Weak and Strong Necessity Modals
On linguistic means of expressing “a primitive concept ought”*

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“‘Ought’ and ‘Must’ — they are contemptible auxiliaries.”
George Eliot†

Abstract
This paper develops an account of the meaning of ‘ought’, and the distinction between weak necessity modals (‘ought’, ‘should’) and strong necessity modals (‘must’, ‘have to’). I argue that there is nothing specially “strong” about strong necessity modals per se: uses of ‘Must ϕ’ predicate the (deontic/epistemic/etc.) necessity of the prejacent ϕ of the actual world (evaluation world). The apparent “weakness” of weak necessity modals derives from their bracketing whether the necessity of the prejacent is verified in the actual world. ‘Ought ϕ’ can be accepted without needing to settle that the relevant considerations (norms, expectations, etc.) that actually apply verify the necessity of ϕ. I call the basic account a modal-past approach to the weak/strong necessity modal distinction (for reasons that become evident). Several ways of implementing the approach in the formal semantics/pragmatics are critically examined. The account systematizes a wide range of linguistic phenomena: it generalizes across flavors of modality; it elucidates a special role that weak necessity modals play in discourse and planning; it captures contrasting logical, expressive, and illocutionary properties of weak and strong necessity modals; and it sheds light on how a notion of ‘ought’ is often expressed in other languages. These phenomena have resisted systematic explanation. In closing I briefly consider how linguistic inquiry into differences among necessity modals may improve theorizing on broader philosophical issues.

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†Mary Garth, in Middlemarch, Bk. 2, Ch. 14. Shamelessly modified from the original.
1 Introduction

A notion of ‘ought’ is central in many areas of philosophical discourse. “A primitive ought,” Gibbard tells us, “is the basic conceptual atom that gives normative concepts their special character” (2006: 738). Yet, historically speaking, surprisingly little attention has been paid among philosophers to the distinctive features of the meaning and use of ‘ought’. ‘Ought’ is often treated as relevantly equivalent to a range of expressions of obligation and necessity. The eponym of Gibbard’s Professorial Chair, Richard Brandt, observed as much half a century ago: “Philosophers often use the following expressions as approximate equivalents: ‘It is X’s duty to do A’; ‘It is obligatory for X to do A’; ‘It would be wrong for X not to do A’; and ‘X ought to do A’” (1964: 374). Here, more recently, is Åqvist:

[D]eontic logic… is the logical study of the normative use of language and… its subject matter is a variety of normative concepts, notably those of obligation (prescription), prohibition (forbiddance), permission and commitment. The first one among these concepts is often expressed by such words as ‘shall’, ‘ought’ and ‘must’, the second by ‘shall not’, ‘ought not’ and ‘must not’… (Åqvist 2002: 148)

There has been extensive work in descriptive linguistics on discourse differences among Åqvist’s “normative-concept-expressing” words. For instance, it’s common to distinguish categories of so-called “weak” necessity modals such as ‘ought’ (‘should’, ‘be supposed to’) and “strong” necessity modals such as ‘must’ (‘have to’, ‘(have) got to’, ‘be required to’).¹ Holding the reading of the modals fixed, (1a) is consistent in a way that (1b) is not. ‘Ought ϕ’ can be followed by ‘Must ϕ’, but not vice versa:

(1) a. I should help the poor, but I don’t have to.
   b. #I must help the poor, but it’s not as if I should.

(2) a. I ought to help the poor. In fact, I must.
   b. I must help the poor. #In fact, I ought to.

There are also conversational differences. Informally:

To say that one ought to take a certain option is merely to provide a nudge in that direction. Its typical uses are to offer guidance, a word to the wise…, to recommend, advise… In contrast, to say that one must take a certain option

is to be quite forceful. Its typical uses are to command, decree, enact, exhort, entreat, require, regulate, legislate, delegate, or warn. (McNamara 1990: 156)

Gibbard hedges his bets on what natural language expression best approximates his favored normative notion:

Ewing's point is not that the English word ‘ought’ exactly captures the notion he has in mind. The word often suggests merely the weight of one set of considerations among others… The word ‘must’ might be better for his purposes and mine [?!!?!?], but it has its own flaws: it suggests greater urgency than Ewing’s ought would have when factors in a decision nearly balance out. (Gibbard 2012: 14–15; more on ‘?!!?!?’ later)

An interesting question for a paper — not this one (cf. Silk 2015b) — is to what extent insensitivity to linguistic differences among ‘ought’, ‘should’, ‘must’, ‘have to’, etc. may be a source of philosophical malaise. On the flip side, perhaps investigating the meaning and discourse function of such words can help clarify the contours of Gibbard’s basic normative “conceptual atom” and improve philosophical theorizing. In this paper I wish to set the stage for such a project by attending to the matters narrowly linguistic: what ‘ought’ — the eponymous expression of Gibbard’s “primitive concept ought” (2012: 204) — itself means. I focus on the distinction in “strength” between ‘ought’ — and its weak-necessity-modal kin — and strong necessity modals such as ‘must’.

Although there has been growing interest in the semantics of ‘ought’, accounts are often developed in ways which bracket differences among necessity modals. Formal accounts of the weak/strong necessity modal distinction are typically developed piecemeal with an eye toward a narrow range of data. Adjudicating among theories can be difficult, if not premature.

The aim of this paper is to provide a more comprehensive theoretical investigation into weak and strong necessity modals. Building on previous work (Silk 2012, 2013b) I develop an account of the meaning of ‘ought’ and the distinction between weak and strong necessity modals. The account systematizes a wide range of semantic and pragmatic phenomena: it generalizes across flavors of modality; it elucidates a special role that weak necessity modals play in discourse and planning; it captures contrasting logical, expressive, and illocutionary properties of weak and strong necessity modals; and it sheds light on how a notion of ‘ought’ is expressed in other languages. These phenomena have resisted systematic explanation.

Roadmap: §2 presents core data illustrating the effects of standing contextual assumptions on the relative felicity of weak vs. strong necessity modals. The §2-examples highlight what I regard as the fundamental difference between the class of weak necessity modals and the class of strong necessity modals.

Footnote 2: Here and throughout: For shame.
§3 presents the basic account of the weak/strong necessity modal distinction. No innovations are introduced in the semantics/pragmatics of strong necessity modals; uses of ‘Must $\phi$’ predicate the (deontic/epistemic) necessity of the prejacent $\phi$ of the actual world. The apparent “weakness” of weak necessity modals derives from their bracketing whether the necessity of the prejacent is verified in the actual world. ‘Ought $\phi$’ can be accepted without accepting that $\phi$ is necessary (deontically, epistemically, etc.). Weak necessity modals afford a means of entertaining and planning for hypothetical extensions of the context in which certain considerations (norms, values, etc.) apply, without needing to commit that the considerations aren’t actually defeated.

§4 examines how weak necessity is expressed cross-linguistically to motivate several formal implementations of the informal §3-account. For reasons that will become clear, I call the family of analyses a modal-past approach to ‘ought’ and the weak/strong necessity modal distinction. I argue that the proposed treatment of the cross-linguistic data improves on the treatment in von Fintel & Iatridou 2008. The account gives precise expression to the informal idea that ‘ought’ is weaker than ‘must’, and captures ways in which ‘ought’, unlike ‘must’, patterns with past-marked modal forms.

§5 applies the formal semantics from §4 to several puzzles of entailingness and performativity with ‘ought’ and ‘must’. Examining these puzzles highlights a second dimension along which modals differ, regarding their tendencies to be used in (what I call) an “endorsing” vs. “non-endorsing” way.

§6 recaps distinctive features of the account, before contrasting it with one prominent alternative in greater detail: the “collective commitment” analysis in Rubinstein 2012.

§7 concludes and raises directions for future research. Potential implications of the linguistic work on modals for broader philosophical theorizing are briefly considered.

For familiarity I follow the literature in labelling modals such as ‘ought’ and ‘should’ as “weak necessity” modals, and labelling modals such as ‘must’, ‘have to’, ‘(have) got to’ as “strong necessity” modals. The terminology of “weak necessity” and “strong necessity” shouldn’t mislead. I am not assuming that uses of the modals in the ‘ought’-family invariably convey a weaker felt conversational force, that the modals express different “kinds” of necessity, or even that they comprise a scale of logical/quantificational strength. A claim such as that strong necessity modals truth-conditionally entail weak necessity modals, which truth-conditionally entail possibility modals constitutes a substantive empirical hypothesis on my terminology. We will see reasons for questioning each of the above claims. (Hereafter I typically use ‘ought’ as my representative of the ‘ought’/‘should’/etc. family, and ‘must’ as my representative of the ‘must’/‘have to’/etc. family.)

I treat ‘$\phi$’, ‘$\psi$’, etc. as schematic letters to be replaced with declarative sentences. For convenience I sometimes refer to the possible-worlds proposition expressed by ‘$\phi$’ by dropping the single quotes, e.g. using ‘$\phi$ is necessary’ for ‘$\{w: [\phi]^w = 1\}$’; I slide between equivalent set-/function-talk.
2 ‘Ought’ and ‘must’ in context

Descriptive and theoretical research on modals highlights various conversational differences among necessity modals. This section focuses on one such difference, concerning the effects of contextual assumptions on the relative felicity of weak vs. strong necessity modals.⁴

A central purpose of conversation is to share and coordinate our expectations, values, and plans. Sometimes we assert propositions outright. We commit to settling on their truth for the remainder of the conversation. But sometimes we don’t wish to impose such a strong restriction on the future course of the conversation. We may want to propose that someone is obligated to do something but be unsure about whether there might be competing norms that could outweigh or cancel her obligation. Or we may want to proceed as if some proposition is true while remaining open to the possibility that our apparent evidence for it is misleading. I suggest that the role of weak necessity modals is to afford a means of making such proposals and expressing such states of mind.

Start with an epistemic case. Suppose we’re working on an art project and I ask you where the colored pencils are. Normally you put them in the drawer with the crayons but sometimes you accidentally put them on the shelf. In this scenario it’s more appropriate for you to use ‘ought’ in responding to my question:

(3)  
Me: Do you know where the colored pencils are?  
You: They ought to (/should/?must/?have to) be in the drawer with the crayons.

