by a budget constraint and the resulting variance of public consumption. Notice the objective function for the government includes the variances of log labor and public and private consumption.

\[
E[U(\tau_w, \tau_c; g)] = \alpha_1 \log(\beta \bar{c}) + \alpha_2 \log((1-\beta)\bar{c}) + \alpha_3 \log(\bar{g}) + \alpha_4 \log(1-\bar{L}) - \frac{(\alpha_1 + \alpha_2)\sigma_{\log(c)}^2}{2} - \frac{\alpha_3 \sigma_{\log(g)}^2}{2} - \frac{\alpha_4 \sigma_{\log(L)}^2}{2}
\]

\[
g = \tau_c \beta \bar{y} + \tau_w \bar{wL} \quad \sigma_g^2 = \tau_c^2 \beta^2 \sigma_y^2 + \tau_w^2 \sigma_{wL}^2 + 2\tau_c \tau_w \beta \sigma_{y,wL}
\]

The first-order conditions for the volatility-conscious government includes the variances of public and private consumption and their derivatives with respect to the tax rates.\(^\text{17}\)

\[
\frac{\partial}{\partial \tau_c} : \frac{\alpha_3}{\bar{g}} \left( \frac{\beta + \tau_c}{\partial \tau_c} \right) \bar{y} - \frac{\alpha_3}{2(\bar{g}^2 + \sigma_g^2)} \frac{\partial \sigma_g^2}{\partial \tau_c} = \frac{\alpha_1 + \alpha_2}{c} \frac{\partial \bar{y}}{\partial \tau_c} + \frac{\alpha_1 + \alpha_2}{2(\bar{c}^2 + \sigma_c^2)} \frac{\partial \sigma_c^2}{\partial \tau_c}
\]

\[
\frac{\partial}{\partial \tau_w} : \frac{\alpha_3}{\bar{g}} \frac{\partial \bar{g}}{\partial \tau_w} - \frac{\alpha_3}{2(\bar{g}^2 + \sigma_g^2)} \frac{\partial \sigma_g^2}{\partial \tau_w} = \frac{\alpha_1 + \alpha_2}{c} \frac{\partial \bar{c}}{\partial \tau_w} + \frac{\alpha_1 + \alpha_2}{2(\bar{c}^2 + \sigma_c^2)} \frac{\partial \sigma_c^2}{\partial \tau_w} + \frac{\alpha_4}{2((1-\bar{L})^2 + \sigma_L^2)} \frac{\partial \sigma_L^2}{\partial \tau_w}
\]

In equation (13) \(\partial \sigma_c^2/\partial \tau_c\) is negative. In equation (13) \(\partial \sigma_g^2/\partial \tau_c\) can be positive or negative. This derivative is negative when the consumption tax is being used relatively less than the income tax. In this case the benefit from hedging income tax-specific risk is high. When \(\partial \sigma_g^2/\partial \tau_c\) is positive the additional terms to equation (13) are both negative. The relative magnitudes of these additional terms quantify the benefits of the government absorbing some of the production risk.

\[\text{17}\]
3.2. **Calibration.** A model period is calibrated to be one year in length. All parameters of the model are set internally using simulated generalized method of moments. There are four utility function parameters, $\alpha_1, \alpha_2, \alpha_3,$ and $\alpha_4,$ and five production function parameters, $\gamma, \mu, \sigma^2_\varepsilon, \sigma^2_u,$ and $\omega.$ The production function parameters define the level of output, share of output to labor and profit, and the variances of the technological shocks. These nine parameters are calibrated using simulated method of moments.

The first moment is the share of private consumption that is taxable, captured in the model by $\beta$. Mikesell (2012) estimates the average taxable share of private consumption to be 46.7 percent between 1970 and 2010 at the state level. In contrast, he finds the taxable share to be 34.5 percent in 2010, representing a significant decrease in the consumption tax base. He also finds considerable heterogeneity across states in their average tax base between 1970 and 2010. Massachusetts has the smallest average tax base, at 27.2 percent, and Hawaii has the largest, at 106 percent. The baseline calibration uses 46.7 percent to constrain $\beta$ and the sensitivity analysis considers $\beta \in [27.2, 106].$

The literature surveyed by Domeij and Floden (2006) on the Frisch labor supply elasticity suggests a range between 0 and 0.5 although the authors argue these estimates are likely to be biased downwards by up to 50 percent suggesting a range between 0 and 1. Kimball and Shapiro (2008) estimate a Frisch elasticity of close to 1, which has been used in other public finance calibrations (e.g., House and Shapiro (2006) and Uhlig and Trabandt (2011)). For the log utility specification used in this paper, a Frisch elasticity close to 1 implies a labor supply of 0.5 because $\eta_{\text{Frisch}} = 1/L - 1.$ The second moment in the baseline calibration uses a labor supply of 0.5.

