The second-order term in the expansion of the state vector is

\[
\left(\langle \psi_0 | H - \mathbf{i} \nabla | \psi_0 \rangle \frac{\partial \mathcal{F}}{\partial \psi_0} \langle \psi_0 | \right) \sum + \left(\langle \psi_0 | \mathbf{i} \nabla | \psi_0 \rangle \frac{\partial \mathcal{F}}{\partial \psi_0} \langle \psi_0 | \right) \sum \psi_{\text{pert}} = \langle \psi_{\text{vac}} | \psi_0 \rangle
\]

and through second order

\[
\frac{\partial \mathcal{F}}{\partial \psi_0} \langle \psi_0 | \mathbf{i} \nabla \psi_0 \rangle \sum + \langle \psi_0 | H \mathbf{i} \nabla | \psi_0 \rangle = \psi_{\text{pert}}
\]

Through first order

\[
\frac{\partial \mathcal{F}}{\partial \psi_0} \langle \psi_0 | \mathbf{i} \nabla \psi_0 \rangle \sum - \langle \psi_0 | \psi_{\text{pert}} \rangle = \langle \psi_{\text{vac}} | \psi_0 \rangle
\]

and collect terms by order.

\[
(\cdots + \langle \phi | + \langle \psi | + \langle \phi | \mathbf{i} \nabla + \mathbf{i} \nabla + \langle \phi | \rangle \left( \mathbf{i} H - \mathbf{i} \nabla + \langle \phi | + \langle \psi | + \langle \phi | \rangle \left( \frac{\partial \mathcal{F}}{\partial \psi_0} \langle \psi_0 | \mathbf{i} \nabla \psi_0 \rangle \sum + \langle \psi_0 | \psi_{\text{pert}} \rangle = \langle \psi_{\text{vac}} | \psi_0 \rangle
\]

The equation (6.7) is expanded:

\[
\cdots + \langle \phi | + \langle \psi | + \langle \phi | \mathbf{i} \nabla + \mathbf{i} \nabla + \langle \phi | \rangle \sum = \psi_{\text{pert}}
\]

Expand:

\[
\langle \psi_{\text{pert}} | \mathbf{i} \nabla | \psi_{\text{pert}} \rangle = \psi_{\text{pert}}
\]

while from Equations (5) and (5.21), the level shift is

\[
\frac{\partial \mathcal{F}}{\partial \psi_0} \langle \psi_0 | \mathbf{i} \nabla \psi_0 \rangle \sum + \langle \psi_0 | \psi_{\text{pert}} \rangle = \langle \psi_{\text{vac}} | \psi_0 \rangle
\]

The states can be computed from

**States Approximation Methods for Bound**

**Chapter 7**

**Albers**

**HW Solutions**

**HW1 : Solution**
\[
(\zeta + \eta)(\xi + \zeta) = \frac{\text{map}_{\psi}}{\gamma \psi} = (\frac{\psi}{\xi} \zeta \psi^* + \frac{\psi}{\xi} \zeta^* \psi) \frac{\text{map}_{\psi}}{\gamma \psi} = \eta \xi + \eta \psi
\]

The last two terms contribute only when \( \psi = 0 \). The remaining term contributes to \( \langle \psi | \psi \rangle \).

The second-order shift of the zero energy level is

\[
\frac{\langle \psi | \psi \rangle}{\delta \xi} = \langle \psi | \psi \rangle \delta \xi = \eta \xi
\]

Problem 7.2: Harmonic Oscillator Perturbation Problem

The second-order shift of the first excited state is given by

\[
\frac{\langle \psi_{1,2} | \psi \rangle}{\delta \xi} = \langle \psi_{1,2} | \psi \rangle \delta \xi = \eta \xi
\]

The second-order shift of the second excited state is

\[
\frac{\langle \psi_{2,2} | \psi \rangle}{\delta \xi} = \langle \psi_{2,2} | \psi \rangle \delta \xi = \eta \xi
\]

The matrix elements needed are

The perturbation connects states whose levels differ by \( \eta \). The perturbation is

\[
\langle \psi \rangle \langle \psi | \psi \rangle = \langle \psi \rangle \delta \xi = \eta \xi
\]

The perturbation energy is\( \eta \xi \).

Problem 7.2: Oscillator with Cubic Perturbation

\[
\frac{\langle \psi | \psi \rangle}{\delta \xi} = \frac{\langle \psi | \psi \rangle}{\delta \xi} \frac{\langle \psi | \psi \rangle}{\delta \xi} = \langle \psi | \psi \rangle \delta \xi = \eta \xi
\]

Quantum
\[ \frac{\epsilon^m}{\mu} = \left[ \frac{u}{1} - \frac{1}{1} \sum_{i=0}^{\infty} \frac{u}{1} \sum_{i=0}^{\infty} \frac{u}{1} \sum_{i=0}^{\infty} \frac{u}{1} \right] \frac{u}{1} = \]

\[ \frac{\epsilon^m}{\mu} = \left[ \frac{u}{1} - \frac{u}{1} \sum_{i=0}^{\infty} \frac{u}{1} \sum_{i=0}^{\infty} \frac{u}{1} \sum_{i=0}^{\infty} \frac{u}{1} \right] \frac{u}{1} = \]