Suppose, alternatively, that we’re looking for the colored pencils together, and you saw something that leads you to conclude that they are in the drawer. Perhaps you noticed that they weren’t on the shelf, and this is the only other place you think they could be. In this scenario it’s more natural for you to use ‘must’:

(4)  
Me: Do you know where the colored pencils are?  
You: They must (/have to/?ought to/?should) be in the drawer with the crayons.

⁴See WOISETSCHLAEGER 1977: ch. 5 and McNAMARA 1990: ch. 3 for prescient early discussion of contextual differences between ‘ought’ and ‘must’. For extensive discussions in descriptive linguistics, see the references in nn. 1, 35, 39. See Rubinstein 2012 and Silk 2012 for recent theoretical emphasis. Rubinstein doesn’t consider epistemic examples; I examine her alternative take on the data in § 6.

For reasons discussed in §§5–6, it’s important not to substitute other strong necessity modals (e.g. ‘have to’) for ‘must’ in examples unless indicated otherwise; speakers who find ‘should’ more natural than ‘ought’ may substitute ‘should’ for ‘ought’ throughout. Judgments concerning some of the examples may be vague for some speakers and may vary given subtle changes in context. This is part of what needs to be explained. The positive account in this paper will crystalize the informal reactions described in this section. I use ‘?’ to indicate that using the marked item is dispreferred; ‘?’ marks a weaker infelicity than ‘#’.
Its following from our evidence that the colored pencils are in the drawer depends on today not being one of the days when you accidentally put them on the shelf. Using ‘must’ is preferred if, and only if, you know that conditions are normal in this way. What is illuminating is that you can use ‘ought’ even if you aren’t in a position to judge that they are. Accepting your ‘ought’-claim doesn’t require us to presuppose that your evidence is undefeated.

Consider a deontic case (n. 4). Suppose I am considering whether to fight in the Resistance or take care of my ailing mother. I mention that the value of family, which supports my helping my mother, is important, and you agree. But the issue is admittedly complex, and we haven’t settled whether there might be more important competing values. Sensitive to this, you may find it more appropriate to express your advice that I help my mother by using ‘ought’ than by using ‘must’:

(5) Me: Family is very important.
You: I agree. You ought to/should (/?must/?have to) tend to your mother.

But if we settle that family is of primary importance, it can become more natural to use ‘must’ and for us to accept that I have to help my mother:

(6) Me: Family is most important — more important than country.
You: I agree. You must/have to (/?ought to/?should) tend to your mother.

My having an obligation to help my mother depends on the value of family being more important\(^5\) in my situation than any competing value. Parallel to the epistemic case, what is illuminating is that you can felicitously use ‘ought’ to express your advice that I help my mother without assuming that this precondition for my having a genuine obligation is satisfied. Accepting your ‘ought’-claim needn’t require us to presuppose that the value of family is more important than other potentially competing values.

Cases such as (3)–(6) highlight what I regard as the fundamental difference between the class of weak necessity modals and the class of strong necessity modals. It’s typical to gloss epistemic notions of necessity as concerning what follows from a body of evidence (knowledge, information), and deontic notions of necessity as concerning what is obligatory.\(^6\) Yet

\(^5\)Or at least not less important; I will bracket complications from incomparabilities and irresolvable dilemmas. For theoretical discussion of dilemmas and the ‘ought’/‘must’ distinction, see SWANSON 2011, SILK 2012, 2013b: ch. 3, 2015b, and references therein.

\(^6\)See, a.m.o., LYONS 1977, COATES 1983, PALMER 1990, 2001, SWEETSER 1990, BYBEE ET AL. 1994, VAN DER AUWERA & PLUNGJAN 1998, NYUTS 2001, HUDDELESTON & PULLUM 2002. In saying that the uses of ‘ought’ and ‘must’ have the same type of reading, I’m not assuming that the modals have the same interpretation or stand in entailment relations (§1). In calling uses “epistemic” I don’t assume that they are factive/entailing, or even that they convey the same kind of doxastic attitude toward the prejacent (contrast YALCIN 2016). What is important about the uses in (3)–(4) is that they both directly address the question of where the colored pencils are and
we can accept your epistemic ‘ought’-claim in (3) without settling that conditions are relevantly normal and thus without settling that our evidence implies that the colored pencils are in the drawer; and we can accept your deontic ‘ought’-claim in (5) without settling that family is the most important relevant value and thus without settling that I have a genuine obligation to help my mother. Accepting ‘Ought $\phi$’ needn’t commit one to accepting that $\phi$ is necessary (epistemically, deontically, etc.).

Whether ‘ought’ or ‘must’ is preferred “depends on context” in the sense of depending on whether certain preconditions for the prejacent to be necessary (in the above sense) are accepted: In (3)–(4), how you express your attitude toward the proposition that the colored pencils are in the drawer depends on your views about the (in)defeasibility of the relevant evidence; in (5)–(6), how you express your advice that I help my mother depends on the status in the context of the value of family vis-à-vis other potentially relevant values. This effect of contextual assumptions on the relative felicity of ‘ought’ and ‘must’ has been generally underappreciated in theoretical accounts (see n. 4 for notable exceptions).\(^7\)

Two clarificatory remarks: First, I said that ‘ought’ is preferred in contexts such as in (5) where it isn’t settled that the precondition for me to have an obligation is satisfied; yet it is worth observing that ‘must’ may be appropriate in certain contexts. If you can be presumed a normative authority on the issue and use ‘must’, I may accommodate by accepting that the value of family takes precedence. This isn’t an isolated phenomenon. Suppose that Alice, a young teen, is considering with her mother, Martha, whether to take the A-train or C-train to a concert. The A is quicker, but the C is safer. Martha regards Alice’s safety as paramount. Although the primacy of safety isn’t common ground, Martha can felicitously say:

(7) You must take the C-train, not the A-train.

The (teleological) necessity of Alice’s taking the C-train depends on the goal of traveling safely taking priority over the goal of traveling quickly. Given Martha’s authority in the context, she expects Alice to accommodate her assumption that this condition is satisfied. Such contexts notwithstanding — contexts in which the speaker doesn’t have or doesn’t wish to exercise the relevant authority — ‘ought’ will be preferred.

Second, saying that accepting ‘Ought $\phi$’ doesn’t conventionally commit one to accepting that $\phi$ is necessary doesn’t amount to the trivial claim that accepting ‘Ought $\phi$’ needn’t convey an assessment of the truth or likelihood of the prejacent given a body of information. In calling uses “deontic” I don’t assume that they are performative or have the same directive force. What is important about the uses in (5)–(6) is that they both directly address the practical question of what I am to do.

\(^7\)For instance, the semantics in Finlay 2009, 2010, Lassiter 2011, Swanson 2011 have no obvious mechanism for capturing this effect of context on uses of ‘ought’ vs. ‘must’. Von Fintel & Iatridou 2008 mentions in passing a possibly relevant role for context (pp. 139–140), but the issue isn’t investigated. See Rubinstein 2012: §2.2 for extensive broader critical discussion of previous comparative and domain restriction approaches to weak necessity modals. (More on von Fintel & Iatridou’s and Rubinstein’s accounts in §§4, 6.)
commit one to accepting ‘Must φ’. One might analyze the examples by positing concepts of distinctive kinds of necessity, and explain accepting ‘Ought φ’ as accepting that the weaker kind of necessity holds of φ: one might posit and formalize a concept of weak epistemic necessity such that accepting that it’s a weak epistemic necessity that d the colored pencils are in the drawer doesn’t require accepting that today is relevantly normal and the evidence implies d; and one might posit and formalize a concept of weak deontic necessity (weak obligation) such that accepting that I have a weak obligation to help my mother doesn’t require accepting that the value of family isn’t defeated. Yet such a move isn’t forced upon us. An alternative is to stick with the single familiar notions of necessity — e.g., epistemic necessity as following from a body of evidence, and deontic necessity as being obligatory and following from a body of norms (n. 6) — and try saying that ‘Ought φ’ can be accepted without accepting that φ is necessary, period. The following sections investigate the prospects for this latter approach. (More on comparisons with the former approach in due course.)

3 The analysis: Preliminary

The core of the approach to the weak/strong necessity modal distinction developed in the following sections is this: There is nothing specially “strong” about the necessity expressed by strong necessity modals. Strong necessity modals are given their usual semantics/pragmatics: ‘Must φ’ is true iff φ is necessary (in the relevant sense, i.e. epistemically, deontically, etc.; §2), and uses of ‘Must φ’ predicate the necessity of φ of the actual world — just as ‘May φ’ is true iff φ is possible, and uses of ‘May φ’ predicate the possibility of φ of the actual world. The apparent “weakness” of weak necessity modals, we can try saying, derives from their bracketing the assumption that the necessity of φ need be verified in the actual world. Accepting ‘Ought φ’ needn’t commit one to accepting that the actual circumstances verify the necessity of φ. Weak necessity modals afford a means of coordinating on the implications of our values, norms, etc. without having to settle precisely how they weigh against one another in particular circumstances, and while remaining open to new evidence about how they apply. This section begins developing these ideas within a standard premise-semantic framework for modals. The next section examines how the account may be implemented more precisely in the formal semantics and pragmatics.

As usual, I follow Kratzer (1977, 1981, 1991) in treating modals as semantically associated with a parameter determining a set of premises (propositions). Since modals can occur in intensional contexts, premise sets are indexed to a world of evaluation. What context

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*I will often omit this parenthetical, but it should be understood.
supplies that determines the reading of a modal is the family of world-indexed premise sets \((P_w)_{w \in W}\), or premise frame: a function \(P\) from worlds \(w\) to premise sets \(P(w)\).  

It is non-trivial how the considerations which seem intuitively relevant in interpreting modals are to be represented in the formal objects in the compositional semantics. First, the expressive, practical, and discourse-managing roles of modals such as ‘ought’ have been central in motivating expressivist theories such as Gibbard’s. However, the account and formalization of the weak/strong necessity modal distinction to follow will be neutral on matters of expressivism (contextualism/relativism/invariantism) — e.g., on what type of psychological state of mind is (conventionally) expressed by certain uses of ‘ought’/‘must’ sentences; whether notions of content or truth(-in-a-context) are given a fundamental explanatory role in explaining semantic properties of sentences or the dynamics of discourse; whether values for parameters such as premise frames figure in the derivation of semantic content, and whether such values are supplied by the context of utterance or a posited context of assessment; and so on. I will speak simply of “context,” and I relativize parameters such as premise frames simply to worlds. The implementations may be adapted along alternative contextualist/relativist/expressivist/invariantist lines (cf. Silk 2016a: 31–32, 68, 144-145, 221–224).  

