1. Winter, 2001

(a) If a function is differentiable, then it is continuous. TRUE FALSE

(b) If a function is continuous, then it is differentiable. TRUE FALSE

(c) If \( f'(x) \) is increasing, then \( f \) is concave up. TRUE FALSE

(d) If \( f''(x) = -3 \), then \( f \) is decreasing. TRUE FALSE

(e) If \( f \) has a critical point at \( x = 3 \), then \( f \) has a local maximum or a local minimum at \( x = 3 \). TRUE FALSE
2. Winter, 2002

(a) If $f'$ is increasing, then $f$ is increasing.  

(b) If $f$ is an exponential function, then $\frac{d}{dx} \ln f(x)$ is constant.  

(c) If $f''(x) = 0$ for all $x$, then $f$ is a constant function.  

(d) There is a function $f$ so that $f(x) > 0$, $f'(x) < 0$, and $f''(x) < 0$ for all $x$.  

(e) If $f''(x) < 0$ for all $x$, then $f(x) \leq f(0) + f'(0)x$.  

(f) If $f'(x) = 0$, then $f$ has either a relative maximum or a relative minimum at $x$.  

TRUE   FALSE  

TRUE    FALSE  

TRUE    FALSE  

TRUE    FALSE  

TRUE    FALSE  

TRUE    FALSE  

TRUE    FALSE
3. Fall 2003

(a) If \( x = 4 \) is a critical point of the function \( f \), then \( f'(4) = 0 \). TRUE FALSE

(b) If \( g'(x) < 0 \) for \( x < 3 \), \( g'(x) > 0 \) for \( x > 3 \), and \( g'(3) = 0 \), then \( g \) has a local minimum at \( x = 3 \). TRUE FALSE

(c) If \( f'(x) \) is defined for all \( x \), then \( f(x) \) is defined for all \( x \). TRUE FALSE

(d) It is possible to have a local minimum of \( f \) at \( x = c \) if \( f''(c) = 0 \). TRUE FALSE

(e) If \( f'(3) = 6.4 \) and \( g'(3) = 2.3 \), then the graph of \( f(x) - g(x) \) has a slope of 4.1 at \( x = 3 \). TRUE FALSE

(f) If \( f(x) \) is increasing for all \( x \), then \( f'(x) \) is increasing. TRUE FALSE

(g) For a revenue function, \( R \), and a cost function, \( C \), if \( R(q_0) > C(q_0) \) and \( MR < MC \) at \( q = q_0 \), a company would be advised to increase \( q \). TRUE FALSE

(h) The profit function is always maximized if marginal revenue equals marginal cost. TRUE FALSE
4. Winter, 2004

(a) Let \( f \) be a continuous function on the interval \([1, 10]\) and differentiable on \((1, 10)\). Suppose that \( f(5) = 3 \) and \( f(2) = 1 \). Then there is a point \( c \) in the interval \((2, 5)\) so that \( f'(c) = \frac{2}{3} \).

(b) If \( g(x) = \frac{1}{f(x)} \), then \( g'(x) = -\frac{1}{f'(x)f(x)} \).

(c) If \( a \) is a local maximum for the function \( f \) on the interval \([2, 50]\), then \( f'(a) = 0 \).

(d) If \( g(x) = f^{-1}(x) \), then \( g'(x) = (-1)f^{-2}(x) \).

(e) The 100th derivative of \( f(x) = x^5 + e^{2x} \) at \( x = 0 \) is \( 2^{100} \).

(f) If \( f(x) = (x-1)(x-2)(x-3)(x-4)(x-5)(x-6) \), then \( f'(x) = (x-1)+(x-2)+(x-3)+(x-4)+(x-5)+(x-6) \).

(g) If \( f \) is continuous on \([a, b]\), then \( f \) has a global maximum and a global minimum on that interval.
5. Winter, 2005

(a) The function $f'$ is continuous everywhere and changes from negative to positive at $x = a$. Which of the following must be true?

