I. Where we were last:

A. Regression as Projection Problem

1. Scalar Vector

\[
\text{Want } x b \perp e \Rightarrow (xb)'e = 0
\]
\[
\Rightarrow b x' e = 0
\]
\[
\Rightarrow b x' (y - x b) = 0
\]
\[
b x' x - b x' x b = 0
\]
\[
x' x - x' x b = 0
\]
\[
x' x b = x' x
\]
\[
b^* = x' x / x' x = (x' x)^{-1} x' x
\]

Notice: Suppose \( x^* = x - \bar{x}, x(\bar{x}) \), then:
\[
x^* x^* = \sum (x_i - \bar{x})^2 = n \bar{v}(x)
\]
\[
A x = 0
\]
(under normality)
\[
\begin{pmatrix}
(I - \frac{1}{n} 1n' ) M
\end{pmatrix}
\]
\[
\begin{pmatrix}
(x_i - \bar{x})(x_i - \bar{x})' \\
(x_i - \bar{x})(x_i - \bar{y})
\end{pmatrix}
\]
\[
= n \cdot C(x, y)
\]
\[
b = \text{Cov}(x, y) / \text{V}(x)
\]
\[
x = x M
\]
\[
I = x A
\]
\[
0 = x M
\]

2. Vector/Matrix

\[
y = X b + e
\]
(\( n \times 1 \))
(\( n \times k \))
(\( k \times 1 \))
(\( n \times k \))
(\( k \times 1 \))
(\( 1 \times k \))
(\( 1 \times 1 \))
(\( 1 \times 1 \))
(\( 1 \times 1 \))

(\( n \times 1 \))
\[
X b \perp e \Rightarrow (Xk)' e = 0
\]
\[
(X b)' (y - X b) = 0
\]
\[
b' (x' x - b' x x b) = 0
\]
\[
\Rightarrow b' (x' x - x' x b) = 0
\]
\[
\Rightarrow b (x' x - x' x b) = 0
\]
\[
\Rightarrow \text{one solution is } b = \beta, \text{ but this is not optimal. One we want is:}
\]
\[
\sum x' x b = X' y
\]
\[
\Rightarrow b^* = (x' x)^{-1} x' y
\]
B. Further Topics:

1. Work with Determinants & Inverses -- "what's in them, intuitively?"

2. Trace: sum of diagonal elements of a matrix:

\[
\text{Trace}(\mathbf{A}) = \sum_{i=1}^{n} a_{ii}
\]

3. Regression Matrices of Importance:

\[
\begin{align*}
\mathbf{Q} &= (\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}' \\
\mathbf{A} &= \mathbf{Q}^{-1} \mathbf{X}' \\
\mathbf{N} &= \mathbf{X}\mathbf{A} \\
\mathbf{M} &= (\mathbf{I} - \mathbf{N})
\end{align*}
\]

4. Partitions: often convenient to partition matrices like, e.g.,

\[
\mathbf{A} = \begin{bmatrix}
1 & 2 \\
3 & 4 \\
1 & 2 \\
3 & 4
\end{bmatrix} = \begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\]

Also, \( \text{Add} (\text{Sub}) \) \& \( \text{Mult} \) as before (but relevant pairs must be conformable).

Block Diagonal: \( A_{ij} = 0 \) \( \forall i \neq j \)

\[
|A| = \prod_{i=1}^{n} |A_{ii}|
\]

5. Kronecker Product (\( \otimes \)) = Matrix A \( \otimes \) Matrix B

6. Characteristic Eqs, Vectors, Roots -- that they exist, that we use them in some configurations