Molecular dynamics simulation of effective thermal conductivity of vapor-filled nanogap and nanocavity

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The effect of adsorption on momentum $a_t$ and thermal $a_T$ accommodation coefficients and effective thermal conductivity $\langle k_f \rangle$ of Ar vapor-filled Pt nanogap and nanocavity is examined using nonequilibrium molecular dynamics (MD) simulations. For the accommodation coefficients, the increase in solid-fluid or fluid-fluid interatomic interactions within adsorbed layer causes transitions in magnitudes of $a_i$ and $a_T$ near Ar triple-point temperature. In the nanogap, $\langle k_f \rangle$ MD results are in close agreement with the available closed-form solution for transition-flow regime, i.e., Knudsen number $0.1 < \text{Kn}_c < 10$, using the MD results of $a_T$. In the nanocavity, the adiabatic sidewalls do not significantly alter the gas mean free path and $\langle k_f \rangle$, and the sparse adsorption coverage on the sidewalls results in a negligibly small fluid circulation by surface diffusion under a temperature gradient. © 2009 American Institute of Physics. [DOI: 10.1063/1.3186043]

I. INTRODUCTION

When gas is confined in spacing with a characteristic length $L$ comparable or smaller than its mean free path $\lambda_f$ (average traveled distance without collision with adjacent particles), the effective thermal conductivity of the gas $\langle k_f \rangle$ is reduced due to the interaction of the gas particles with the confining surfaces.\(^1\)\(^-\)\(^3\)

In nonconfined gas (continuum regime, $\text{Kn}_L < 0.001$, Knudsen number $\text{Kn}_L = \lambda_f/L$), thermal conductivity is related to a number density of fluid particles $n_f$, specific heat capacity $c_{f,v}$, mean thermal velocity $\langle u_f \rangle$, and $\lambda_f$\(^1\)\(^-\)\(^3\) as

$$k_f = \frac{\pi n_f c_{f,v} \langle u_f \rangle \lambda_f}{3}.$$  

However, in the severely confined gas (free-molecular regime, $\text{Kn}_L > 10$), the effective thermal conductivity $\langle k_f, fm \rangle$ is\(^1\)

$$\frac{\langle k_f, fm \rangle}{L_z} = \frac{\rho \langle c_{f,v} f + R^f/2 \rangle}{a_T,1 + a_T,2 - 1} \left( \frac{2 \pi M R_f T_f}{3} \right)^{1/2},$$

where $L_z$ is the gap size along the $z$ direction (it is used for $\text{Kn}_L$), $p$ is the gas pressure, $R^f$ is the ideal gas constant, $M$ is the gas molecular weight, $T_f$ is the gas temperature, $a_T,1$ and $a_T,2$ are the thermal accommodation coefficients for two confining surfaces. In moderately confined gas (transition regime, $0.1 < \text{Kn}_L < 10$), the effective thermal conductivity $\langle k_f, t \rangle$ is\(^2\)\(^-\)\(^3\)

$$\langle k_f, t \rangle = \langle k_f, fm \rangle \left( 1 + \frac{4}{15} \left( \frac{5}{a_T,1 \lambda_f + a_T,2 - \lambda_f a_T,1 a_T,2} \right) \right)^{-1}.$$  

The thermal accommodation coefficient for monatomic gas is\(^4\)

$$a_T = \frac{\langle T_f \rangle - \langle T_f \rangle'}{\langle T_f \rangle - \langle T_s \rangle},$$

where $\langle T_f \rangle$ and $\langle T_f \rangle'$ are the temperatures of incident and reflected particles, respectively, and $\langle T_s \rangle$ is the solid surface temperature. When $a_T = 1$, fluid particles are completely accommodated with the solid surface (i.e., particle reflects with the surface temperature). Similarly, the tangential momentum accommodation coefficient is\(^1\)

$$a_u = \frac{\langle u_{f,t} \rangle - \langle u_{f,t} \rangle'}{\langle u_{f,t} \rangle'},$$

where $\langle u_{f,t} \rangle$ and $\langle u_{f,t} \rangle'$ are the tangential components of incident and reflected particles, respectively.