- $a$ is a critical point of $f$. TRUE FALSE
- $f(a)$ is a local maximum of $f$. TRUE FALSE
- $f(a)$ is a local minimum of $f$. TRUE FALSE
- $f'(a)$ is a local maximum. TRUE FALSE
- $f'(a)$ is a local minimum. TRUE FALSE

(b) A function $g$ is defined on all points of a closed interval. Which of the following must be true?

- $g$ must have both a global maximum and a global minimum. TRUE FALSE
- $g$ is differentiable on the interval. TRUE FALSE
- $g$ has no critical points. TRUE FALSE
- $g$ is continuous on the interval. TRUE FALSE
- None of the above statements must be true. TRUE FALSE

(c) For the graph of a cubic polynomial $ax^3 + bx^2 + cx + d, (a > 0)$, the signs of $f'(0)$, $f''(0)$ and $f'''(0)$ (respectively) could be which of the following? (Circle all that are possible.)

- $-, 0, +$ TRUE FALSE
- $-, 0, -$ TRUE FALSE
- $+, +, +$ TRUE FALSE
- $-, +, -$ TRUE FALSE
- $+, -, +$ TRUE FALSE
(d) The graph of $y = h(x)$ has a local max at $x = 3$ on the closed interval $[0, 5]$.
Which of the following must be true?

- $h'(3)$ is equal to zero or $h(3)$ is an end point.  
  TRUE  FALSE

- $h$ has a critical point at $x = 3$.  
  TRUE  FALSE

- $h''(3)$ is positive.  
  TRUE  FALSE

- $h''(3)$ is negative.  
  TRUE  FALSE

- None of the statements must be true.  
  TRUE  FALSE
(a) If \( f(x) \) is increasing, then \( f'(x) \) is increasing. \hspace{2cm} \text{TRUE} \hspace{1cm} \text{FALSE}

(b) Suppose \( f'(a) \geq f'(b) \) whenever \( a \leq b \). Then \( f \) has no points of inflection. \hspace{2cm} \text{TRUE} \hspace{1cm} \text{FALSE}

(c) If \( f(x) \) is defined for all \( x \) then \( f'(x) \) is defined for all \( x \). \hspace{2cm} \text{TRUE} \hspace{1cm} \text{FALSE}

(d) If \( f \) and \( g \) are functions whose second derivatives are defined, then \( (fg)' = fg'' + f''g \). \hspace{2cm} \text{TRUE} \hspace{1cm} \text{FALSE}

(e) If the radius of a circle is increasing at a constant rate, then so is the area. \hspace{2cm} \text{TRUE} \hspace{1cm} \text{FALSE}

(f) If \( f(x) \) has an inverse function, then the derivative of the inverse function is \( \frac{1}{f'(x)} \). \hspace{2cm} \text{TRUE} \hspace{1cm} \text{FALSE}

(g) If \( f'(1) = -3.4 \) and \( g'(1) = 4.1 \), then the function \( h(x) = f(x) + g(x) \) is increasing at \( x = 1 \). \hspace{2cm} \text{TRUE} \hspace{1cm} \text{FALSE}

(h) The graph of \( y = xe^{-0.1x} \) has an inflection point at \( x = 20 \). \hspace{2cm} \text{TRUE} \hspace{1cm} \text{FALSE}
7. Winter 2007

(a) If \( y(x) \) is a twice differentiable function, then \( \frac{d^2y}{dx^2} = \frac{(dy/dx)^2}{2} \).

(b) There exists a function \( f(x) \) such that \( f(x) > 0, f'(x) < 0, \) and \( f''(x) > 0 \) for all real values of \( x \).

(c) If \( h \) is differentiable for all \( x \) and \( h'(a) = 0 \), then \( h(x) \) has a local minimum or local maximum at \( x = a \).