The Bureau of Economic Analysis reports that state and local expenditures are 11 percent of gross domestic product. In the model, gross domestic product is given by the sum of public and private consumption. Therefore the third moment sets the ratio of public consumption to total
consumption to be .11 and characterizes $\alpha_3$ relative to $\alpha_1$ and $\alpha_2$. The fourth moment normalizes the utility parameters to be shares by setting their sum to one.

The following five moments calibrate production in the model using data from the Bureau of Economic Analysis. First, wage income per person is calculated to be $15,214.59, which is the mean wage income from the Bureau of Economic Analysis Personal Income and Outlays section in real terms chained to 2005 dollars. Similarly, profit income is calculated to be $8860.90 per person. These two moments inform the values of the labor share of production, $\gamma$, and the level of technology state, $\mu$. The variance and covariance of wage and profit income are used as moments to calibrate the variance of the shocks $\sigma_u^2$ and $\sigma_\epsilon^2$ and the parameter $\omega$. The wage income coefficient of variation (the ratio of the standard deviation to the mean) is 0.034. The coefficient of variation for profit income is 0.083, meaning income from profits is more volatile than wage income. The correlation between wage and profit income is 0.227.

All but one simulated moment, listed in Table 1, is within 3 percent of its target. Five of the nine simulated moments are within 1 percent of their target: the labor supply, utility normalization, the coefficient of variation for wage and profit income, and the correlation between wage income and profits. The share of government expenditures to total consumption is 8 percent below its target simulated moment and is the furthest moment from its target. The simulated wage income is 1.8 percent higher than its target and the simulated profit is 1.8 percent below its target. Finally, $\beta$ is 2.2 percent higher than its target.

The parameters from the calibration are given in Table 2. When the consumption tax rate is zero $\beta$ equals the ratio of $\alpha_1$ to the sum of $\alpha_1$ and $\alpha_2$. Therefore the nondistorted $\beta$ is 0.496, which is 6 percent larger than the target distorted $\beta$. The labor production share, $\gamma$ is close to its stylized fact value of 0.66. The coefficient of variation of the production technology is 0.0017 which implies that a one standard deviation shock to production is 0.17 percent (less than one
Table 1. Calibrated Targets and Moments

<table>
<thead>
<tr>
<th>Moment</th>
<th>Symbol</th>
<th>Target Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Share private consumption</td>
<td>$\beta$</td>
<td>0.467 0.4775</td>
</tr>
<tr>
<td>Labor supply</td>
<td>$L$</td>
<td>0.5000 .5000</td>
</tr>
<tr>
<td>Share public consumption</td>
<td>$g/(c + g)$</td>
<td>0.11 0.1013</td>
</tr>
<tr>
<td>Utility normalization</td>
<td>$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4$</td>
<td>1 1</td>
</tr>
<tr>
<td>Wage income</td>
<td>$wL$</td>
<td>$15, 214$ $15, 490$</td>
</tr>
<tr>
<td>Profit income</td>
<td>$\pi$</td>
<td>$8, 860$ $8, 700$</td>
</tr>
<tr>
<td>Coefficient of variation wage income</td>
<td>$c_v(wL)$</td>
<td>$\sigma_{wL}/\bar{wL}$ 0.0340 0.0341</td>
</tr>
<tr>
<td>Coefficient of variation profit income</td>
<td>$c_v(\pi)$</td>
<td>$\sigma_\pi/\bar{\pi}$ 0.083 0.0826</td>
</tr>
<tr>
<td>Correlation of wage and profit income</td>
<td>$\sigma_{\pi, wL}/(\sigma_\pi \sigma_{wL})$</td>
<td>0.2265 0.2266</td>
</tr>
</tbody>
</table>

