The scattering length is a (the Bonnard limit). The threshold cross section is zero.

The scattering amplitude \( \langle \phi | \theta \rangle f \) approaches the Born approximation formula as \( \theta \to 0 \) while being finite at \( \theta = 0 \). The

\[
\left( \frac{e^{i\theta} + e^{\theta}}{8} \right) \frac{\theta}{9} = \frac{\Sigma_{\theta}^{\theta} - \theta}{9} = \frac{\Sigma_{\theta}^{\theta}}{9} \frac{\theta}{9} = \Sigma_{\theta}^{\theta} \frac{\theta}{9} \frac{\theta}{9} = \langle \phi | \theta \rangle f
\]

\[
\left( \frac{e^{i\theta} + e^{\theta}}{I} \right) \frac{\theta}{9} = \frac{\Sigma_{\theta}^{\theta} - \theta}{I} = \frac{\Sigma_{\theta}^{\theta}}{I} \frac{\theta}{9} = \langle \phi | \theta \rangle f
\]

The double internal factors. The first factor is an integral from the pure Coulomb case.

\[
\begin{aligned}
\Sigma_{\theta}^{\theta} \left( \frac{e^{i\theta} + e^{\theta}}{I} \right) \frac{\theta}{9} = \frac{\Sigma_{\theta}^{\theta}}{I} \frac{\theta}{9} = \langle \phi | \theta \rangle f
\end{aligned}
\]

The Born approximation formula.

\[
\Sigma_{\theta}^{\theta} \left( \frac{e^{i\theta} + e^{\theta}}{I} \right) \frac{\theta}{9} = \frac{\Sigma_{\theta}^{\theta}}{I} \frac{\theta}{9} = \langle \phi | \theta \rangle f
\]
The threshold total cross section is

\[ \sigma = \frac{\gamma}{\tan(\gamma)} \left. \frac{d\sigma}{d\gamma} \right|_{\gamma = 0} \]

The scatter length is

\[ + \frac{\gamma}{\phi(\theta)} f \]

As \( \gamma \to 0 \), the partial wave expansion (8.3) becomes

\[ \phi(\theta) f = \frac{\sin \theta}{\cos \theta + \sin \theta} \]

For the problem,

\[ \alpha_{\text{in}}(\theta) = \frac{\sin \theta}{\cos \theta + \sin \theta} \]

Outside the well, \( \alpha \) is proportional to \( \frac{\alpha_{\text{in}}(\theta)}{\alpha_{\text{in}}(\theta)} \)

Setting the well, \( \alpha \) is proportional to \( \alpha_{\text{in}}(\theta) \), so

The total cross section is

\[ \left( \frac{e^p}{\pi} + \varepsilon + \varepsilon g + P \right) \frac{\gamma + 1}{\gamma} \frac{\varepsilon g + 1}{\varepsilon} = \sigma \]

The integrated total cross section is

\[ \frac{e^p}{\pi} \int_{\varepsilon}^{\varepsilon_g} \frac{\gamma + 1}{\gamma} \frac{\varepsilon g + 1}{\varepsilon} = \sigma \]

Again, for large \( \gamma \), this becomes the Rutherford cross section.

\[ \left( \frac{e^p}{\pi} + \varepsilon + \varepsilon g + P \right) \frac{\gamma + 1}{\gamma} \frac{\varepsilon g + 1}{\varepsilon} \sigma = \sigma(\phi, \theta) f = \frac{1}{\varepsilon} \frac{\partial \sigma}{\partial \varepsilon} \]
\[
\tan \phi = \infty \Rightarrow \phi = 90^\circ. \text{ So when } \phi = \frac{\pi}{2}, \text{ the minimum value, } \phi_0 = \frac{\pi}{2}. \]  

Length is \( L = \sqrt{\frac{1}{\tan \phi_0}} \), where \( \phi_0 = \frac{\pi}{2} \).

**Problem 8.9** From Problem 8.7, the scatter length is \( A \) when \( \phi = 90^\circ \), so \( \phi_0 \in [0, \pi/2] \). The range of \( A \) is bounded between \( a \) and \( A \).

There is no value of \( A \) within this range that can be close to giving the observed value of \( A \).

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**Part 9**. The scatter length is

**Problem 8.9 A Neutron-Proton Scattering Model—1**

Bound states begin to appear.

For small \( \phi \), this is the same as the limit of the exact solution (8.8.10). But when \( \phi \) becomes comparable to \( 90^\circ/2 \), the Born approximation ceases to be good. That is the region where the pattern shows deviations from the Born approximation made in (8.8.10). But when \( \phi \) becomes comparable to \( 90^\circ/2 \), the Born approximation ceases to be good. That is the region where the pattern shows deviations from the Born approximation made in (8.8.10). But when \( \phi \)