(d) If \( f \) and \( g \) are positive and increasing on an interval \( I \), then \( f \) times \( g \) is increasing on \( I \).
8. Winter 2008

(a) Let \( x = c \) be an inflection point of \( f \). Assume \( f' \) is defined at \( c \).

- If \( L \) is the linear approximation to \( f \) near \( c \), then \( L(x) > f(x) \) for \( x > c \). \text{TRUE} \quad \text{FALSE}

- The tangent line to the graph of \( f \) at \( x = c \) is above the graph on one side of \( c \) and below the graph on the other side. \text{TRUE} \quad \text{FALSE}

(b) The differentiable function \( g \) has a critical point at \( x = a \).

- If \( g''(a) > 0 \), then \( a \) is a local minimum. \text{TRUE} \quad \text{FALSE}

- If \( a \) is a local maximum, then \( g''(a) < 0 \). \text{TRUE} \quad \text{FALSE}

(c) The derivative of \( g(x) = (e^x + \cos x)^2 \) is

- \( g'(x) = 2(e^x - \sin x)(e^x + \cos x) \). \text{TRUE} \quad \text{FALSE}

- \( g'(x) = 2e^{2x} + 2(e^x \cos x - e^x \sin x) \). \text{TRUE} \quad \text{FALSE}

(d) A continuous function \( f \) is defined on the closed interval \([a, b] \).

- \( f \) has a global maximum on \([a, b] \). \text{TRUE} \quad \text{FALSE}

- \( f \) has a global minimum on \([a, b] \). \text{TRUE} \quad \text{FALSE}

(e) Consider the family of functions \( e^{-(x-a)^2} \).

- Every function in this family has a critical point at \( x = 0 \). \text{TRUE} \quad \text{FALSE}

- Some function in this family has a local maximum at \( x = 2 \). \text{TRUE} \quad \text{FALSE}
(a) If $f$ is differentiable and $f'(p) = 0$ or $f'(p)$ is undefined, then $f(p)$ is either a local maximum or a local minimum.  TRUE  FALSE

(b) For $f$ a twice differentiable function, if $f'$ is increasing, then $f$ is concave up and increasing.  TRUE  FALSE

(c) The global maximum of $f(x) = x^2$ on every closed interval is at one of the endpoints of the interval.  TRUE  FALSE

(d) if $f(x)$ has an inverse function $g(x)$, then $g'(x) = 1/f'(x)$.  TRUE  FALSE

(e) If a function is periodic with period $c$, then so is its derivative.  TRUE  FALSE

(f) If $C(q)$ represents the cost of producing a quantity $q$ of goods, then $C'(0)$ represents the fixed costs.  TRUE  FALSE

(g) If a differentiable function $f(x)$ has a global maximum on the interval $0 \leq x \leq 10$ at $x = 0$, then $f'(x) \leq 0$ for $0 \leq x \leq 10$.  TRUE  FALSE

(h) If $f(x)$ is differentiable and concave up, then $f'(a) \leq \frac{f(b)-f(a)}{b-a}$ for $a < b$.  TRUE  FALSE

(i) If you zoom in with your calculator on the graph of $y = f(x)$ in a very small interval around $x = 10$ and see a straight line, then the slope of that line equals the derivative $f'(10)$.  TRUE  FALSE

(j) If $f'(x) \geq 0$ for all $x$, then $f(a) \leq f(b)$ whenever $a \leq b$.  TRUE  FALSE
(a) If \( j'(x) \) is continuous everywhere and changes from negative to positive at \( x = a \), then \( j \) has a local minimum at \( x = a \).  

(b) If \( f \) and \( g \) are differentiable increasing functions and \( g(x) \) is never equal to 0, then the function \( h(x) = \frac{f(x)}{g(x)} \) is also a differentiable increasing function.  

(c) If \( k \) is a differentiable function with exactly one critical point, then \( k \) has either a global minimum or global maximum at that point.  

(d) If \( F \) and \( F' \) are differentiable functions and \( F''(2) = 0 \), then \( F \) has a point of inflection at \( x = 2 \).  