Table 2. Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Meaning</th>
<th>Value</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>Share taxable consumption</td>
<td>0.2945</td>
<td>Mikesell (2012)</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>Share untaxable consumption</td>
<td>0.2992</td>
<td>Mikesell (2012)</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>Share public consumption</td>
<td>0.0421</td>
<td>Government expenditures (BEA)</td>
</tr>
<tr>
<td>$\alpha_4$</td>
<td>Share leisure</td>
<td>0.3642</td>
<td>Frisch elasticity</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Share labor</td>
<td>0.6406</td>
<td>Wage and profit income (BEA)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Production technology</td>
<td>37,692</td>
<td>Wage and profit income (BEA)</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>Persistent shock</td>
<td>1,148.8</td>
<td>Variance wage and profit income (BEA)</td>
</tr>
<tr>
<td>$\sigma_\epsilon^2$</td>
<td>Temporary shock</td>
<td>2,539.8</td>
<td>Variance wage and profit income (BEA)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Wage smoothing</td>
<td>0.0226</td>
<td>Correlation wage and profit income (BEA)</td>
</tr>
</tbody>
</table>

Therefore, the calibrated shocks in the model are moderate to small.

---

18Coefficient of variation equals standard deviation over mean. The standard deviation of the production technology shock is equal to the $\sigma_\theta = \sqrt{\sigma_v^2 + \sigma_\epsilon^2 + 2\sigma_u \epsilon}$. 

percent) of the mean production technology.
4. Results

4.1. **Calculating Welfare Costs.** This section quantifies the costs of volatility and compares it to deadweight loss with two separate methods. The first method compares utility with and without the specific cost. For example, the cost of volatility is quantified comparing utility with and without production risk. The second method compares utility given tax rates that are calculated considering different costs (e.g., with or without volatility costs and/or deadweight loss). The first method follows cost estimates from Richard Musgrave (1957), Martin Feldstein (2008), and others in tax incidence.\(^\text{19}\) The second method quantifies the costs from policy makers ignoring important aspects of optimal taxation, specifically deadweight loss and volatility.

Table 3 lists the six utilities calculated and the five comparisons of utilities that quantify the costs of volatility and deadweight loss. The first comparison quantifies the cost of volatility comparing utility with and without production risk ((\(U_1\)) and (\(U_2\)) respectively). In both cases

\(^{19}\text{Effects of Taxes on Economic Behavior}^*\) Feldstein, 2008.
the tax rates are set using the optimal tax rates calculated using the first-order conditions (13) and (14), but the tax rates differ because the setting differs. The second comparison quantifies the cost of deadweight loss comparing utility with lump-sum taxes \(U_3\) and utility with distortionary taxes \(U_2\), both in a setting without production risk. These comparisons are between utility with and without volatility in the first case, and between utility with and without deadweight loss in the second case.

The third comparison quantifies the cost of volatility comparing utility with the efficient tax rates (taking into account volatility costs and deadweight loss) and tax rates set considering deadweight loss only (ignoring the costs from volatility). The fourth comparison quantifies the cost of deadweight loss comparing utility with the efficient tax rates and tax rates set considering volatility costs only (ignoring deadweight loss). The tax rates in these two cases are calculated accounting for different sets of terms in the first-order conditions from the volatility-conscious government in equations (15) and (16).

\[
\begin{align*}
\partial \tau_c : & \quad \frac{\alpha_3}{g} y \tau_c \frac{\partial \beta}{\partial \tau_c} + \frac{\alpha_3}{g} \beta y - \frac{\alpha_3}{2(\bar{\sigma}_c^2 + \sigma_c^2)} \frac{\partial \sigma_c^2}{\partial \tau_c} = \frac{\alpha_1 + \alpha_2}{c} \beta y + \frac{\alpha_1 + \alpha_2}{2(\bar{\sigma}_c^2 + \sigma_c^2)} \frac{\partial \sigma_c^2}{\partial \tau_c} \\
\partial \tau_w : & \quad \frac{\alpha_3}{g} \tau_w \frac{\partial L}{\partial \tau_w} + \frac{\alpha_3}{g} \left[wL - \tau_w \frac{\partial w}{\partial \tau_w} + \tau_c \beta \frac{\partial y}{\partial \tau_w}\right] - \frac{\alpha_3}{2(\bar{\sigma}_w^2 + \sigma_w^2)} \frac{\partial \sigma_w^2}{\partial \tau_w} \\
& \quad = \frac{\alpha_1 + \alpha_2}{c} \left[wL - \frac{\partial c}{\partial \pi} \frac{\partial \pi}{\partial \tau_w} + \frac{\partial c}{\partial w} \frac{\partial w}{\partial \tau_w}\right] + \frac{\alpha_1 + \alpha_2}{2(\bar{\sigma}_c^2 + \sigma_c^2)} \frac{\partial \sigma_c^2}{\partial \tau_w} + \frac{\alpha_4}{2((1 - \bar{\sigma}_c^2)^2 + \sigma_c^2)} \frac{\partial \sigma_c^2}{\partial \tau_w} \\
& \quad + \frac{\alpha_1 + \alpha_2}{2((1 - \bar{\sigma}_c^2)^2 + \sigma_c^2)} \frac{\partial \sigma_c^2}{\partial \tau_w} + \frac{\alpha_4}{2((1 - \bar{\sigma}_c^2)^2 + \sigma_c^2)} \frac{\partial \sigma_c^2}{\partial \tau_w} \\
\end{align*}
\]

