Textbook problems: Ch. 8: 8.6, 8.8

8.6 A resonant cavity of copper consists of a hollow, right circular cylinder of inner radius \( R \) and length \( L \), with flat end faces.

\( a) \) Determine the resonant frequencies of the cavity for all types of waves. With \( (1/\sqrt{\mu\epsilon R}) \) as a unit of frequency, plot the lowest four resonant frequencies of each type as a function of \( R/L \) for \( 0 < R/L < 2 \). Does the same mode have the lowest frequency for all \( R/L \)?

\( b) \) If \( R = 2 \) cm, \( L = 3 \) cm, and the cavity is made of pure copper, what is the numerical value of \( Q \) for the lowest resonant mode?

8.8 For the Schumann resonances of Section 8.9 calculate the \( Q \) values on the assumption that the earth has a conductivity \( \sigma_e \) and the ionosphere has a conductivity \( \sigma_i \), with corresponding skin depths \( \delta_e \) and \( \delta_i \).

\( a) \) Show that to lowest order in \( h/a \) the \( Q \) value is given by \( Q = Nh/(\delta_e + \delta_i) \) and determine the numerical factor \( N \) for all \( l \).

\( b) \) For the lowest Schumann resonance evaluate the \( Q \) value assuming \( \sigma_e = 0.1 \) \((\Omega m)^{-1}\), \( \sigma_i = 10^{-5} \) \((\Omega m)^{-1}\), \( h = 10^2 \) km.

\( c) \) Discuss the validity of the approximations used in part \( a) \) for the range of parameters used in part \( b) \).