1. Basic Principle of Counting

2. Generalized Basic Principle of Counting

3. **Permutation**: \( n \) distinct objects can be rearranged in \( n! \) Ways

4. **Permutation**: \( n \) objects of which \( n_1 \) are alike; \( n_2 \) are alike; ......; \( n_r \) are alike; such that \( n_1 + n_2 + n_2 + ....... + n_r = n \). Then there are \[ \frac{n!}{(n_1!)(n_2!)(n_3!) \ldots \ldots (n_r!)} \] permutations of these \( n \) objects.

5. Number of ways of selecting \( r \) objects from \( n \) objects and the order of selection is **relevant** is \[ \binom{n}{r} (n-1\cdot(n-2) \ldots \ldots \ldots (n-r+1) = \frac{n!}{(n-r)!} \]

6. Number of ways of selecting \( r \) objects from \( n \) objects and the order of selection is **NOT relevant** is \[ \binom{n}{r} = \frac{n!}{(n-r)! \cdot r!} \]

7. **Binomial Expansion**: 
\[ (x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} \]

8. A set of \( n \) distinct items is to be divided into \( r \) distinct groups of respective sizes \( n_1, n_2, n_3, \ldots, n_r \), where \( \sum n_i = n \). This can be done in \[ \binom{n}{n_1, n_2, n_3, \ldots, n_r} = \frac{n!}{(n_1!)(n_2!)(n_3! \ldots \ldots (n_r)!} \] ways

9. **Multinomial Expansion**: 
\[ (x_1+x_2+\ldots+x_r)^n = \sum_{n_1+n_2+\ldots+n_r=n} \binom{n}{n_1, n_2, \ldots, n_r} x_1^{n_1} x_2^{n_2} \ldots x_r^{n_r} \]
10. There are \( \binom{n-1}{r-1} \) distinct positive integer-valued vectors \( (x_1, x_2, \ldots, x_r) \) satisfying the equation

\[ x_1 + x_2 + \ldots + x_r = n \]

11. There are \( \binom{n+r-1}{r-1} \) distinct non-negative integer-valued vectors \( (x_1, x_2, \ldots, x_r) \) satisfying the equation

\[ x_1 + x_2 + \ldots + x_r = n \]
Summary of Important Formulas - Chapter 2 – Axioms of Probability

12. A Sample Space S is the set of ALL possible outcomes of an experiment.

13. An event E is ANY subset of Sample Space S

14. Basic Laws:
   a. Commutative Laws: \( E \cup F = F \cup E \) and \( E \cap F = F \cap E \) \((EF = FE)\)
   b. Associative Laws: \(( E \cup F ) \cup G = E \cup (F \cup G)\) and \(( E \cap F ) \cap G = E \cap ( F \cap G) \)
   c. Distributive Laws: \(( E \cup F ) \cap G = ( E \cap G ) \cup (F \cap G)\) and \(( E \cap F ) \cup G = ( E \cup G ) \cap ( F \cup G) \)

15. DeMorgan’s Laws:
   a. DML 1: \(( \bigcup_{i=1}^{n} E_i \bigcup c = \bigcap_{i=1}^{n} E_i ^c \bigcup \bigcup \bigcup \)
   b. DML 1: \((E \cup F ) \cap G = E \cap ( F \cap G)\) and \(( E \cap F ) \cup G = E \cup ( F \cup G) \)

16. Axioms of Probability:
   a. Axiom 1: \( 0 \leq P(E) \leq 1 \)
   b. Axiom 2: \( P(S) = 1 \)
   c. Axiom 3: If \( E_i \) are mutually exclusive events, for \( i = 1,2,\ldots, \infty \) (i.e. \( E_i \cap E_j = \phi \) for all \( i\neq j \)), then \( P( \bigcup_{i=1}^{\infty} E_i ) = \sum_{i=1}^{\infty} P(E_i) \)

17. Some Important Results:
   a. \( P( \emptyset ) = 0 \)
   b. If \( E_i \) are mutually exclusive events for \( i = 1 \) to \( n \), then \( P( \bigcup_{i=1}^{n} E_i ) = \sum_{i=1}^{n} P(E_i) \)
   c. If \( E \subseteq F \), then \( P( E ) \leq P( F ) \)
   d. \( P( E^c ) = 1 - P( E ) \)
   e. For any two \( E \) and \( F \), \( P( E \cup F ) = P( E ) + P( F ) - P( EF ) \)
f. For any three $E, F,$ and $G$, $P(E \cup F \cup G) = P(E) + P(F) + P(G) - P(EF) - P(EG) - P(FG) + P(EFG)$

g. Inclusion-Exclusion Identity

$$P(\bigcup_{i=1}^{n} E_i) = \sum_{i=1}^{n} (-1)^{i+1} \sum_{1 \leq i_1 < i_2 < \cdots < i_r} P(E_{i_1} E_{i_2} \cdots E_{i_r})$$

18. If $\{ E_n, \ n \geq 1 \}$ is an increasing sequence of events, then we define a new event

$$\lim_{n \to \infty} E_n = \bigcup_{i=1}^{\infty} E_i$$

If $\{ E_n, \ n \geq 1 \}$ is an decreasing sequence of events, then we define a new event

$$\lim_{n \to \infty} E_n = \bigcap_{i=1}^{\infty} E_i$$

19. Probability as a Continuous Function:

If $\{ E_n, \ n \geq 1 \}$ is either an Increasing or a Decreasing sequence of events, then

$$\lim_{n \to \infty} P(E_n) = P(\lim_{n \to \infty} E_n)$$