Each of these utility comparisons can be stated in terms of private consumption by finding the additional private consumption needed to provide the same level of utility. The calculation below demonstrates this calculation for utility with distortionary taxes \(U_2\) and utility with lump-sum taxes \(U_3\). The percentage of additional private consumption needed in the case of distortionary

\(\text{\footnotesize{20}}\)The first-order conditions (13) and (14) reduce to the first-order conditions (11) and (12) in the absence of production risk.
taxes to equal the utility with lump-sum taxes is given by $x$. A similar calculation is made for each of the utility comparisons.

$$U_2(c(1 + x), \beta, L; g) = U_3(c, \beta, L; g)$$

$$(\alpha_1 + \alpha_2) \log(1 + x) = U_3(c, \beta, L; g) - U_{distort}(c, \beta, L; g)$$

$$x = \exp \left( \frac{U_3(c, \beta, L; g) - U_2(c, \beta, L; g)}{(\alpha_1 + \alpha_2)} \right) - 1$$

$$\Rightarrow xc = U_3(c, \beta, L; g) - U_2(c, \beta, L; g)$$

Table 3 reports the welfare cost of volatility and deadweight loss for both the traditional and new methodologies. Using the traditional method, the welfare cost of volatility and deadweight loss are both approximately $600$ billion dollars a year. Using the new method, the welfare cost of volatility is $587$ billion dollars per year and the cost of deadweight loss is $113$ billion dollars per year. The difference between the optimal tax rates and the tax rates set while ignoring the costs of volatility is larger than the difference between the optimal tax rates and the tax rates set while ignoring the costs of deadweight loss. Because the deviation from the optimal tax rates caused by ignoring the costs of volatility is larger than the deviation from ignoring deadweight loss, the welfare cost is larger for volatility than deadweight loss.

While both methods quantify the costs of volatility and deadweight loss they ask fundamentally different questions. The traditional method asks what ‘would the benefit to society be if volatility and deadweight loss were eliminated’. The welfare estimates of approximately $600$ billion for volatility and deadweight loss demonstrate that both of these are significant costs in the economy. The new method asks what ‘would the benefit to society be from having policy makers set tax rates considering the costs of deadweight loss and volatility’. These welfare estimates demonstrate that policymakers should be more concerned with the costs of volatility than deadweight loss because the potential welfare costs from ignoring volatility are 4 times larger than the costs of ignoring deadweight loss.
The final comparison given in Table 3 quantifies the benefit of distributing the production risk across public and private consumption. This compares optimal tax rates, accounting for both volatility and deadweight loss, with lump-sum taxes, both in a setting with production risk. The optimal tax rates distribute some of the risk between public and private consumption but produce deadweight loss by using distortionary taxes. In contrast, lump-sum taxes concentrate the production risk in private consumption but produce no deadweight loss. The result demonstrates that the government’s optimal tax rates provide substantial, $225 billion per year, benefit over lump-sum taxes, reinforcing the result that volatility costs are larger in magnitude than deadweight loss.

4.2. Sensitivity Analysis. The sensitivity of the calibration to the weighting matrix, in the simulated method of moments, is determined by running the calibration with 1000 random weighting matrices where each weight is allowed to take on a value between 1 and 100. The penalty function for each of the moments differs with their relative weights. For example, allowing the weighting function to differ from the identity matrix (the baseline weighting function) by changing the first value on the diagonal to equal 100, as opposed to 1, increases the penalty function on only the first moment. The resulting simulated method of moments decreases the error between the simulated and target moment for the first moment at the cost of allowing other simulated moments to differ more from their targets. The simulated moments remained within 15 percent of their target in all of the 1000 random weighting functions used and the resulting calibrations were qualitatively similar.