(e) If \( f \) is a differentiable function with \( f(a) = b \) and \( f' \) is always positive, then \( f'(a) \left( (f^{-1})'(b) \right) = 1 \).
(a) Suppose that $f$ is a function whose second derivative is both continuous and positive everywhere. Then

$$f(2 + \Delta x) > f(2) + f'(2)\Delta x.$$  

(b) Suppose that $g$ is a continuous function and $g'$ is defined for all $x$. Then $g''$ is also defined for all $x$.

(c) If a continuous function $H$ has exactly one local maximum and two local minima, then there are exactly three distinct values of $x$ such that $H'(x) = 0$.

(d) Suppose that $A$ and $B$ are two continuous functions such that $A'(x) = B'(x)$ for all $x$. Then $A(x) = B(x)$ for all $x$.

(e) Suppose $P(x)$ is a continuous function satisfying $P'(x) = 0$ whenever $x > 0$. Then $P(a) = P(b)$ whenever $0 < a < b$.

(f) If the functions $R$ and $S$ are inverses of each other, then $R'$ and $S'$ are inverses of each other.
(a) If \( y = h'(x)m(x) - h(x)m'(x) \), then \( \frac{dy}{dx} = h''(x)m(x) - \frac{1}{h(x)m''(x)} \).  
TRUE FALSE

(b) If \( m''(a) = 0 \), then \( m(x) \) has an inflection point at \( x = a \).  
TRUE FALSE

(c) If \( h''(x) > 0 \) on the interval \([a, b]\) and \( h(a) > h(b) \), then \( h(a) \) is the absolute maximum value of \( h(x) \) on \([a, b]\).  
TRUE FALSE

(d) There exists a continuous function \( f(x) \) which is not differentiable at \( x = 0 \) with a local maximum at \((0, 5)\).  
TRUE FALSE

(e) The function \( g(x) = e^{-(x-a)^2/b} \) has a local maximum at \( x = b \).  
TRUE FALSE
13. Fall, 2011. Each part of this problem has four statements. For each part, circle TRUE for all statements which are always true and FALSE for all other statements.

(a) Let \( q(t) = A \cos(Bt) + C \sin(Bt) \), with \( A, B, \) and \( C \) constants.

- \( q''(t) = -B^2 q(t) \). TRUE FALSE
- The function \( q(t) \) is concave down everywhere. TRUE FALSE
- The value of \( q'(\frac{\pi}{2B}) \) is \( AB \). TRUE FALSE
- If \( q'(0) = \pi \) and \( C = 2 \), then \( q(t) = q(t + 4) \) for all values of \( t \). TRUE FALSE

(b) Let \( f(x) \) be a function defined on the closed interval \([0,4]\), such that \( f''(x) > 0 \) on the entire interval, and \( f'(x) \) is zero only at \( x = 3 \).

- \( f(1) > f(4) \). TRUE FALSE
- \( f'(1) < f'(3) \). TRUE FALSE
- The point \((3, f(3))\) is a local maximum. TRUE FALSE
- Either one or both of \( f(4) \) and \( f(0) \) are a global maximum. TRUE FALSE
(a) If \( f(x) \) is a function with a local maximum at \( x = c \), then \( f'(c) = 0 \).

(b) If \( g'(55) = g'(65) = 0 \), then \( g(x) \) is constant on the interval \( 55 \leq x \leq 65 \).

(c) The point \( (\pi, 1) \) is on the curve defined by the implicit function \( 5 \sin(xy) = \ln(y) \).

(d) The function \( A(x) = \frac{1}{R^2} \cos(Rx) + \frac{1}{2}x^2 \) has an inflection point at \( x = 0 \), where \( R \) is a nonzero constant.

(e) If \( h'(x) < 0 \) for all \( x \) in the interval \( [2, 8] \), then the global maximum of \( h(x) \) on that interval occurs at \( x = 2 \).
## Answers

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