Premise frames afford a natural way of encoding the contents of bodies of norms, preferences, etc. Call a conditional norm, preference, expectation, etc. a consideration. A contextually supplied premise frame \(P\) encodes the content of a body of considerations. The premises in a premise set \(P(w)\) represent what follows from a body of considerations given the relevant circumstances in \(w\). For instance, suppose you want to go for a run given that it’s sunny, that you didn’t just eat a burrito, and so on. The content of your preference can be encoded in a premise frame which assigns a premise set including the proposition that you go for a run to worlds where it’s sunny, etc. Similarly the normative import of a value of charity might be encoded in a deontic premise frame which assigns a premise set including the

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Footnotes:

9 See also van Fraassen 1973, Lewis 1973, 1981, Veltman 1976. Kratzer calls premise frames ‘conversational backgrounds’. Kratzer’s (1981, 1991) semantics uses two premise sets: a “modal base” \(F(w)\) that represents a set of relevant background facts in \(w\), and a (possibly inconsistent) “ordering source” \(G(w)\) that represents the content of some ideal in \(w\). Making the limit assumption (Lewis 1973: 19–20), Kratzer’s semantics treats ‘Must \(\phi\)’ as true at \(w\) iff \(\phi\) follows from every maximally consistent subset of \(F(w) \cup G(w)\) that includes \(F(w)\) (equivalently (Lewis 1981), iff every \(\leq_{G(w)}\)-minimal world in \(\bigcap F(w)\) is a \(\phi\)-world.). These complications won’t be relevant here; I simplify by treating modals as evaluated with respect to a single finite, consistent premise set. Nothing will turn on views about the limit assumption, or debates about so-called “weak vs. strong” semantics for epistemic ‘must’ (whether epistemic ‘must’ takes a non-empty ordering source).


11 See Silk 2017b (cf. 2015a, 2016a: §5.6) for developments of the following approach to interpreting the formal premise-/ordering-semantic apparatus.
proposition $d$ that you donate to charity to worlds where you have means of supporting your family, etc. — as with $P_d$ in (8), for relevant worlds characterized with respect to whether or not you have a job, there are reputable charitable organizations, and there is a local soup kitchen, and where $u$ is the proposition that you undermine local aid organizations, and $h$ is the proposition that you help at a local soup kitchen.

\begin{align*}
(8) & \quad P_d(JRS) = \{d, h, \ldots\} \\
& \quad P_d(JRS) = \{u, h, \ldots\} \\
& \quad P_d(JRS) = \{d, \ldots\} \\
& \quad P_d(JRS) = \{h, \ldots\} \\
& \quad \vdots
\end{align*}

The normative importance of helping those in need is reflected in $P_d$’s assigning a premise set that includes $h$ to worlds where there is a local soup kitchen. The normative importance of charitable giving is reflected in $P_d$’s assigning a premise set that includes $d$ to certain worlds where available aid organizations are reputable. However, the latter norm isn’t unconditional: it applies only in worlds where you can support your family, and where the organizations will put the donations to good use. The relative importance of supporting your family over helping others is reflected in $P_d$’s assigning a premise set that doesn’t include $d$ to worlds such as $JRS$ where the aid organizations are trustworthy but you don’t have a job.

With this way of understanding premise frames at hand, let’s return to the semantics and diagnoses of the §2-examples. I give strong necessity modals their usual semantics of necessity: ‘Must $\phi$’ is true at $w$, given a contextually supplied premise frame $P$, iff the prejacent proposition $\phi$ follows from $P(w)$, as in (9) (nn. 3, 9). The truth of ‘Must $\phi$’ depends on the value of $P$ at the evaluation world. Asserting ‘Must $\phi$’ commits one to accepting that $\phi$ follows from what the relevant considerations enjoin given the facts, $P(w)$ (n. 3).¹²

\begin{align*}
(9) & \quad [\text{Must } \phi]^{c,w} = 1 \text{ iff } \cap P_c(w) \subseteq [\phi]^c \\
(10) & \quad \text{A sentence } S \text{ is accepted in } c \text{ iff for every } w \in c, [S]^{c,w} = 1
\end{align*}

What distinguishes weak necessity modals, I have said, is that they bracket whether the necessity claim is verified in the actual world. We can adopt the following constraint on a semantics and pragmatics for ‘ought’, for some relevant body of considerations $P$:

\begin{align*}
(11) & \quad \text{It’s not the case that: ‘Ought } \phi’ \text{ is accepted in } c \text{ only if for every } w \in c, \cap P(w) \subseteq [\phi]^c
\end{align*}

¹²Subscripts on premise frames are used simply to indicate the intended contextually determined assignment. The familiar definition of acceptance in (10) blurs the distinction between contexts and the context sets (Stalnaker 1978) they determine. The context set is the set of live possibilities, the set of worlds compatible with what is accepted for purposes of conversation (Stalnaker 1978).
In a manner to be made precise, uses of ‘Ought $\varphi$’ present the possibility that $\varphi$ follows from the relevant considerations given certain circumstances, but without committing that such circumstances obtain or that the considerations actually apply. (We will examine what accepting ‘Ought $\varphi$’ does commit one to shortly.)

Let’s apply the preliminary analyses thus far to our §2-examples. Recall (3)–(4), reproduced in (12)–(13).

(12)  
\begin{align*}
  \text{Me:} & \quad \text{Where are the colored pencils?} \\
  \text{You:} & \quad \text{They ought to be in the drawer with the crayons.}
\end{align*}

(13)  
\begin{align*}
  \text{Me:} & \quad \text{Where are the colored pencils?} \\
  \text{You:} & \quad \text{They must be in the drawer with the crayons.}
\end{align*}

As with norms and preferences, one’s expectations given a body of evidence can be conditional — e.g., conditional on things being normal in the relevant respects. Let $w_N$ be a world in which your routine proceeds as normal, and let $w_N^\neq$ be a world where something abnormal happens to disrupt your routine.\(^{13}\) The conditional expectations concerning the colored pencils’ location can be encoded in a premise frame $P_e$ which (inter alia) assigns to $w_N$ a premise set $P_e(w_N)$ including the proposition $d$ that the colored pencils are in the drawer, and which assigns $w_N^\neq$ a premise set $P_e(w_N^\neq)$ including $\lnot d$. Given $P_e$, $d$ is an epistemic necessity at $w_N$ and not at $w_N^\neq$. So in order for $d$ to be accepted as epistemically necessary, per (9)–(10), the context set must be restricted to worlds like $w_N$ — worlds where you didn’t get distracted before putting the colored pencils away, no one played a trick on us and moved them, etc. If I’m unsure whether you are in a position to assume that nothing unusual led you to place the colored pencils somewhere else, I may challenge your assumption and raise a possibility that is incompatible with the epistemic necessity of $d$, as in (14).

(14)  
\begin{align*}
  \text{You:} & \quad \text{The colored pencils must be in the drawer with the crayons.} \\
  \text{Me:} & \quad \text{Really? I see that they aren’t on the shelf. But don’t you sometimes accidentally put them in the cabinet with the glue sticks?} \\
  \text{You:} & \quad \text{No, I never put them there. (/Oh, I forgot about that.)}
\end{align*}

But if you use ‘ought’, you aren’t committing to conditions being normal, as reflected in (15); hence my mentioning such alternative possibilities may be beside the point, as in (16).

(15)  
\begin{align*}
  \text{You:} & \quad \text{The colored pencils ought to be in the drawer with the crayons.} \\
  \text{Me:} & \quad \text{I checked and they aren’t there.} \\
  \text{You:} & \quad \text{Oh, then I’m not sure where they are. I would have expected them to be there.}
\end{align*}

(16)  
\begin{align*}
  \text{You:} & \quad \text{The colored pencils ought to be in the drawer with the crayons.}
\end{align*}

\(^{13}\)As above, $w_N$ and $w_N^\neq$ may be understood as representatives of relevant equivalence classes of worlds.
Me: #Really? I see that they aren't on the shelf. But don't you sometimes accidentally put them in the cabinet with the glue sticks?
You: I know; that's why I said ought!

Epistemic ‘Ought’ can be accepted even if it isn’t settled that certain conditions relevant to the epistemic necessity of \( \phi \) are satisfied.

Turn to our deontic modal examples in (5)–(6). My having an obligation to take care of my mother depends on the value of family being more important in my situation than other potentially competing values. Hence in order for the proposition \( m \) that I tend to my mother to follow from \( P(w) \) — what the normative considerations enjoin given the circumstances — it must be the case that the value of family takes precedence in my situation in \( w \). In (5), unlike (6), after my assertion is accepted it still isn’t settled whether this condition is satisfied. So, were you to use ‘must’ you would imply that you are foreclosing certain possibilities that I have left open. Unless you are in a position to do so (cf. (7)), your using ‘must’ is dispreferred. By accepting ‘Ought’ \( m \), we can provisionally proceed as if my helping my mother is required without needing to settle that the value of family is more important than other competing values we accept or may come to accept.

These examples highlight a critical role for weak necessity modals in discourse and deliberation. Take the deontic case. There are typically a range of interests, values, and norms potentially relevant for determining what to do. How the normatively relevant factors, whatever they are, interact is often highly complex. (One needs only a foray into deontic logic or normative ethics to convince oneself of this.) There may be uncertainty about the facts determining which considerations ultimately apply. For instance, given (8), one might not know the details about someone’s financial or family situation. Other empirical factors — how donations are used, short- and long-term impacts on those in need, etc. — can be even more difficult to assess. Hence one might not be in a position to commit to being in a world like JRS/JRȘ where norms of charity aren’t outweighed or defeated. Scenarios such as in (5)–(6) compound these challenges for normative and empirical evaluation. Using deontic ‘Must’ \( \phi \) may thus be inapt: one might not be in a position to commit to being in a world where \( \phi \) follows from what the relevant norms enjoin given the facts. Deontic ‘ought’ affords a means of guiding our deliberations and plans — indeed our conditional plans, as emphasized throughout Gibbard’s work on the psychology of normative judgment — while remaining open to new evidence about what values are at stake, and how they interact with one another and with the relevant facts.