The sensitivity of the calculated utilities to the calibration is determined by calculating the utilities 3000 times with varying calibration. The utilities are calculated using parameters drawn from a normal distribution with a mean equal to their calibrated value and a standard deviation.

---

21 The baseline calibration is determined by running simulated method of moments with the identity matrix as the weighting matrix.
equal to five percent of their calibrated value.\textsuperscript{22} The standard deviation of the resulting 3000 utility calculations is 1.2 percent of the baseline calculation. Therefore the calibration is relatively robust to the weighting matrix used and the utilities are relatively robust to errors in the calibration.

5. Conclusion

Costs from volatility have largely been ignored in the optimal taxation literature because the unique characteristics of the U.S. federal government make these costs negligible. However, for other governments, especially state governments, volatility is a real and important cost. This paper demonstrates theoretically the importance of considering volatility, in both public and private consumption, for governments setting tax rates. Optimally governments tradeoff costs from volatility and deadweight loss, but this paper shows that, of these two considerations, only volatility is of first-order importance.

The magnitude of the costs from volatility is estimated using a model calibrated to the United States using data from 1970 - 2010. Although volatility is of first-order importance and deadweight loss is of second-order importance, either of these two costs could have had a larger magnitude than the other. The results from the calibrated model demonstrate that the magnitude of the cost from volatility is larger than the cost due to deadweight loss. In terms of private consumption, the magnitude of the cost from setting tax rates ignoring the costs of volatility is $600 billion. Therefore, volatility is of utmost importance for policy makers to consider when setting tax policy.

This paper focuses on the tradeoff between volatility and deadweight loss; however, there are other important tradeoffs considered in the optimal taxation literature. One of these tradeoffs is distributional concerns across heterogenous individuals. The effect of tax revenue volatility \textsuperscript{22} The parameters are constrained in two ways; first, to be positive if necessary, and in the case of the utility parameters to sum to one.
across individuals in the income distribution may be heterogeneous depending on how governments respond to shocks to their revenue streams. Empirically, how governments respond to these shocks and the resulting distributional effects remain an open question in the literature. Therefore, although I’ve demonstrated the importance of tax revenue volatility in setting tax policy there are more aspects of the interplay between volatility and tax policy to be explored.
References


The first order conditions for the volatility-unaware government.

\[
\partial \tau_c : \left( \frac{\alpha_1}{\beta} - \frac{\alpha_2}{1-\beta} \right) \frac{\partial \beta}{\partial \tau_c} + \frac{\alpha_1 + \alpha_2}{c} \frac{\partial c}{\partial \tau_c} + \frac{\alpha_3}{g} \frac{\partial g}{\partial \tau_c} = 0
\]

\[
\partial \tau_w : \frac{\alpha_1 + \alpha_2}{c} \frac{\partial c}{\partial \tau_w} + \frac{\alpha_3}{g} \frac{\partial g}{\partial \tau_w} - \frac{\alpha_4}{I} \frac{\partial I}{\partial \tau_w} = 0
\]

\[
\partial \tau_c : \left( \frac{\alpha_1}{\beta} - \frac{\alpha_2}{1-\beta} \right) \frac{\partial \beta}{\partial \tau_c} - \frac{1}{c} \left( \beta + \tau_c \frac{\partial \beta}{\partial \tau_c} \right) y + \frac{\alpha_3}{g} \frac{\partial g}{\partial \tau_c} = 0
\]

\[
= 0 \text{ envelope theorem}
\]

\[
\frac{\alpha_3}{g} \left( \beta + \tau_c \frac{\partial \beta}{\partial \tau_c} \right) y = \frac{\alpha_1 + \alpha_2}{c} \beta y
\]

The use of the envelope theorem in simplifying the first order condition of the volatility-unaware government provides additional intuition for why deadweight loss is of second order importance. The effects of raising the consumption tax rate can be split into an income effect, transferring income from the individual to the government and a substitution effect due to changes in the individual’s consumption behavior, captured by \( \partial \beta \partial \tau_c \). The individual’s first order condition with respect to \( \beta \) is the term that multiplies \( \partial \beta \partial \tau_c \) causing this term to be zero, at least to a first order approximation. Therefore, the welfare cost of raising the consumption tax rate due to
behavioral changes in consumption is mitigated by the individual’s maximization and drops out of the first order condition for the government.