Near the saturation temperature, condensable gas adsorbs onto confining surface by interatomic/molecular potential and this adsorbed layer enhances thermal $a_T$ and momentum $a_u$ accommodation coefficients due to an increase in the fluid particle-confining surface interactions. This increased thermal accommodation, in turn, enhances the effective thermal conductivity, and the larger momentum accommodation increases viscous shear stress at the confining surface. This adsorption-affected heat transfer is significant in gas-filled nanopores, such as thermal insulation aerogel\(^7\)\(^-\)\(^8\) and polymer electrolyte of fuel cells\(^9\)\(^-\)\(^10\) where the effective thermal conductivity and accommodation coefficients must be addressed. However, the effect of adsorption had not been addressed in the derivation of the closed-form solutions of the effective thermal conductivities in the molecular- and transition-flow regimes [Eqs. (2) and (3)] (Refs. 1–3) and in the treatment of the momentum\(^11\)\(^-\)\(^13\) and thermal\(^6\)\(^-\)\(^12\)\(^-\)\(^14\) accommodation coefficients.

Here we examine the effects of adsorption on the momentum $a_u$, thermal $a_T$, and effective thermal conductivity $\langle k_f \rangle$ of Ar vapor-filled Pt nanogaps and nanocavities (nanogaps with enclosing adiabatic sidewalls) using a nonequilibrium molecular dynamics (MD) simulation and in temperature and pressure ranges which include the triple point.

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II. MD SIMULATIONS

The simulation box size is 7.8 × 6.8 × 20 nm³, and for the nanogap, periodic boundary conditions are used for the x and y directions. The Newton equation of motion is integrated by the Verlet algorithm with a time step of Δt = 2 fs, and the initial temperature is controlled using the direct potential scaling method.¹⁵,¹⁶ For the Ar–Ar interactions, the Lennard-Jones potential is used

\[ \varphi_{ij}(r) = 4 \sigma_{ij}^{12} / \left( r_{ij}^{12} - \left( \sigma_{ij} / r_{ij} \right)^{6} \right), \]

where the \( \sigma_{ij} = 10.42 \) meV, the \( \sigma_{ij} = 0.3405 \) nm, and the cutoff distance is 3.5\( \sigma_{ij} \).¹⁷,¹⁸ The mass of Ar atom is \( m_{f} = 6.636 \times 10^{-24} \) g, and \( N_{f} = 100–1536 \) Ar atoms are used in the simulations.

For the confining surfaces, a three-layered face-centered-cubic (111) plane solid Pt is modeled using the harmonic potential with \( \Delta r_{ij} = 0.28 \) nm, the friction \( \gamma = 5.184 \times 10^{-12} \) kg/s, and the random excitation force of the Gaussian distribution with the standard deviation \( \sigma_{T} = (2 k_{B}T_{s}/\Delta t)^{1/2} \). The temperature difference between the two surfaces is from 10 to 120 K for the accommodation coefficients (so it is larger than the temperature fluctuation of the solid), while 10 K is used for the effective thermal conductivity calculations. The temperatures and magnitudes of KnT for MD simulations are summarized in Table I.

For the Ar–Pt interactions, Lennard-Jones potential [Eq. (6)] is used in the nanogap, whereas a one-dimensional potential between Ar and the adiabatic sidewalls is used in the nanocavity. This is

\[ \varphi_{ij}(l) = 4 \pi r_{ij}^{3} / 15 \left( \sigma_{ij} / l \right)^{6} - 2 \left( \sigma_{ij} / l \right)^{10} - 5 \left( \sigma_{ij} / l \right)^{12}, \]

where \( \epsilon_{ij} = 6.82 \) meV, \( \sigma_{ij} = 0.309 \) nm, \( l \) is the distance between the fluid particles and sidewalls, and the cutoff distance is 3.5\( \sigma_{ij} \). To reach steady state, \( 5 \times 10^6 \) time steps are simulated, then we calculate ensemble-averaged thermodynamic properties, \( a_{p}, a_{T}, \) and \( \langle k_{B} \rangle \) over another \( 5 \times 10^6 \) time steps. The gas pressure is calculated using the ideal gas law.

The accommodation coefficients are evaluated using Eqs. (4) and (5), following the recipes described in Refs. 12 and 13. An imaginary line is used to separate incident and reflecting particles, and is placed at the cutoff distance of the solid–fluid interaction (1 nm from the surface). The selection of distance is based on two requirements.¹²,¹³ One is that the reflected particles must not be interfered with surface forces, and the other is that the reflected particles should not collide with the adjacent fluid particles before passing through this imaginary line. For \( N_{f} > 1000 \), the second requirement is violated due to small mean free path (\( \lambda_{m} \approx 2 \) nm). Thus, we use \( N_{f} = 1536 \) only near the triple-point temperature, where adsorption significantly reduces the number of gaseous particles (to about 100).