I have proposed that what makes weak necessity modals “weak” is that they bracket whether the necessity of the prejacent is verified in the actual world: One can accept ‘Ought’ \( \phi \) without presupposing that \( \phi \) follows from what the relevant considerations \( P \) enjoin given the facts — e.g., without committing that all preconditions for \( \phi \) to be a genuine obligation are
satisfied, that one’s evidence for $\phi$ isn’t misleading, and so on. This feature of weak necessity modals isn’t the only dimension along which necessity modals differ (more on which in §§5–6). However, I claim that it does distinguish the class of weak necessity modals from the class of strong necessity modals. Previous accounts of weak necessity modals have often been developed by considering a limited range of modal flavors, in a limited range of contexts; extensions to other readings, to the extent that they are discussed at all, are often strained (e.g. Copley 2006, Swanson 2011, Rubinstein 2012, Charlow 2013, Ridge 2014, Portner & Rubinstein 2016, Yalcin 2016). The account in this paper, by contrast, generalizes across flavors of modality, and captures a precise sense in which the relative felicity of ‘ought’ and ‘must’ depends on standing assumptions. Weak necessity modals afford a means of coordinating on the implications of our values, expectations, etc. without needing to settle precisely how they apply and weigh against one another in particular circumstances (n. 7).

The preliminary account thus far raises many questions. Yet even at the present level of abstraction, we can see that the approach to the weak/strong necessity modal distinction in this paper differs crucially from other main approaches in the literature — e.g. probabilistic and comparative possibility approaches (Finlay 2009, 2010, 2014, Lassiter 2011) and domain restriction approaches (Copley 2006, von Fintel & Iatridou 2008, Swanson 2011, Rubinstein 2012, Charlow 2013). For instance, domain restriction accounts maintain that accepting ‘Ought $\phi$’ requires accepting that $\phi$ is a necessity, and that the truth of ‘Ought $\phi$’ at $w$ requires that $\phi$ is a necessity at $w$; what distinguishes ‘ought’ from ‘must’ is the logical strength of the necessity — roughly, implication of the prejacent by a superset of premises (equivalently, truth of the prejacent throughout a subdomain of worlds). Weak necessity modals are treated as expressing a logically weaker kind of necessity. The present approach rejects these claims (§2).

4 Weak necessity and the modal past

Our project is to develop an account to the weak/strong necessity modal distinction that systematizes a broader range of linguistic phenomena (§1). Taking on this more demanding goal will require sustained investigations into diverse domains. This section examines how notions of “ought” are expressed cross-linguistically to motivate several ways of implementing the foregoing approach to the weak/strong necessity modal distinction and formalizing the semantics of ‘ought’. §5 shows how the resulting account helps explain various seemingly unrelated semantic/pragmatic properties of ‘ought’ and ‘must’.
4.1 Data

Past forms of modals — in English, ‘would’ for ‘will’, ‘could’ for ‘can’, ‘might’ for ‘may’ — are often used not to indicate past time reference, but to express tentativeness or politeness and weaken the apparent force, as in (17)–(19). These forms are also the forms that appear in the consequents of subjunctive conditionals, as in (20).

(17)  
\begin{enumerate}
\item a. I will add one point to this discussion.
\item b. I would add one point to this discussion.
\end{enumerate}

(18)  
\begin{enumerate}
\item a. Alice will/may/can’t be at home now.
\item b. Alice would/might/couldn’t be at home now.
\end{enumerate}

(19)  
\begin{enumerate}
\item a. May/Can I comment on your proposal?
\item b. Might/Could I comment on your proposal?
\end{enumerate}

(20)  
If you took the flight tomorrow, you would/could/might get there in time.

Palmer 2001 dubs such uses of past tense the “modal past.”

Strikingly, ‘ought’ patterns with the past-marked modal forms. First, ‘ought’ weakens the apparent force of ‘must’ — hence the common label “weak necessity modal.” Second, ‘ought’, unlike ‘must’, can appear in subjunctive conditionals:

(21)  
\begin{enumerate}
\item a. If Alice came to the party tomorrow, Bert ought to leave.
\item b. #If Alice came to the party tomorrow, Bert must(ed) leave.
\end{enumerate}

Third, ‘ought’ is non-entailing. For simple clauses ’ϕ’, ’Ought ϕ’, unlike ’Must ϕ’, is compatible with ’¬ϕ’:

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(i)  
I wanted to ask you a question.  
(Bybee 1995: ex. 21)

(ii)  
I thought/was thinking about asking you to dinner.  
(Fleischman 1989: 8)

(iii)  
\begin{enumerate}
\item a. A: How old is John?
B: He’d be about sixty.
\item b. (If John were still alive,) he’d be about sixty.
\end{enumerate}  
(implicit CF)  
(weakness interpretation)  

I am not presupposing that all modal-past forms necessarily convey weakness/tentativeness relative to their nonpast counterparts (cf. epistemic ‘might’ in contemporary English; Coates 1983, Palmer 1990, Huddleston & Pullum 2002, Collins 2007). My descriptive use of Palmer’s label makes no theoretical assumptions about how the modal/weakness/tentativeness interpretations arise or are related to temporal interpretations of past morphology (n. 18).
I could give to Oxfam, but I won’t.

a. Alice ought to be here by now, but she isn’t.
b. #Alice must be here by now, but she isn’t.

Indeed, when used with the perfect, ‘ought’ implicates the negation of the prejacent.

I could have given to Oxfam.  \(\text{(Implicates: I didn’t)}\)

We ought to have given to Oxfam.  \(\text{(Implicates: we didn’t)}\)

b. #We must have given to Oxfam (but we didn’t).

‘Must’ cannot even receive a deontic reading when used with past time reference. ‘Ought’, unlike ‘must’, can be used to communicate that an obligation held in the past. That ‘ought’ can take scope under the perfect in (25a) is a fourth respect in which ‘ought’ patterns with past-marked modal forms (Condoravdi 2002).

In sum, although ‘must’ doesn’t have a past form, ‘ought’, we can try saying, functions notionally as its modal past (cf. Palmer 1990, 2001). This is surprising. But it becomes less surprising when we examine other languages. Let’s use ‘ought’ for the notion which in English is expressed with weak necessity modals such as ‘ought’ (‘should’, etc.), and use ‘must’ for the notion which in English is expressed with strong necessity modals such as ‘must’ (‘have to’, etc.). As emphasized in von Fintel & Iatridou’s (2008) seminal discussion of weak necessity modals, it’s cross-linguistically common to mark the semantic distinction between ought and must morphologically rather than lexically (see also Palmer 2001, McGregor & Wagner 2006, Van Linden & Verstraete 2008, Matthewson 2010). A notion of ought is often expressed not by using a different word — like ‘ought’ in English — but by using the modal-past form of a strong necessity modal, i.e. the form of a strong necessity modal that is used in counterfactuals.\(^1\)

von Fintel & Iatridou (2008) approach the cross-linguistic data in a domain restriction account of weak necessity modals. They treat weak necessity modals as quantifying over “the best of the best” worlds — roughly, the relevant \(P(w)\)-compatible worlds that are compatible with an additional premise set representing a secondary ideal (in Kratzer’s terminology, a secondary ordering source).\(^2\) In passing, von Fintel & Iatridou speculate that “the counter-

\(^1\)As von Fintel & Iatridou note (2008: 126n.22), ‘ought’ fits the cross-linguistic pattern historically; it was formerly the past subjunctive of the verb ‘owe’. Modal-past forms in other languages may be derived from various elements, not simply past tense (e.g. Iatridou 2000).

\(^2\)More precisely (see n. 9): For a set of worlds \(W\) and premise set \(S\), let the \(S\)-minimal worlds in \(W\) be the worlds \(u \in W\) such that no world \(v \in W\) satisfies a proper superset of propositions \(p \in S\). (For finite consistent premise sets the set of \(S\)-minimal worlds is \(W \cap S\).) Let \(s\) be a set of relevant worlds (as e.g. determined by a Kratzerian modal base), \(G_1(w)\) be a premise set representing some primary ideal, and \(G_2(w)\) be a premise set representing some secondary ideal. On von Fintel & Iatridou’s semantics, whereas ‘must’ universally quantifies
factual marking is co-opted here in a somewhat meta-linguistic kind of way: ‘if we were in a context in which the secondary ordering source was promoted [to primary status], then it would be a strong necessity that...’ (2008: 139). The tentativeness associated with counterfactual marking is attributed to the fact that the premises in the secondary premise set needn’t apply: “The choice of whether to really promote the secondary ordering source is left open” (2008: 139).

I find these suggestions about the cross-linguistic data unsatisfying (see Rubinstein 2012 for further criticisms). First, von Fintel & Iatridou briefly suggest treating the secondary ordering source for epistemic readings as representing what is normally the case, and the secondary ordering source for deontic readings as representing “less coercive sets of rules and principles (2008: 119)”; however, no general story is given about what primary vs. secondary ordering sources represent per se, or how an ordering source is determined as primary or secondary for different types of readings across contexts. Absent an independent understanding of what makes it the case about a speaker that she is counterfactually promoting a secondary ordering source, the proposed story about the role of the counterfactual marking seems ad hoc. A worry is that the explanation of the tentativeness associated with counterfactual morphology redescribes what needs to be explained. The tentativeness is “explained” by introducing a parameter representing “shakier assumptions” (2008: 119n.9) that may not apply — uncharitably put, a parameter representing considerations one may only be tentatively committed to. Finally, it’s unclear how von Fintel & Iatridou’s explanation would generalize to tentativeness effects of counterfactual morphology on other lexical items — e.g., possibility modals like ‘might’, or desire verbs like ‘wish’, which as von Fintel & Iatridou note, are in many languages expressed via counterfactual morphology on the word for ‘want’ (see Iatridou 2000, n. 14). For instance, interpreting modal-past forms of possibility modals with respect to a secondary ordering source incorrectly predicts a strengthening effect. The tentative use of ‘might’/’could’ in (19) cannot be derived by counterfactually “promoting” a secondary ordering source and then evaluating the possibility of the prejacent.  

4.2 Implementations: A modal-past approach

This subsection examines how independent work on the semantics/pragmatics of counterfactual marking may be incorporated into our analyses of ‘ought’ and ‘must’. I want to be over the $G_i(w)$-minimal $s$-worlds, ‘ought’ universally quantifies over a subset of these worlds: the $G_i(w)$-minimal worlds among the $G_j(w)$-minimal $s$-worlds. ‘Ought’ quantifies over the $G_i(w)$-minimal worlds that ‘must’ quantifies over. See Sloman 1970, Williams 1981, McNamara 1990 for informal precedents. I return to general worries for domain restriction analyses below.  