In agreement with equation (32.4f), the sum can be calculated as follows:

\[ \frac{\epsilon^m}{\mu} \sum_{i=0}^{\infty} \frac{u}{1} \sum_{i=0}^{\infty} \frac{u}{1} = (\nabla) \]

Provided both \( u \) and \( m \) are odd, and zero otherwise. So there is still only one singularity.

\[ \frac{\epsilon^m}{\mu} = u_{m}^H \]

The diagonal matrix elements are also independent of \( u \).

\[ \frac{\epsilon^m}{\mu} = \frac{u}{1} \frac{\epsilon^m}{\mu} = u_{n}^H \]

For even \( n \), the wave functions are odd, so there is no singularity. For odd \( n \),

\[ \epsilon(0)^{u}\frac{\epsilon^m}{\mu} u_{n}^H = u_{n}^H = u_{n}^H \]

The first-order singularity are

\[ \frac{\epsilon^m}{\mu} \sum_{i=0}^{\infty} u_{n}^H \]

The unperturbed energies are

\[ \text{Problem 7.4: Infinite Well with Delta Function Reversal} \]

In agreement with the second-order perturbation contribution,

\[ \left[ \frac{\epsilon^m}{\mu} - \left( \frac{\epsilon^m}{\mu} \right)^2 + \frac{\epsilon^m}{\mu} + 1 \right] \mu y = \left( \frac{\epsilon^m}{\mu} + 1 \right) \frac{\epsilon^m}{\mu} \frac{\epsilon^m}{\mu} + u = u_{m}^H \]

The exact eigenvalues are

\[ \epsilon^m \mu y \left( \frac{\epsilon^m}{\mu} + 1 \right) = \left( \frac{\epsilon^m}{\mu} - u_{m}^H \right) \frac{\epsilon^m}{\mu} \frac{\epsilon^m}{\mu} + 1 \]

\[ \left( \frac{\epsilon^m}{\mu} + 1 \right) - \left( 1 - u_{m}^H \right) \]

\[ \epsilon^m = \left[ \frac{\epsilon^m}{\mu} + 1 \right] \frac{\epsilon^m}{\mu} \frac{\epsilon^m}{\mu} + 1 \]

\[ \epsilon^m = \left( \frac{\epsilon^m}{\mu} + 1 \right) \frac{\epsilon^m}{\mu} \frac{\epsilon^m}{\mu} + 1 \]

Thus, \( u_{m}^H \) is automatically zero when \( m = 0 \) or \( n = 1 \). Thus

\[ \frac{\epsilon^m}{\mu} = \left( u_{m}^H \right) \frac{\epsilon^m}{\mu} \frac{\epsilon^m}{\mu} = \frac{u_{m}^H}{\mu} \]

where

\[ \text{Chapter V1: Approximation Methods for Bound States} \]
\[
\langle m,w|Z - Z\rangle = \langle m,w|H|Z - Z\rangle
\]
\[
\langle m,w|Z + Z\rangle = \langle m,w|H|Z + Z\rangle
\]

In particular, from the first equation the first section (Section 2.3) of the first section is obtained. So do not bother to work out any other components. The first section of the first section is obtained by combining \(\langle Z\rangle \text{ and } \langle 0\rangle\) components of \(Z\) and \(0\).

\[
\frac{\delta^2}{\delta t^2} = \frac{\delta^2}{\delta x^2}
\]

which is also the second order of the first order through second order in \(\mathcal{A}\) as usual.

\[
\langle 0|\nabla\mathcal{A} = (\nabla,\mathcal{A})
\]

and

\[
\frac{\delta^2}{\delta t^2} = \frac{\delta^2}{\delta x^2}
\]

The first-order perturbation is zero. The second-order perturbation is zero. The unperturbed eignenvalues are

\[
\frac{2}{\varepsilon_0 + 1 \omega} + \mathcal{A} = \mathcal{E}
\]

To this order the energies are

\[
\frac{2}{\varepsilon_0 + 1 \omega} = \langle \Phi_0|, H|\Phi_0\rangle
\]

which is the first-order perturbation.

\[
\frac{2}{\varepsilon_0 + 1 \omega} = \langle \Phi_0|, H|\Phi_0\rangle
\]

The second-order perturbation is zero. The unperturbed eignenvalues are

\[
\frac{2}{\varepsilon_0 + 1 \omega} = \langle \Phi_0|, H|\Phi_0\rangle
\]

When \(\varepsilon_0 \gg |\mathcal{V}|\), the unperturbed eignenvalues are

\[
\varepsilon_0 + \mathcal{A} = \mathcal{E}
\]

or

\[
0 = \varepsilon_0 - (\mathcal{A} - \mathcal{A})(\mathcal{A} - \varepsilon_0 + \mathcal{A})
\]

The exact eignenvalues are solutions to the characteristic equation

\[
\begin{pmatrix}
\varepsilon_0 + \mathcal{A} & \mathcal{A} \\
\mathcal{A} & \varepsilon_0 - \mathcal{A}
\end{pmatrix} = \mathcal{E}
\]

The final Hamiltonian is

\[
\mathcal{A} = \text{Two-State System}
\]