$$\partial \tau \frac{w}{c} : \frac{\alpha_1 + \alpha_2}{c} \left[ \frac{\partial c}{\partial I} \frac{\partial I}{\partial \tau_w} + \frac{\partial c}{\partial \pi} \frac{\partial \pi}{\partial \tau_w} + \frac{\partial c}{\partial w} \frac{\partial w}{\partial \tau_w} - (1 - \tau \beta) w I \right] + \frac{\alpha_3}{g} \frac{\partial g}{\partial \tau_w} - \frac{\alpha_4}{I} \frac{\partial I}{\partial \tau_w} = 0$$

$$\alpha_1 + \frac{\alpha_2}{c} \frac{\partial I}{\partial \tau_w} = 0$$ envelope theorem

$$\alpha_1 + \frac{\alpha_2}{c} \frac{\partial I}{\partial \tau_w} = \alpha_1 + \frac{\alpha_2}{c} \left[ \frac{\partial c}{\partial \pi} \frac{\partial \pi}{\partial \tau_w} + \frac{\partial c}{\partial w} \frac{\partial w}{\partial \tau_w} \right] - \alpha_1 + \frac{\alpha_2}{c} (1 - \tau \beta) w I + \frac{\alpha_3}{g} \frac{\partial g}{\partial \tau_w} = 0$$ GE effects

$$\alpha_1 + \frac{\alpha_2}{c} \left[ \frac{\partial c}{\partial \pi} \frac{\partial \pi}{\partial \tau_w} + \frac{\partial c}{\partial w} \frac{\partial w}{\partial \tau_w} \right] - \alpha_1 + \frac{\alpha_2}{c} (1 - \tau \beta) w I + \frac{\alpha_3}{g} \frac{\partial w I}{\partial \tau_w} + \frac{\alpha_3}{\tau_w} \frac{\partial I}{\partial \tau_w} = 0$$ Leakage

$$\alpha_1 + \frac{\alpha_2}{c} \left[ \frac{\partial c}{\partial \pi} \frac{\partial \pi}{\partial \tau_w} + \frac{\partial c}{\partial w} \frac{\partial w}{\partial \tau_w} \right] - \alpha_1 + \frac{\alpha_2}{c} (1 - \tau \beta) w I + \frac{\alpha_3}{g} \frac{\partial w I}{\partial \tau_w} + \frac{\alpha_3}{\tau_w} \frac{\partial I}{\partial \tau_w} = 0$$ Horizontal Externality

7. Appendix: Consumption Base Decomposition

In the text private consumption of the representative agent is decomposed into consumption that is taxed and consumption that is untaxed such that the fraction $\beta$ of total consumption is taxed and $(1 - \beta)$ is untaxed. This decomposition changes the variables from two consumption goods into total consumption and the fraction spent on taxable items. This section demonstrates the change of variables and its benefits.

First, start with two goods $B, N$ such that the consumption of $B$ is taxed and the consumption of $N$ is not taxed and the representative agent has utility $U(B, N)$ over the two goods. By definition $B = \beta c$ and $N = (1 - \beta) c$. The utility function can be written as a function of $\beta$ and $c$ by substituting these equations in for $B$ and $N$. The budget constraint is given below written

---

23If the utility function is homothetic then the utility function can be written as $U(B, N) = v(\beta) U(c)$ otherwise $U(B, N) = U(c, \beta)$. 
both as a function of $B$ and $N$ and $\beta$ and $c$.

$$W = (1 + \tau c)B + N$$

$$= (1 + \tau c)\beta c + (1 - \beta)c$$

$$= c(1 + \beta \tau c)$$

Now we want to know the welfare impact of a tax change. We can separate the impact into the income effect and the substitution effect where the substitution effect is the deadweight loss from the behavioral responses.

$$\frac{\partial U(B, N)}{\partial \tau c} = U_1 \frac{\partial B}{\partial \tau c} + U_2 \frac{\partial N}{\partial \tau c}$$

$$= U_1(S_{B, \tau c} - \frac{\partial B}{\partial W}B) + U_2(S_{N, \tau c} - \frac{\partial N}{\partial W}B)$$  Slutsky Decomposition

$$= \frac{U_1S_{B, \tau c} + U_2S_{N, \tau c}}{\text{Substitution Effect}} - \left( \frac{U_1}{W} \frac{\partial B}{\partial W} B + \frac{U_2}{W} \frac{\partial N}{\partial W} B \right)$$  Income Effect
The benefit of writing the utility in terms of $\beta$ and $c$ is that $U_1 \frac{\partial c}{\partial \tau_c}$ captures the income effect and $U_2 \frac{\partial \beta}{\partial \tau_c}$ captures the behavioral response and deadweight loss.