For a set of relevant worlds $s$ and primary and secondary ordering sources $G_i(w), G_j(w)$, let $i$ be the set of $G_i(w)$-minimal $s$-worlds, and $j$ be the set of $G_j(w)$-minimal $i$-worlds. Since $j \subseteq i$, that there is a $\phi$-world in $j$ asymmetrically implies that there is a $\phi$-world in $i$ (n. 16).  

15
clear that I am not assuming that lexicalized weak necessity modals are “decomposed” into a strong necessity modal and counterfactual features (schematically STRONG+CF). Methodologically, considering the cross-linguistic data may provide insight into the semantics of lexicalized expressions of ought. But we should be careful not to “read off” a semantics for ‘ought’ from a semantics for ‘must’ and counterfactual morphology. ‘Ought’ doesn’t mean ‘would have to’ (more on which in § 7.1).

It’s generally agreed that counterfactual marking signals that the worlds being talked about (“topic worlds”) needn’t be candidates for actuality. There are various ways of formalizing this signal and deriving it in the grammar. To fix ideas I assume that counterfactual marking cancels a presupposition (e.g. associated with indicative) that the modal’s domain of quantification, or set of topic worlds more generally, is a subset of the context set.1⁸

I have said that no innovations are introduced into the semantics/pragmatics of strong necessity modals. ‘Must ϕ’ is given its familiar semantics of necessity — ‘Must ϕ’ is true at w iff ϕ follows from P(w) — and uses of ‘Must ϕ’ carry the usual indicative presupposition that the worlds being talked about are in the context set. One way of implementing the general indicative presupposition is as restricting the domain of the interpretation function to proper points of evaluation — contexts c and worlds w such that w ∈ c. Applying this to the case of a necessity modal yields:1⁹

\[
\text{Definition 1. } [\text{Must } \phi]^c = \lambda w : w \in c . (\cap P(w) \subseteq [\phi]^c)
\]

Uttering ‘Must ϕ’ predicates the necessity of ϕ throughout the context set, i.e. predicates the proposition that ϕ follows from P(w) of every world in the context set — just as uttering ‘May ϕ’ predicates the possibility of ϕ throughout the context set, or uttering ‘ϕ’ predicates ϕ throughout the context set. The treatment of ‘must’ vis-à-vis necessity is parallel to the treatment of ‘may’, ‘can’, etc. vis-à-vis possibility:

\[
\text{Definition 2. } [\text{May } \phi]^c = \lambda w : w \in c . (\cap (P(w) \cup ([\phi]^c)) \neq \emptyset)
\]

It’s in this sense that there is nothing special or distinctively “strong” in the semantics/pragmatics of (so-called) strong necessity modals.

Turn to weak necessity modals. A natural idea is that the counterfactual marking in the relevant languages cancels an assumption that the relevant worlds at which the prejacent is

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1⁸ An alternative is to treat counterfactual marking as positively presupposing that the set of topic worlds isn’t included in the context set. For discussion and technical implementations see e.g. Stalnaker 1975, von Fintel 1998, Iatridou 2000, Ippolito 2003, Schlenker 2005, Arregui 2009, Bittner 2011.

1⁹ In a framework with object-language world-variables, the presupposition would be that the assignment function maps the given world-variable to a world in the context set (cf. Schlenker 2005).
necessary are in the context set\textsuperscript{20} — hence the §2-observation and constraint in (11) that accepting ‘Ought \( \phi \)’ doesn’t require accepting that all the preconditions for \( \phi \) to be necessary are satisfied. The apparent “weakness” of uses of ‘Ought \( \phi \)’, compared to uses of ‘Must \( \phi \)’, derives from failing to presuppose that the topic worlds where \( \phi \) is necessary are in the context set. Call this approach a modal-past approach to weak necessity modals and the weak/strong necessity modal distinction. The remainder of this section presents several ways of implementing a modal-past approach in the formal semantics and begins investigating their costs and benefits.

One straightforward way of treating ‘ought’ as the semantic modal past of ‘must’ would be to treat ‘ought’ as having an ordinary semantics of necessity, like ‘must’, but lacking the presupposition that the worlds being talked about — the relevant worlds at which \( \phi \) is necessary — are in the context set, as reflected in Definition 3.

\textbf{Definition 3.} \([\text{Ought } \phi]^{c} = \lambda w . \cap P(w) \subseteq [\phi]^{c}\) \hfill (v1)

Informally, uttering ‘Ought \( \phi \)’ places the necessity claim on the “conversational table,” but doesn’t conventionally commit one to its truth (cf. Silk \textit{2016b}). Implementing a modal-past approach as in Definition 3 faces pressing challenges in the discourse dynamics and compositional semantics. On a standard Stalnakerian theory of conversation, assertions propose to restrict the context set to worlds where the asserted content is true (Stalnaker 1978). But for any \( w \in c \), ‘Ought \( \phi \)’ is true at \( w \) according to Definition 3 iff ‘Must \( \phi \)’ is true at \( w \) according to Definition 1.\textsuperscript{21} Some alternative mechanism would be needed to distinguish how ‘ought’ and ‘must’ update context, e.g. allowing uses of ‘ought’ to distinguish among worlds outside the context set or have some non-eliminative effect. Second, it isn’t obvious how the relative weakness of ‘ought’ vis-à-vis ‘must’ would carry over in embedded environments that shift the evaluation world, e.g., indicative conditionals or attitude ascriptions.

One way of avoiding these challenges is to build a counterfactual element into the semantics of ‘ought’. Consider Definition 4, for contextually supplied selection function \( h \).

\textbf{Definition 4.} \([\text{Ought } \phi]^{c,w} = 1 \text{ iff } \forall w' \in h(c)(w) : \cap P_{c}(w') \subseteq [\phi]^{c}\) \hfill (v2)

To a first approximation, one can think of \( h(w) \) as picking out a set of relevant worlds that are minimal/“preferred” in some contextually relevant sense — most normal, expected, de-

\textsuperscript{20}On this I disagree with Arregui \textit{2010}. Though Arregui associates ‘should’ with a past morphology feature, she denies that the feature is interpreted with ‘should’. Arregui maintains that ‘Should \( \phi \)’ presupposes that the modal’s quantificational domain is included in the context set (for non-stative ‘\( \phi \)’). We have seen that this is incorrect. Non-entailing uses ‘(Should \( \phi \) \& \( \neg \phi \)’) are consistent (more on which in §5):

(i) Alice should give to charity but she won’t.

\textsuperscript{21}The right-to-left direction is obvious. Left-to-right: if \([\text{Ought } \phi]^{c}(w) = 1\), then \( \cap P(w) \subseteq [\phi]^{c} \); but then, \([\text{Must } \phi]^{c}(w) = 1\) since, by hypothesis, \( w \in c \).
sirable, etc., depending on the context (cf. Starr 2010: 167, Grosz 2012).

In (12), \( h(w) \) might select the maximally \( w \)-normal worlds where you don't get distracted before putting the colored pencils away, no one hides them, etc. — worlds like \( w_N \). In (26), \( h(w) \) might select the maximally \( w \)-similar worlds where you have a job, the available charities are trustworthy, etc. — worlds like JRS from (8).

(26) You ought to donate to charity.

‘Ought \( \phi \)’ is true iff these worlds \( w' \) verify the necessity of \( \phi \), i.e. iff \( \phi \) follows from the relevant considerations \( P \) at every \( w' \in h(w) \). The point from (11) that accepting ‘Ought \( \phi \)’ doesn’t require settling that \( \phi \) is actually necessary is captured via \( h \): the set of worlds \( h(w) \) at which the necessity of \( \phi \) is evaluated might include the evaluation world \( w \), but it might not.

Definition 4 uses a simple selection function to determine the worlds at which the necessity of the prejacent is evaluated. The semantics could be complicated by deriving the set of selected worlds from more basic elements, such as independently represented orderings/premise sets of the relevant types (normality, desirability, etc.). Even bracketing such structure, the selection of worlds could be treated as explicitly depending on the prejacent \( \phi \) or a contextually relevant set of circumstances \( C \), i.e. \( h(w, \phi, C) \). This would reflect the intuitive idea that in interpreting ‘Ought \( \phi \)’ one evaluates what follows from \( P \) at worlds satisfying certain conditions plausibly relevant to whether \( \phi \) is a necessity.\(^{23}\) In (12) one looks at worlds \( u \) satisfying what is normally the case in matters concerning where the colored pencils are, and one checks whether, conditional on such facts, the relevant information (represented by the given epistemic premise frame \( P_e \)) implies that the colored pencils are in the drawer, i.e. whether \( \bigcap P_e(u) \subseteq \text{drawer} \); in (26) one looks at worlds \( v \) satisfying what is normally or preferably the case in matters concerning whether to donate, and one checks whether, conditional on such facts, the relevant norms (represented by the given deontic premise frame \( P_d \)) enjoin you to donate, i.e. whether \( \bigcap P_d(v) \subseteq \text{donate} \). Alternatively, we might simply use a world-indexed function \( h \), and appeal to such additional factors in an extra-semantic account of how \( h \) is determined in concrete discourse contexts.

The approach in Definition 4 raises the question of what distinguishes ‘Ought \( \phi \)’ from counterfactual necessity sentences ‘If \( \chi \), it would have to be that \( \phi \)’ (more on which shortly). A comparative semantics such as Definition 5 may avoid this issue — where \( \prec_w \) is a partial order on propositions along a contextually relevant dimension (likelihood, normality, desirability,

\(^{22}\)I often use double quotes around ‘preferred’ as a reminder that what is intended is the generalized notion of minimality, not necessarily bouletic or deontic (more in §7.1).

\(^{23}\)My 2012 implemented these ideas by analyzing weak necessity modals as expressing a kind of conditional necessity (cf. Wertheimer 1972). I no longer endorse this as a way of capturing the points in §§2–3 or understanding a modal-past approach. Conditional necessity analyses make pressing the challenge of distinguishing uses of ‘ought’ from (implicit or explicit) conditional necessity claims (see below and §7.1).
etc.), and $s(w, p)$ is the set of closest $p$-worlds to $w$, understanding the relevant closeness relation as the relation that would figure in interpreting a counterfactual.

