Exercise 82

Consider the following nonseparable intertemporal preferences:

\[ U(c_1, c_2) = \ln(c_1) + \beta \ln(c_2 - kc_1). \]

- Interpret \( k \) depending on whether it is positive or negative.
- Show that the optimal consumptions \( c_1 \) and \( c_2 \) are respectively decreasing and increasing with \( k \).

Solution 82

Part 1: If \( k \) is positive, then consumption is habit-forming, and \( k \) represents the degree of habit formation. This means that, in the second period you care about the growth in consumption. Intuitively, this would cause a consumer to consume less in the first-period, since they suffer a penalty from that consumption in the second-period.

If \( k \) is negative, then consumption is durable, and \( k \) represents the degree of durability. This means that you enjoy a boost in the second-period from the first-period consumption. Intuitively, in the two-period model, this will cause an increase in first-period consumption since you enjoy part of that consumption again in the second period.

Part 2: The solution requires explicating a budget constraint. Let \( c_1 + c_2 \leq \overline{C} \). It is clear that the constraint will bind. Substituting \( c_2 = \overline{C} - c_1 \), we can consider the unconstrained problem:

\[ \max_{c_1} \ln(c_1) + \beta \ln(\overline{C} - c_1 - kc_1). \]

This yields a basic first-order condition in terms of \( c_1 \). We know that this will define a global maximum by the strict concavity of the objective function:

\[ FOC : \frac{1}{c_1} + \frac{\beta (1 + k)}{\overline{C} - (1 + k)c_1} = 0 \Rightarrow \]

\[ \overline{C} - (1 + k)c_1 = -\beta (1 + k)c_1 \Rightarrow \]

\[ c_1 = \frac{\overline{C}}{(1 - \beta)(1 + k)}. \]

Denote this optimal first-period consumption as \( c_1^* \). We can determine the marginal change in \( c_1^* \) from a change in \( k \) by simple differentiation:

\[ \frac{\partial c_1^*}{\partial k} = \frac{\overline{C}}{(1 - \beta)(1 + k)^2} < 0. \]
The sign of this derivative follows directly, assuming that $\beta < 1$. A rise in $k$ causes a *decrease* in first-period consumption. Since the budget constraint will always bind, a rise in $k$ must therefore correspond to an *increase* in second-period consumption.