The ensemble-averaged heat flux is calculated using¹⁸,²⁰

\[ \langle q_{k,s} \rangle = \frac{1}{V} \left[ \sum_{i} \frac{1}{2} m_{i} \left( \mathbf{u}_{i} \cdot \mathbf{u}_{i} \right) u_{i,z} + \sum_{i} \varphi_{i} u_{i,z} \right] + \frac{1}{2} \sum_{i} \sum_{j} z_{ij} \left( \mathbf{u}_{i} \cdot \mathbf{F}_{ij} \right), \]

where \( V \) is the volume, \( m_{i} \) is the mass of particle \( i \), \( \mathbf{u}_{i} \) is the velocity, \( u_{i,z} \) is the \( z \)-component of the velocity, \( \varphi_{i} \) is the potential energy, \( z_{ij} \) is the \( z \)-component of the displacement separation, and \( F_{ij} \) is the interaction force between \( i \) and \( j \) particles. Note that \( \langle \cdot \rangle \) represents the ensemble average. Then, the effective thermal conductivity is evaluated from the Fourier law, \( \langle k_{s} \rangle = -(\langle q_{k,s} \rangle / \langle \partial T / \partial z \rangle) \). Note that \( \langle q_{k,s} \rangle \) is the same as the heat flowing into the hot and leaving the cold surface \( \langle q_{k,c} \rangle \), which are calculated integrating the net damping and random forces with respect to time in the Langevin equation. Radiation heat transfer is neglected among the walls.

III. RESULTS AND DISCUSSIONS

A. Momentum and thermal accommodation coefficients

A snapshot of MD simulation in the nanogap, including adsorption, at \( \langle T_{s} \rangle = 85 \) K with \( N_{f} = 600 \), is shown in Figs. 1(a) and 1(b). The cold surface is below the triple-point temperature \( T_{g} = 85 \) K (Ref. 21), \( T_{w} = 80 \) K, whereas the hot surface is above it, \( T_{h} = 90 \) K. Noting that the triple-point temperature of Ar confined in Vycor glass nanopore is only 2 K larger than that for the bulk Ar,²¹ thus we expect the equilibrium phase diagram of the confined Ar to be different. Here we only use the bulk triple-point temperature as a reference when addressing phase change in the Ar–Pt nanogap.

Near surfaces, thermodynamic equilibrium (chemical potential) between gas and adsorbed phase determines the surface coverage. Figure 2 shows MD adsorption results, the variation in the number of adsorbed Ar atoms per unit surface area with respect to the surface temperature, 85 < \( T < 1320 \) K, i.e., 0.06 < \( \epsilon_{ij} / k_{B}T_{g} < 0.931 \). A full surface coverage by a monolayer is also marked. Note that the surface temperature ranges from the triple-point temperature to above the critical-point temperature of Ar, where \( \epsilon_{ij} / k_{B}T_{g} = 0.525 \) at the critical-point temperature of Ar, and \( \epsilon_{ij} / k_{B}T_{g} = 0.931 \) at the triple-point temperature. Minor adsorption is found at high
temperatures due to large kinetic energy compared with surface force, with an exponential increase with decreasing temperature due to a decrease in kinetic energy compared to surface force (similar to type III isobar\cite{22}). This is associated with a relatively weak fluid-solid interatomic interaction.\cite{22} This weak interaction results in a small decrease in the adsorption at high pressures; however, once fluid particles adsorb onto the surface, interatomic interactions (potential) promote further adsorption. This is more pronounced at high pressure, \( N_f=900 \). A transition occurs at \( \epsilon_{sf}/k_BT = 0.7 \) for \( N_f=600 \), whereas at \( \epsilon_{sf}/k_BT = 0.6 \) for \( N_f=900 \). This transition, in turn, affects the accommodation coefficients and the effective thermal conductivity. Near the triple-point temperature, a fully covered adsorbed layer is found. For \( N_f=900 \), the number of adsorbed atoms continue to decrease as the temperature increases, whereas for \( N_f=600 \), those are nearly constant above the critical temperature. This residual adsorption at high pressures leads to pressure-dependent \( a_u \) and \( a_T \) but is expected to disappear at very high temperature (large gas superheat). Note that \( K_n \) changes with adsorption as shown in Table I.