Definition 5. \[ \text{Ought } \phi \] \text{cf } w = 1 \text{ iff } \{ u : u \in s(w, \bigcap P_c(u) \subseteq [\phi]') \} \prec_w \{ v : v \in s(w, \bigcap P_c(v) \nsubseteq [\phi]') \} \text{ (v3)}

This treats 'Ought $\phi$' as saying that it would be better (in a relevant sense) if $\phi$ was necessary (in a relevant sense). Like Definition 4, Definition 5 avoids treating the truth of 'Ought $\phi$' at $w$ as requiring that $\phi$ be a necessity at $w$. The closest worlds $u$ at which $\phi$ is necessary needn't be in the context set. Yet we can still see how uses of 'ought' may bear on interlocutors' views about what is necessary. 'Ought $\phi$' introduces the possibility that $\phi$ is necessary and comments on it. The attitudinal comment is that the (closest) worlds in which $\phi$ is necessary are $<_w$-better — more desirable, normal, expected, etc., depending on the context. A potential worry is why the semantics for 'ought' and 'must' should be so dissimilar. One must provide a precise sense in which 'ought' is weaker than 'must' (logically, conversationally). Further, it isn't obvious how a comparative semantics like Definition 5 might derive from STRONG+CF or shed light how ought-interpretations of STRONG+CF arise in other languages.

Definitions 3–5 provide several avenues for developing a modal-past implementation of the core ideas from §2. The semantics avoid analyzing accepting 'Ought $\phi$' in terms of $\phi$ being a necessity (in any sense) at every $w$ in the context set, and they offer precise representations of 'ought' as the notional modal-past of 'must'. 'Ought' is interpreted with respect to the same body of considerations $P_c$ as 'must'; and 'introduces' the proposition $[\lambda w. \bigcap P_c(w) \subseteq [\phi]']$ that the prejacent is a necessity relative to $P_c$; what distinguishes 'ought' is that attitude taken toward that proposition needn't be acceptance — in Definition 3, by allowing improper points of evaluation; in Definition 4, by including an additional layer of modality and evaluating the truth of $[\lambda w. \bigcap P_c(w) \subseteq [\phi]']$ throughout a set of possibly counterfactual worlds; and in Definition 5, by commenting on the preferredness (in a relevant sense) of $[\lambda w. \bigcap P_c(w) \subseteq [\phi]']$. It will be useful in what follows to have a particular analysis at hand. To fix ideas I will generally assume the semantics in Definition 4. It's less clear how a comparative meaning such as Definition 5 might come to be given to ought-expressing uses of STRONG+CF in other languages, and I think the ideas motivating Definition 3 are more perspicuously developed in a dynamic setting (see Silk 2016b). I leave more thorough comparisons for future work.

\[ \text{Definition 5 could be refined depending on one's views on the semantics of comparative possibility (Lassiter 2011, Kratzer 2012), but the basic idea should be clear enough. Contrast the comparative probability semantics for 'ought' in Finlay 2009, 2010, 2014, which treats 'Ought $\phi$' as saying (roughly) that $\phi$ is more likely than any relevant alternative to $\phi$. In contrast Definition 5 treats 'Ought $\phi$' as making a comparative claim about the necessity of $\phi$, rather than a comparative claim about $\phi$, and it generalizes the relevant comparative notion via the context-dependent parameter $<$. (See Rubinstein 2014, Portner & Rubinstein 2016) for additional discussion of connections between weak necessity modals and comparatives/gradability.} \]
4.3 Entailments?

An initially plausible thought is that — as the (not-necessarily-innocent) labels might suggest — strong necessity modals entail weak necessity modals, which entail possibility modals. However, a worry for the modal-past analyses is that they seem to predict the consistency of sentences ‘Ought \( \phi \) and must \( \neg \phi \)’ and ‘Ought \( \phi \) and \( \neg \)may \( \phi \)’, as in (27).

(27) #I mustn't/can't/may not lie to Alice, but I ought to.

The ‘must’/‘may’ conjunct is true at \( w \) iff the necessity of \( \neg \phi \) is verified at \( w \); the ‘ought’ conjunct is true at \( w \), assuming the semantics in Definition 4, iff the necessity of \( \phi \) is verified at every “preferred” (desirable, expected, etc.) world \( w' \in h(w) \). Accepting (27) requires restricting the context set to worlds \( w \) such that \( P(w) \) implies \( \neg \phi \) and, for every \( w' \in h(w) \), \( P(w') \) implies \( \phi \). Absent further constraints on \( h \), such conditions are consistent.

I think this prediction is actually a feature, not a bug. ‘Must,’ ‘ought,’ and ‘may’ cannot be ordered by logical strength simply in virtue of their conventional meanings. ‘Must \( \phi \)’ \( \models \) ‘May \( \phi \)’, no doubt about that; but ‘Must \( \phi \)’ \( \not\models \) ‘Ought \( \phi \)’ and ‘Ought \( \phi \)’ \( \not\models \) ‘May \( \phi \)’.

Epistemic ‘Must \( \phi \)’ commits the speaker to high credence in \( \phi \) and epistemic ‘May \( \phi \)’ commits the speaker to some credence in \( \phi \). Epistemic ‘must’ entails epistemic ‘may’. But epistemic ‘Ought \( \phi \)’ doesn’t commit the speaker to any credence in \( \phi \). This invalidates the entailments between epistemic ‘ought’ and epistemic ‘must’/‘may’ (cf. Copley 2006, Silk 2016b, Swanson 2016a, Yalcın 2016; see n. 6):25

(28) #Alice can’t be home yet; she hasn’t called, and she always calls right away to let us know she got back safely. She must be home already. I hope there wasn’t an accident.

(29) Alice must not/can’t be home yet; she hasn’t called, and she always calls right away to let us know she got back safely. She ought to be home already. I hope there wasn’t an accident.

Although epistemic ‘Ought \( \phi \)’ doesn’t commit the speaker to any unconditional credence in \( \phi \), it still conveys a doxastic attitude. Informally, it conveys a conditional attitude about the necessity of \( \phi \) given a body of evidence, on the assumption that conditions are relevantly normal (§§ 2–4; cf. Groenema 1995: 72–73; Wedgwood 2007: 118–119; Yalcın 2016).

One may infer from the denial of ‘\( \phi \)’ that conditions aren’t normal. Yet since ‘ought’ can be

25I use ‘can’t’ for the external negation (\( \neg < \text{can} \)), since with epistemic ‘may not’ the negation is internal. With ‘mustn’t’ the negation is internal (\( \text{must} < \neg \)). With deontic ‘may not’ the negation is external (\( \neg < \text{can/may} \)).
used to talk about the modal status of $\phi$ at non-actual possibilities, epistemic ‘Ought $\phi$’ can still be true and accepted.

It’s hard to come up with coherent deontic examples analogous to (29). Hard, but perhaps not impossible:

(30) I must tell my wife about the affair. I know I shouldn’t; it’ll only hurt her. But I must.
(31) I know I shouldn’t tell my wife about the affair; it’ll only hurt her. But I can’t lie to her.
(32) #I must tell my wife about the affair. I know I can’t; it’ll only hurt her. But I must.

It isn’t immediately obvious what to say about these examples. It’s interesting that entailments from ‘must’ to ‘ought’ and from ‘ought’ to ‘may’ seem more compelling with non-epistemic readings. Yet insofar as we want unified semantics for modal verbs that generalize across readings, we should prefer an account that avoids treating the entailments as semantically valid.

5 “Weakness” in weak necessity modals

§4 developed the §3-account by treating ‘ought’ as the notional modal past of ‘must’ and incorporating general insights about the semantics/pragmatics of counterfactual marking. This section shows how a modal-past account of the “weakness” of weak necessity modals systematizes several seemingly unrelated puzzles of entailingsness and performativity with ‘ought’ and ‘must’.

5.1 Diagnosing weakness

A modal-past analysis gives precise expression to the informal intuition that ‘ought’ is weaker and more tentative than ‘must’. In uttering ‘Ought $\phi$’ the speaker fails to mark the necessity claim as being about worlds that are candidates for actuality. However, as Stalnaker notes, “normally a speaker is concerned only with possible worlds within the context set, since this set is defined as the set of possible worlds among which the speaker wishes to distinguish” (1975: 69). So, uttering ‘Ought $\phi$’ implicates that one isn’t in a position to commit to $\phi$’s being a necessity throughout the set of live possibilities. Grice’s first quantity maxim — “Make your contribution as informative as is required” (Grice 1989: 26) — can then be exploited to generate a familiar upper-bounding implicature (Horn 1972, Gazdar 1979; n. 26): Using ‘ought’ implicates that for all one knows — better, for all one is willing to presuppose in the conversation — ‘Must $\phi$’ is false. This implicature has the usual properties of implicatures; it’s reinforceable, cancelable, and suspendable:
I ought to help the poor, but I don’t have to.

I ought to help the poor. In fact, I must.

I ought to help the poor. Maybe I have to.

In (2a) the speaker first conveys that the worlds in which my helping the poor is deontically necessary needn’t be live possibilities, and then commits that what holds in the former worlds also holds in the actual world. The implicature data with ‘ought’ can be treated analogously to implicature data with subjunctive conditionals.

a. If you had the flu, you would have exactly the symptoms you have now.

(bf. Anderson 1951: 53)

b. If you had the flu, you would have very different symptoms from the symptoms you have now.

c. If you had the flu, you would be sick. Maybe you do have the flu; you are pretty congested.

Likewise we can assimilate the tentativeness of ‘ought’ to the tentativeness of modal-past forms generally, as in non-counterfactual subjunctive conditionals (“future-less-vivid” conditionals) such as (20) above and (35).

If you came to our party tomorrow — and I’m not saying that you will — you would have a great time.

Using the past form highlights the possibility that the marked clause might not ultimately be accepted. The basis of the scale between ‘ought’ and ‘must’ isn’t fundamentally logical but epistemic strength (§4). ‘Ought’ and ‘must’ are ordered, not in terms of e.g. subset/superset relations in their domains of quantification, as per domain restriction accounts (§§2–3), but in terms of epistemic attitude toward the proposition that ϕ is necessary.26

5.2 Entailingness and directive force

Though many authors have claimed that epistemic ‘Ought ϕ’ expresses that ϕ is probable,27 we have seen that this isn’t quite right (§4): Epistemic ‘Ought ϕ’ doesn’t commit the speaker to any unconditional credence in ϕ, as reflected in (23), reproduced in (36).

a. Alice ought to be here by now, but she isn’t.


b. #Alice must be (/may be, /is probably) here by now, but she isn’t.

