c) We are to determine the capacitance of two concentric conducting cylinders of length $L$ with radii $a$ and $b$ where $b > a$.

If we chose a Gaussian region that completely encloses one the inner cylinder, then the surface integral of the electric field will give $E \cdot A$ where $E$ is the magnitude of the electric field and $A = 2\pi r L$, the area of the boundary of the region.

Using Gauss' law, we see that the surface integral is equal to the total charge contained within the region divided by $\varepsilon_0$. Specifically, we have that

$$E \cdot 2\pi r L = \frac{Q}{\varepsilon_0} \implies E = \frac{Q}{2\pi \varepsilon_0 L r},$$

where $a < r < b$ and $Q$ is the charge on one of the cylinders.

The magnitude of the voltage difference between the two cylinders is equal to the line integral of the electric field from one to the other. Specifically,

$$\Delta V = \int_a^b Ed\ell = \frac{Q}{2\pi \varepsilon_0 L} \int_a^b \frac{dr}{r} = \frac{Q}{4\pi \varepsilon_0} \log \left( \frac{b}{a} \right).$$

Therefore, using the definition of capacitance, we see that

$$\therefore C = \frac{2\pi \varepsilon_0 L}{\log \left( \frac{b}{a} \right)}.$$

3. Capacitance II

We are to approximate the capacitance per unit length of two parallel, cylindrical conductors with radii $a_1$ and $a_2$ which are separated by a distance $d$.

Let us work within the coordinate system such that the center of the first cylinder, with radius $a_1$, is located at $r = 0$ and the second cylinder, with radius $a_2$, is located at $r = d$.

Because the electric field is linear, we can consider the field caused by each of the conductors separately. Specifically, for points between the two cylinders, we can add the electric fields produced by each cylinder separately. We can determine the electric field per unit length induced by each cylinder by imagining a Gaussian region that completely encloses a unit length of either cylinder. Therefore, for a point collinear with the centers of each cylinder, we have that

$$E = \frac{Q}{2\pi \varepsilon_0 r} + \frac{Q}{2\pi \varepsilon_0 (d - r)}.$$  

The magnitude of the voltage difference between the two cylinders is equal to the line integral of the electric field from one to the other. Specifically,

$$\Delta V = \frac{Q}{2\pi \varepsilon_0} \int_{a_1}^{d - a_2} \left( \frac{1}{r} + \frac{1}{d - r} \right) dr,$$

$$= \frac{Q}{2\pi \varepsilon_0} \left[ \log \left( \frac{d - a_2}{a_1} \right) - \log \left( \frac{a_2}{d - a_1} \right) \right],$$

$$= \frac{Q}{2\pi \varepsilon_0} \log \left( \frac{(d - a_2)(d - a_1)}{a_1 a_2} \right),$$

$$\approx \frac{Q}{2\pi \varepsilon_0} \log \left( \frac{d}{a_1 a_2} \right),$$

$$= \frac{Q}{\pi \varepsilon_0} \log \left( \frac{d}{a_1 a_2} \right).$$

Therefore, using the definition of capacitance per unit length, we see that

$$\therefore C \approx \frac{\pi \varepsilon_0}{\log \left( \frac{d}{a_1 a_2} \right)}.$$