\[
\begin{align*}
- \left( U_1 \frac{\partial B}{\partial W} B + U_2 \frac{\partial N}{\partial W} B \right) &= U_1 \beta \frac{\partial c}{\partial \tau_c} + U_2 \frac{B(1 - \beta)}{c\beta} \frac{\partial c}{\partial \tau_c} \\
\text{Income Effect} &= U_1 \frac{c}{\tau_c} \\
\text{Income Effect} &= U_1 \beta \frac{\partial c}{\partial \tau_c} + U_2 (1 - \beta) \frac{\partial c}{\partial \tau_c} \\
&= U_1 \frac{c}{\tau_c}
\end{align*}
\]

The first equality holds because of the following.

\[
\frac{\partial B}{\partial W} = \frac{\partial \beta c}{\partial W}
\]

\[
= \beta \frac{\partial c}{\partial W}
\]

\[
= \frac{\beta}{1 + \tau_c \beta}
\]

where $c = \frac{W}{1 + \tau_c \beta}$

\[
= -\frac{\partial c}{\partial \tau_c} \frac{1}{c}
\]

where $\frac{\partial c}{\partial \tau_c} = -\frac{W \beta}{(1 + \tau_c \beta)^2} = -\frac{c \beta}{(1 + \tau_c \beta)}$
\[
\frac{\partial N}{\partial W} = \frac{\partial (1 - \beta)c}{\partial W}
\]

\[
= (1 - \beta) \frac{\partial c}{\partial W}
\]

\[
= \frac{(1 - \beta)}{1 + \tau c \beta} \quad \text{where} \quad c = \frac{W}{1 + \tau c \beta}
\]

\[
= -\frac{\partial c}{\partial \tau_c} \frac{(1 - \beta)}{\beta c} \quad \text{where} \quad \frac{\partial c}{\partial \tau_c} = -\frac{W \beta}{(1 + \tau c \beta)^2} = -\frac{c \beta}{(1 + \tau c \beta)}
\]

The last equality holds because of the following.

\[
U_1 = U_1 \frac{\partial B}{\partial c} + U_2 \frac{\partial N}{\partial c} = U_1 \beta + U_2 (1 - \beta)
\]
Now show the deadweight loss calculation.

\[
\Upsilon_1 S_{B, \tau_e} + \Upsilon_2 S_{N, \tau_e} = 0
\]

Substitution Effect

\[
U_2 \frac{\partial \beta}{\partial \tau_e} = \frac{\partial \beta}{\partial \tau_e} \left( U_1 \frac{c}{1 + \beta \tau_e} - \frac{U_2 c(1 + \tau_e)}{1 + \beta \tau_e} \right)
\]

\[
= \frac{\partial \beta}{\partial \tau_e} \left( U_1 \left[ \frac{c}{1 + \beta \tau_e} - \frac{U_2 c(1 + \tau_e)}{U_1 (1 + \beta \tau_e)} \right] \right)
\]

\[
= \frac{\partial \beta}{\partial \tau_e} \left( U_1 \left[ \frac{c}{1 + \beta \tau_e} - \frac{c}{1 + \beta \tau_e} \right] \right)
\]

\[
= 0
\]

where from totally differentiating the budget constraint \( \frac{\partial B}{\partial \beta} = \frac{c}{1 + \beta \tau_e} \), \( \frac{\partial N}{\partial \beta} = -\frac{c(1 + \tau_e)}{1 + \beta \tau_e} \), and from the individual’s optimization \( \frac{\Upsilon_2}{\Upsilon_1} = \frac{1}{1 + \tau_e} \).

\[
\frac{\partial B}{\partial \beta} = c + \beta \frac{\partial c}{\partial \beta}
\]

Total Differentiate B.C. \( 0 = (1 + \tau_e \beta)dc + \tau_e cd\beta \)

\[
= c - c \frac{\beta \tau_e}{1 + \beta \tau_e}
\]

\[
dc/d\beta = -\tau_e c / (1 + \tau_e \beta)
\]

\[
= \frac{c}{1 + \beta \tau_e}
\]
\[
\frac{\partial N}{\partial \beta} = -c + (1 - \beta) \frac{c}{\beta} \\
= -c - c \frac{(1 - \beta) \tau_c}{1 + \beta \tau_c} \\
= -\frac{c(1 + \tau_c)}{1 + \beta \tau_c}
\]

8. Appendix: Deadweight Loss

This appendix produces the deadweight loss calculations in the text.