Epistemic ‘(Must $\phi$) $\land \neg \phi$’ is anomalous in a way that epistemic ‘(Ought $\phi$) $\land \neg \phi$’ is not. Surprisingly, there is robust evidence that this is the case with deontic readings as well. 28 When one wishes to convey that one thinks a given obligation won’t be satisfied, one typically uses ‘ought’/’should’ rather than ‘must’.

(37)  
a. He should/ought to come tomorrow, but he won’t.

b. *He must come tomorrow, but he won’t.

(Palmer 1990: 123; judgment Palmer’s)

(38)  
a. You ought to help your mother, but you won’t (/I know you won’t).

b. ?You must help your mother, but you won’t (/I know you won’t).

As Eric Campbell insightfully puts it, “The idea that one must (not) do something… does not generally indicate that one option is simply better than another option, but that the other is out of bounds,” “off the table,” “closed off”; alternative possibilities become “unthinkable” (Campbell 2014: 463–465). Of course obligations can go unfulfilled. What is interesting is that speakers appear to assume otherwise, at least for the purposes of conversation, when expressing obligations with ‘must’. 29

Call data concerning sentences of the form ‘(MODAL $\phi$) $\land \neg \phi$’ entailingness data (though see n. 30). Our discussion in §4 suggests a natural way of capturing entailness data such as (36)–(38). Evaluating ‘Ought $\phi$’ can take us to worlds outside the context set when assessing the necessity of $\phi$. There is no requirement that the value of the premise frame $P$ at worlds outside the context set be compatible with the common ground. So, ‘Ought $\phi$’ can be true at a world $w$ in the context set even if every world in the context set is a $\neg \phi$-world; hence the coherence of ‘(Ought $\phi$) $\land \neg \phi$’. Since ‘Must $\phi$’ doesn’t have a counterfactual element to its meaning, the context set must include $\bigcap P(w)$, for all worlds $w$ in the context set. So, if


29 Cf.: “The basic strong obligation component common to [‘got to’, ‘have to’, and ‘must’] is ‘I can’t think: X will not V’; note that this does not mean that the obligated event will inevitably take place but rather that the speaker is operating on this assumption” (Myhill 1996: 348–349).
'Must $\phi$' is accepted, $\neg\phi$ cannot be satisfied throughout the context set; hence the incoherence of $(\text{Must } \phi) \land \neg\phi$ on any reading (qualifications shortly).\(^{30,31}\)

How exactly to derive these points about the relation between the modals’ premise sets and the context set depends on general issues of presupposition and verbal mood (see §4). For instance, with modals that lack a counterfactual meaning component, there might be a context-set presupposition on the embedded clause that it denotes a proposition defined only at worlds in the context set (e.g., due to an indicative presupposition transmitted to the embedded clause, or a presupposition in the modal’s lexical semantics regarding the evaluation worlds for its clausal argument). So, for ‘must $\phi$’ to be true at $w$, $\bigcap P(w)$ must be a subset of $\{ u: [\lambda v: v \in c. [\phi]_{c:v} = 1](u) = 1 \}$, hence $\bigcap P(w)$ must be a subset of the context set. Alternatively, there might be a general presupposition on the values of quantifier domain variables, such as premise frames, that they be compatible with the common ground at worlds in the context set. Either way, what is important here is that non-counterfactual modal constructions (modal sentences, indicative conditionals, etc.) presuppose that the do-

\(^{30}\)This explanation leaves open whether ‘$\neg\phi$’ may be true at some worlds in the context set, and thus seems to predict that accepting deontic ‘Must $\phi$’ is compatible with with accepting the epistemic possibility of $\neg\phi$. This prediction appears to be borne out by corpus data.\(^{31}\)Verstraete 2007 cites the naturally occurring example in (i) with an imperative, adapted with ‘must’ in (ii).

(i) You’ve got to take a stand Tom. You’ve got to do it mate. […] Don’t stand for it, because if you do you’ll just get trampled on. (CB ukspok)

(ii) You mustn’t stand for it, because if you do you’ll just get trampled on.

Though (iii) strikes me as somewhat anomalous, this is arguably due to a general norm of cooperative conversation that interlocutors do what they can to make the actual world be among the preferred/best worlds (cf. Portner 2007: 358). On this diagnosis, (iii) would be anomalous to the extent that it’s anomalous to commit someone to help see to it that $\phi$ while expressly admitting the possibility of $\neg\phi$.

(iii) ?You must go to confession, but maybe you won’t (/you might not).

\(^{31}\)This of course isn’t the only possible diagnosis. One might attempt to explain the entaillingness data by positing additional features of the conventional meaning of ‘must’. For instance, first, one might say that ‘must’ is only interpreted with respect to a modal base (or the union of several modal bases), not an ordering source (n. 9; cf. Hacquard 2009: 290n.9). (As far as the interpretation of ‘must’ goes, “all laws are natural laws” (cf. Piaget 1962: 340.) Since modal bases consist of propositions true at the evaluation world, ‘Must $\phi$’ would entail ‘$\phi$’. Second, following Swanson 2016a one might include in the semantic entry for ‘must’ but not ‘ought’ a constraint requiring high credence in the prejacent. Third, one might appeal to more basic performative properties of ‘ought’ and ‘must’ (see n. 32). However, it would be theoretically preferable if we could explain the data in terms of independent features of the semantics of ‘ought’ and ‘must’, as I am arguing we can. There are reasons for introducing ordering sources into the semantics, both empirical and conceptual (e.g., for handling inconsistent premise sets and gradability, and distinguishing dynamic and deontic modality) (Kratzer 1981, 1991; cf. Grove 1988, Spohn 1988, Kaufmann 2005). I argue below that the analysis in the main text derives the modals’ contrasting performative properties without needing to take them as basic. We can explain the entaillingness data without ad hoc stipulations about ‘must’. For general discussion of entaillingness with root modals in the perfective, see Hacquard 2009.
main of quantification is included in the context set. ‘Ought \( \phi \)’ lacks this presupposition. Parallel to our points in §3, there is nothing distinctively “strong” posited about ‘must’ vis-
à-vis entailingsness: The incoherence of accepting ‘\((\text{Must } \phi) \land \lnot \phi\)’ follows from the modal’s ordinary semantics of necessity and general context-set presuppositions associated with indicative. It’s the non-entailingsness of ‘ought’, and the consistency of ‘\((\text{Ought } \phi) \land \lnot \phi\)’, that is given special explanation.

Deontic ‘ought’ and ‘must’ are often thought to differ in illocutionary force. Paul McNamara characterizes the phenomena well:

To say that one ought to take a certain option is merely to provide a nudge in that direction. Its typical uses are to offer guidance, a word to the wise (“counsel of wisdom”), to recommend, advise or prescribe a course of action… In contrast, to say that one must take a certain option is to be quite forceful. Its typical uses are to command, decree, enact, exhort, entreat, require, regulate, legislate, delegate, or warn. Its directive force is quite strong. (McNamara 1990: 156)

Previous accounts often capture this contrast by stipulating an ad hoc performative element in the lexical semantics of ‘must’, such as an independent dynamic entry.\(^{32}\) Our account of entailingsness data suggests a strategy for deriving the contrasting speech-act properties of deontic ‘ought’ and ‘must’ from their static semantics and general pragmatic considerations. Accepting ‘Must \( \phi \)’ is incompatible with denying ‘\( \phi \)’; so, if the truth of ‘\( \phi \)’ is assumed to depend on the actions of the addressee, updating with ‘Must \( \phi \)’ will commit her to seeing to it that \( \phi \) (or, in the general case, commit the interlocutors to presupposing that the subject of the obligation is committed to seeing to it that \( \phi \)) (cf. Bybee et al. 1994). So, it’s no surprise that ‘must’ should often be thought to be conventionally directive. By contrast, since accepting ‘Ought \( \phi \)’ is compatible with denying ‘\( \phi \)’, updating with ‘Ought \( \phi \)’ needn’t commit anyone to seeing to it that \( \phi \). Even if deontic ‘ought’ can be used to perform a directive speech act in certain contexts, it doesn’t do so as a matter of its conventional meaning. Yet given our discussion of the conversational role of ‘ought’ (§§2–3), it’s unsurprising that utterances of deontic ‘Ought \( \phi \)’ should often perform more moderate speech acts of recommending or advising. Uttering ‘Ought \( \phi \)’ can convey one’s preference that ‘\( \phi \)’ be accepted, but without imposing the truth of ‘\( \phi \)’ on the common ground. As Gibbard writes, following Stevenson, the speaker “is making a conversational demand. He is demanding that the audience accept what he says, that it share the state of mind he expresses” (Gibbard 1990: 172), though in

a “more subtle, less fully conscious way” than by issuing “an imperative” (Stevenson 1937: 25) — or, we might say, than by using ‘must’. Deontic ‘ought’ generally provides a less face-threatening alternative to deontic ‘must’, in particular in contexts where the speaker might be construed as imposing on the addressee or relevant subject.33

Our treatments of differences in “strength” between ‘ought’ and ‘must’ are compatible with the observation that uses of ‘must’ may carry an intuitively weaker conversational force in certain contexts. Consider (39).

(39) [Context: You’re hosting a party. You wish to offer the guests a cake that you baked yourself. You say:]
   a. You must have some of this cake.
   b. You should/ought to have some of this cake.
   c. You may have some of this cake. (Lakoff 1972a: 910)

As Lakoff observes, using ‘must’ would be most polite, conveying an offer, while using ‘should’ would be less polite and using ‘may’ would be flat-out rude. It’s an interesting question what conversational factors are responsible for such apparent reversals of the modals’ felt forces. Given our focus on the weak/strong necessity modal distinction I put the issue aside, since the phenomenon generalizes to to intuitively “weak” uses of imperatives, as in permission/invitation uses such as (40) (von Fintel & Iatridou 2015), and intuitively “strong” uses of possibility modals, as in (39c) and command uses such as (41).

(40) Here, have some of this cake!
(41) [Context: Celebrity to entourage:]
   You may/can leave now.