Adsortion influences the surface momentum and energy exchange as manifested through \( a_u \) and \( a_T \) accommodation coefficients. In Figs. 3 and 4, variations in MD simulated \( a_u \) and \( a_T \) are shown with respect to surface temperatures, \( 75<T<1320 \) K, i.e., \( 0.06<\epsilon_{sf}/k_BT<1.13 \). \( a_u \) has a low value \( a_u=0.33 \) at high temperature \( \epsilon_{sf}/k_BT=0 \),
and increases toward unity (diffuse and no temperature slip) with decreasing temperature $\epsilon_{ij}/k_BT\rightarrow\infty$. At low temperatures, the kinetic energies of incident particles are small enough for significant solid-fluid interactions (adsorption), i.e., the adsorbed particles fluctuate several times due to the attractive surface force before escaping out of the potential well. In addition, the adsorbed particles exchange kinetic energies with the neighboring adsorbed particles. These cause the increased $a_u$ and result in the transition at $\epsilon_{ij}/k_BT=0.7$. This is related to the temperature where significant adsorption disappears. At high temperatures (above the critical point), incident particles have dominant kinetic energy (over solid-fluid potential energy) and reflect without significant accommodation. In addition, this transition appears even at low temperatures, $\epsilon_{ij}/k_BT=0.931$, i.e., the hot surface is set above the triple-point temperature and the cold is below it, as marked with a solid circle in Fig. 3 (the simulated snapshot is also shown in Fig. 1). A fully covered adsorption layer is found at the cold surface, which, in turn, results in large $a_u$ (also shown in Fig. 2), whereas the sparse coverage at the hot surface leads to small $a_u$. In addition, the simulated results at high pressure, $N_f=900$, show increased $a_u$ compared to $N_f=600$, with nearly 30% increased surface adsorbed layer coverage at high temperatures, whereas $a_u$ values are nearly the same results for $N_f=600$ and 1536 due to a full coverage at low temperatures, as shown in Fig. 2. The transition point also shifts toward higher temperature $\epsilon_{ij}/k_BT=0.6$, due to the increase in a surface coverage. The magnitudes of $a_u$ for monatomic gases are found to be nearly independent of the pressure at high temperatures from a number of experiments. This is related to rough surfaces where a relatively larger adsorption is observed. We use ideal (smooth) solid surfaces, and this may cause a pressure dependence in the temperature range considered.

The MD results are compared to other available MD results, and a difference is found especially at high temperatures. This is related to the increased effective solid-fluid potential due to a rough surface (larger adsorption), as discussed in Ref. 11 ($\sigma_{ij}/a$, where $\sigma_{ij}$ is the separation constant between the solid and the fluid and $a$ is the lattice constant of the solid). Note that in Ref. 11 a rough surface ($\sigma_{ij}/a=0.6$) is modeled compared to the surface in this study ($\sigma_{ij}/a=1.07$), which provides a large effective solid-fluid interaction potential (larger solid surface area). This, in turn, results in larger adsorption and larger $a_u$. This causes transition even at higher temperatures (above $\epsilon_{ij}/k_BT=0.3$).

Adsortion-affected transition is also found in MD results for the thermal accommodation coefficient $a_T$ and is more pronounced at $\epsilon_{ij}/k_BT=0.7$ and $\epsilon_{ij}/k_BT=0.931$. Moderate pressure dependence of $a_T$, $N_f=900$ (due to nearly 30% increased adsorption surface coverage) is also found at high temperature, whereas no pressure dependence appears at low pressure (due to nearly full adsorption coverage), as shown in Fig. 2. The pressure dependence of $a_u$ at high temperatures is expected to disappear at very large temperatures. This is because $a_T$ is evaluated using normal and tangential velocity components [Eq. (4)], whereas $a_u$ uses only the tangential velocity components [Eq. (5)]. The MD results are also compared to the available experiment at the high temperatures, and a moderate agreement is found. This difference is related to the significant adsorbed layer caused by the rough or contaminated surface in the experiment. Eventually, it is expected to disappear at high temperatures $\epsilon_{ij}/k_BT\rightarrow0$, i.e., no adsorption. In the experiment, there is also a transition at $\epsilon_{ij}/k_BT=0.3$.

### B. Effective thermal conductivity in the nanogap and nanocavity

In the nanogap, the effective thermal conductivity in the transition regime is calculated between the triple-point and the critical-point temperatures and is compared to Eq. (3) using the predicted $a_T$, as shown in Fig. 5. Overall, the MD results are in good agreement with that solution within 20% uncertainty even at low temperatures (significant adsorption). Note that Eq. (3) had been derived using the Maxwell integral equations of transfer, the Maxwell velocity distribution function of fluid particles, and the gas temperatures near the confining surfaces (predicted by the accommodation coefficients and confining surface temperatures, neglecting adsorption) as the boundary conditions. This approach is still valid even with adsorption since the thermal accommodation coefficients account for the effective fluid-wall molecular interactions. The MD results show that high temperatures allow for a large number density of gaseous particles, $n_f$ (small $K_{nf}$) without significant adsorption and large mean thermal velocity, which increase $\langle k_f\rangle$. However, the absence of adsorption results in low $a_T$, limiting $\langle k_f\rangle$. At low temperatures, small kinetic energy leads to significant adsorption, which, in turn, increases $\langle k_f\rangle$ by increasing $a_T$, whereas it causes large $K_{nf}$ and small mean thermal velocity which reduces $\langle k_f\rangle$. The calculated $\langle k_f\rangle$ is smaller than the bulk $k_f$ [Eq. (1)] due to large $K_{nf}$ and small $a_T$.