Harberger Expenditure Function

\[
L(p + t, p, u) = E(p + t, u) - E(p, u) - T(p + t, p, u)
\approx E(p + t, u) + \frac{\partial E(p + t, u)}{p}((p + t) - (p + t)) + \frac{1}{2} \frac{\partial^2 E(p + t, u)}{p}((p + t) - (p + t))^2
- E(p + t, u) + \frac{\partial E(p + t, u)}{p}(p - (p + t)) + \frac{1}{2} \frac{\partial^2 E(p + t, u)}{p}(p - (p + t))^2
- T(p + t, p, u)
\]

\[
= \frac{\partial E(p + t, u)}{p}((p + t) - p) + \frac{1}{2} \frac{\partial^2 E(p + t, u)}{p}((p + t) - p)^2 - T(p + t, p, u)
= \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} s_{i,j} t_i t_j
\]

Utility Representation
\( \hat{L} = U(x_2, y_2) - U(x_1, y_1) - G(R_1, R_2) \)
\[
\approx \left( U_1 \frac{\partial x_1}{\partial p} + U_2 \frac{\partial y_1}{\partial p} \right) (p_2 - p_1) - G(R_1, R_2)
\]
\[
= (p_2 - p_1) \left( U_1 s_x + U_1 \frac{\partial x_1}{\partial m} x_1 + U_2 \frac{\partial y_1}{\partial m} y_1 \right) - G(R_1, R_2)
\]
Slutsky Decomposition
(18)
\[
= (p_2 - p_1)(U_1 s_x + U_2 s_y) + (p_2 - p_1)(U_1 \frac{\partial x_1}{\partial m} x_1 + U_2 \frac{\partial y_1}{\partial m} y_1) - G(R_1, R_2)
\]
Substitution Effect
\[
= (p_2 - p_1)U_1(s_x + \frac{U_2}{U_1} s_y) = (p_2 - p_1)U_1(s_x - s_x) = 0
\]
Total derivative budget constraint

Expected Utility Representation
\[
\bar{L} = M(c_2, \sigma_{c,2}^2, \beta_2) + G(R_2, \sigma_{R,2}^2) - M(c_1, \sigma_{c,1}^2, \beta_1) - G(R_1, \sigma_{R,1}^2)
\]
\[
\approx M(c_2, \sigma_{c,2}^2, \beta_2) + G(R_2, \sigma_{R,2}^2) + (p_2 - p_1) \left( M_1 \frac{\partial c_2}{\partial p} + M_2 \frac{\partial \sigma_{c,2}^2}{\partial p} + M_3 \frac{\partial \beta_2}{\partial p} + G_1 \frac{\partial R_2}{\partial p} + G_2 \frac{\partial \sigma_{R,2}^2}{\partial p} \right)
\]
\[
- M(c_2, \sigma_{c,2}^2, \beta_2) + G(R_2, \sigma_{R,2}^2) + (p_1 - p_2) \left( M_1 \frac{\partial c_2}{\partial p} + M_2 \frac{\partial \sigma_{c,2}^2}{\partial p} + M_3 \frac{\partial \beta_2}{\partial p} + G_1 \frac{\partial R_2}{\partial p} + G_2 \frac{\partial \sigma_{R,2}^2}{\partial p} \right)
\]
\[
= (p_2 - p_1) \left( M_1 \frac{\partial c_2}{\partial p} + M_2 \frac{\partial \sigma_{c,2}^2}{\partial p} + M_3 \frac{\partial \beta_2}{\partial p} + G_1 \frac{\partial R_2}{\partial p} + G_2 \frac{\partial \sigma_{R,2}^2}{\partial p} \right)
\]
\[
= (p_2 - p_1) \left( \frac{\partial \beta_2}{\partial p} + M_2 \frac{\partial \sigma_{c,2}^2}{\partial p} + G_1 \frac{\partial R_2}{\partial p} + G_2 \frac{\partial \sigma_{R,2}^2}{\partial p} \right)
\]
Risk effect

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