Problem suggested in class: For what conditions does the following hold?

\[ E\ddot{x}u'(s + \dddot{x}) = 0 \Rightarrow Eu'(s + \dddot{x}) \leq u'(s) \]

A figure helps to establish the forbidden region (see figure 1 on next page).

Applying the diffidence theorem,

\[ Eu'(s + \dddot{x}) - u'(s) \leq mE\ddot{x}u'(s + \dddot{x}) \]

If true for all \( \dddot{x} \), then our necessary and sufficient condition is

\[ u'(s + x) - u'(s) \leq mxxu'(s + x) \]

Taking derivatives with respect to \( x \) yields

\[
\begin{align*}
  u''(s + x)|_{x=0} &= m[u'(s + x) + xu''(s + x)]|_{x=0} \\
  u''(s) &= m[u'(s)] \\
  m &= \frac{u''(s)}{u'(s)} \leq 0
\end{align*}
\]

Now take the third derivative:

\[
\begin{align*}
  u'''(s + x)|_{x=0} &\leq m[2u''(s + x) + xu'''(s + x)]|_{x=0} \\
  u'''(s) &\leq m * 2u''(s) \\
  u'''(s) &\leq 2u''(s) \frac{u''(s)}{u'(s)} \\
  \frac{u'''(s)}{u''(s)} &\geq \frac{2u''(s)}{u'(s)}
\end{align*}
\]

Which gives our local necessary condition. Plugging our solution to \( m \) back into the necessary and sufficient condition:

\[
\begin{align*}
  u'(s + x) - u'(s) &\leq xu'(s + x) \frac{u''(s)}{u'(s)} \\
  \frac{1}{u'(s)} - \frac{1}{u'(s + x)} &\leq x \frac{u''(s)}{[u'(s)]^2} \\
  \frac{1}{u'(s)} - \frac{x u''(s)}{[u'(s)]^2} &\leq \frac{1}{u'(s + x)}
\end{align*}
\]

As seen from the above equation, \( \frac{1}{u'(s + x)} \) must be locally convex. See figure 2 on the next page for a graphical representation. Thus for the initial statement to be true for all \( s \), \( \frac{1}{u'(s + x)} \) must be globally convex.