In the nanocavity, a snapshot of MD simulation, at $T_g=90$ K with $N_f=600$, is shown in Figs. 6(a) and 6(b). The nanocavity creates significant adsorption around the cor-

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**FIG. 5.** (Color online) MD results for the effective thermal conductivity $\langle k_f\rangle$ in the nanogap with respect to vapor pressure $p$, and $K_{nf}$ at a few vapor temperatures $T_g$. Comparisons with the available closed-form solution [Eq. (3)] using predicted $a_T$ are shown. The thermal conductivity of nonconfined (bulk) vapor $k_f$ [Eq. (1)] is also shown.
nners where the surface forces overlap, which, in turn, results in scattered adsorption on the hot, cold, and sidewall surfaces. In Fig. 7, a reasonable agreement is also found between the MD results and Eq. (3), even at low temperatures (close to the triple-point temperature). The adiabatic sidewalls do not significantly influence the mean free path due to the elastic collisions between the fluid particles and sidewalls. In addition, a significant adsorption is found around the edges (corners) of sidewalls, where the surface forces overlap, as shown in Figs. 6(a) and 6(b). However, due to scattered, partial coverage, this is not sufficient enough to produce significant surface diffusion along the sidewalls and a circulation within the cavity under a temperature gradient.

IV. CONCLUSIONS

For the transition-flow regime (0.1 < Kn< 10), momentum \( a_u \) and thermal \( a_T \) accommodation coefficients, and effective thermal conductivity \( \langle k_f \rangle \) are calculated for Ar vapor filled in the Pt nanogap and nanocavity using nonequilibrium MD simulation over triple-to critical-point temperature range. This allows for examining the effect of adsorption on the surface interaction kinetics as well as the gaseous thermal transport.

For \( a_u \) and \( a_T \), we find transition at near triple point, \( \varepsilon_{s,f}/k_BT = 0.7 \) for \( N_f=600 \) and \( \varepsilon_{s,f}/k_BT = 0.6 \) for \( N_f=900 \), while it varies with the solid surface roughness and surface coverage by adsorption. These coefficients increase with decreasing temperature and eventually reach unity. This transition is caused by adsorption, i.e., increased solid-fluid and fluid-fluid interactions within the adsorbed layer. This transition also depends on the pressure. Larger accommodation coefficients are also found at high pressures, where significant adsorption occurs. The adsorption-affected transition is more pronounced in \( a_T \) because \( a_T \) is evaluated using both normal and tangential velocity components, whereas \( a_u \) uses only the tangential velocity components.

In the nanogap, the MD results for the effective thermal conductivity are in agreement with the available closed-form solution for the transitional-flow regime. This agreement extends even in the presence of adsorption at low temperatures.

In the nanocavity, the calculated thermal conductivity is nearly the same as that of the nanogap since the adiabatic sidewalls do not significantly alter the fluid particle mean free path, i.e., because of the elastic collision with the wall. In addition, adsorption occurs mainly around the edges where the surface forces overlap, which, in turn, leads to dispersed adsorption on the hot and cold, sidewall surfaces. This does not produce significant surface diffusion for the surface return of the particles along the adiabatic sidewalls. Thus, any circulation with the nanocavity under a temperature gradient would require very high pressure and low temperature for significant adsorption coverage.

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FIG. 6. (Color online) (a) 3D snapshot of simulated Ar atoms (\( N_f=600 \), \( Kn=0.73 \)) in the nanocavity at \( (T_g)=90K \) and (b) a stereoview of top and bottom corners showing larger adsorption at the edges and corners.

FIG. 7. (Color online) MD results for the effective thermal conductivity \( \langle k_f \rangle \) in the nanocavity as a function of vapor pressure \( (p_f) \) and \( Kn=0.1 \), at a few vapor temperatures \( (T_g) \). Comparisons with the available closed-form solution [Eq. (3)] using predicted \( a_T \) are shown. The thermal conductivity of nonconfined (bulk) vapor \( k_f \) is also shown.