5.3 Qualification: Endorsing and non-endorsing use

I have said that ‘(Must ϕ) ∧ ¬ϕ’ cannot be coherently accepted on any reading. This claim needs to be qualified. Though there is robust data attesting to the general anomalousness of deontic ‘(Must ϕ) ∧ ¬ϕ’ (n. 28), some speakers report being able to hear sincere uses as consistent in certain contexts.34 However, a critical observation is that even speakers who can hear examples such as (42) as consistent agree that it would be more natural to use a strong necessity modal such as ‘have to’ or ‘be required to’, as in (43).

33Cf. “Bradshaw said, he must be taught to rest. Bradshaw said they must be separated. ‘Must,’ ‘must,’ why ‘must’? What power had Bradshaw over him? ‘What right has Bradshaw to say “must” to me?’ he demanded” (Mrs Dalloway, Virginia Woolf). See also n. 36.
34I haven't seen this judgment expressed in published work, though I’ve heard it voiced in personal conversation. Thanks to Jan Dowell for discussion.
I must go to confession; I’m a Catholic. But I’m not going to. I haven’t practiced for years.

I have to (/I’m required to) go to confession; I’m a Catholic. But I’m not going to. I haven’t practiced for years.

Intuitively, in (43) it’s consistent for the speaker to dismiss going to confession because she isn’t endorsing the norms which imply that she is obligated to do so. She is simply reporting what these norms require.

This observation suggests that one can hear examples such as (42) as felicitous to the extent that one accepts “objective” uses of ‘must’ in (roughly) the sense of Lyons (1977, 1995). Adapting Lyons’s terminology, say that a modal is used endorsingly in an utterance of ‘MODAL ϕ’ if the utterance presents the speaker as endorsing/accepting the considerations with respect to which the modal is interpreted; and say that the modal is used non-endorseringly if it doesn’t. Among strong necessity modals, ‘be required to’ is typically used non-endorseringly; ‘have to’ and (‘have) got to’ are more flexible, with ‘have to’ tending more toward the non-endorsering side of the spectrum and (‘have) got to’ toward the endorsing side; and ‘must’ is typically used endorseringly.³⁵ It’s easier to hear sincere utterances with (e.g.) ‘have to’/’be required to’ as compatible with the speaker’s rejecting or being indifferent about the considerations that would verify the modal claim:

[Context: Some friends are deciding whether to go home or stay out late for a party.]

a. You must get home by 11, but I don’t care whether you do.
b. Bert must get home by 11. Aren’t his parents stupid? I would stay out if I were him.

a. You have to (/are required to) get home by 11, but I don’t care whether you do.
b. Bert has to (/is required to) get home by 11. Aren’t his parents stupid? I would stay out if I were him.

When one wishes to describe relevant norms without necessarily expressing endorsement of them, one typically uses (e.g.) ‘have to’ rather than ‘must’.

So, the promised qualification is this: *endorsing* uses of ‘STRONG \( \phi \)' are incompatible with a denial of ‘\( \phi \)’. It’s only with endorsing uses of strong necessity modals that \( \cap P(w) \) must be included in the context set. What makes the claims about entailingsness and performativity particularly compelling in the case of ‘must’ (versus e.g. ‘have to’) — is that ‘must’ is typically used endorsingly.\(^{36}\) But to the extent to which one finds non-endorsing uses of ‘must’ acceptable, to that same extent one is predicted to find uses of deontic ‘Must \( \phi \)’ to be non-entailing and lack directive force.

What is distinctive about weak necessity modals is that even when they are used endorsesingly, they are non-entailing and may lack imperative force. ‘Ought’ and ‘should’ are like ‘must’ in typically being used endorsesingly (n. 35).\(^{37}\) Parallel to (44)–(45), the claims in (46) with ‘ought’ would be more naturally expressed with e.g. ‘supposed to’, as in (47).

\[(46)\]
\begin{enumerate}
\item #You ought to get home by 11, but I don’t care whether you do.
\item #Bert ought to get home by 11. Aren’t his parents stupid? I’d stay out if I were him.
\end{enumerate}

\[(47)\]
\begin{enumerate}
\item You’re supposed to get home by 11, but I don’t care whether you do.
\item Bert is supposed to get home by 11. Aren’t his parents stupid? I’d stay out if I were him.
\end{enumerate}

Yet a characteristic use of ‘Ought \( \phi \)’ is with an explicit or implicated denial of ‘\( \phi \)’ (§4).

6  Negotiability and collective commitment

Along the way we noted various ways in which the approach to the weak/strong necessity modal distinction in this paper differs from other approaches in the literature (§§2–5). For instance, alternative approaches generally agree in treating uses of ‘Ought \( \phi \)’ as predicating a distinctive kind of necessity, namely weak necessity, of the prejacent \( \phi \) at the actual (evaluation) world. Very roughly: On domain restriction accounts (Copley 2006, von Fintel & Iatridou 2008, Swanson 2011, Rubinstein 2012, Charlow 2013), \( \phi \) is a weak necessity if \( \phi \) is true throughout a certain set \( S \) of worlds, where \( S \) is a subdomain of the set of worlds quantified over by ‘must’. On probabilistic/comparative possibility accounts (Finlay 2009, 2010, 2014, Lassiter 2011), \( \phi \) is a weak necessity if \( \phi \) is sufficiently likely/desirable, or more likely/desirable than any relevant alternative to \( \phi \). Although Yalcin’s (2016) normality-based


\(^{37}\)Hence why I framed the discussion primarily with these modals: doing so abstracts away from independent issues concerning (non-)endorsingness.
semantics denies that ‘ought’ and ‘must’ are logically related, ‘Ought φ’ is still interpreted by evaluating the truth of φ throughout a set of minimal worlds relative to the actual world.

The approach in this paper rejects treating acceptance of ‘Ought φ’ in terms of φ being a necessity, in any posited sense of necessity, at every candidate for the actual world. The apparent “weakness” of (so-called) weak necessity modals is diagnosed instead in terms of a failure to presuppose that the relevant worlds at which the prejacent is a necessity are in the context set (§4). For example, the semantics in Definition 4 adds a layer of modality, predicking the ordinary necessity claim \[ (\lambda w. \bigcap P(w) \subseteq [\phi]^c) \] of every world in a certain set of possibly counterfactual worlds. The proposed “modal-past approach” (even if not implemented in precisely this way) captures contrasting discourse properties of ‘ought’ and ‘must’ such as differences in conversational force and relations to standing contextual assumptions; it captures logical properties such as differences in entailment and the lack of entailments between ‘ought’ and ‘must’/‘may’; it captures relations between weak necessity modals and broader modal-past phenomena; and it generalizes across flavors of modality.

There are insights from previous accounts which are preserved in the account developed here. The semantics in §4 allow for connections between weak necessity modals and common ground assumptions (n. 4); mood, counterfactuality, and conditionality (nn. 15, 20, 23); and notions of comparison (n. 24), normality (Makinson 1993, Frank 1996, Yalcin 2016), and probability (though not unconditional probability; n. 27). Previous accounts are often developed with an eye toward one or few of these issues to the exclusion of others. Although we haven’t examined each of the connections in equal depth, I hope the discussion has illustrated the fruitfulness of a modal-past approach and its potential for systematizing diverse linguistic phenomena.

Before concluding I would like to compare in a bit more detail the account of the weak/strong necessity modal distinction in this paper with the particular domain restriction account in Rubinstein 2012, since it constitutes the most extensively developed alternative from the literature (see also Rubinstein 2014, Portner & Rubinstein 2012, 2016). We have already observed general points of disagreement between the modal-past approach and domain restriction accounts (above, also §§3, 5). Here I focus on the distinctive feature of Rubinstein’s account: the appeal to collective commitment to a body of “priorities” (norms, goals, ideals).³⁸

Rubinstein’s main innovation is to supplement von Fintel & Iatridou’s domain restriction semantics with a substantive account of the primary vs. secondary ordering source distinction (§4): what makes a primary ordering source “primary” is that it includes premises which

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³⁸Rubinstein doesn’t examine epistemic modals. I leave open how the account would be extended to epistemic readings. As discussed previously, there will be general worries regarding the lack of entailments between epistemic ‘ought’ and ‘may’/‘must’, and the fact that epistemic ‘ought’ doesn’t in general quantify over a set of worlds that are regarded as epistemically possible (§§4–5). See Silk 2018 for critical discussion of Portner & Rubinstein’s (2012) appeal to collective commitment in an account of mood selection.
the interlocutors are presupposed to be collectively committed to; what makes a secondary ordering source “secondary” is that it includes premises which the interlocutors are presupposed not to be collectively committed to. ‘Must’ is interpreted only with respect to primary premises; ‘ought’ is logically weaker in being interpreted with respect to these premises as well as secondary premises which are presupposed not to be collectively endorsed:

\[
\text{[S]trong necessity modals are only sensitive to prioritizing premises that the conversational participants are presupposed to be collectively committed to…} \\
\text{[I]f any participant in the conversation were given the chance to defend these priorities, it is assumed in the context of the conversation that they would do so. Weak necessity modals take into account all these premises plus some more. For these additional premises, lack of collective commitment is presupposed… [A] speaker uses a weak modal when he or she believes (perhaps mistakenly) that the secondary priorities it depends on are still up for discussion. (Rubinstein 2012: 51–52)}
\]

In (48) although the speaker is committed to a priority favoring cost-effectiveness, it isn’t presupposed that Alice is committed to it (cf. Rubinstein 2012: 55–60). This lack of collective commitment is what calls for using ‘ought’, on Rubinstein’s view.

\[
\text{(48) [Context: Alice is considering whether to take the subway or a cab to a concert. The subway is cheaper; the cab is quicker. You say:]} \\
\text{You ought to (/should, /?have to, /?must) take the subway.}
\]

For clarity use ‘commitment\textsubscript{R}’ for the notion of commitment in Rubinstein’s analyses. Rubinstein is explicit in identifying commitment\textsubscript{R} to a priority p with endorsing that p is desirable (e.g. 2012: 78; also Portner & Rubinstein 2012: 471, 475, 477–481). What determines whether p is a “primary” or “secondary” priority is the interlocutors’ mutual presuppositions about the desirability of p. Yet, as we have seen, the weak/strong necessity modal distinction crosscuts the distinction between necessity modals that express endorsement in this sense and those that don’t, and hence crosscuts the distinction between necessity modals that express collective commitment\textsubscript{R} and those that express a lack of commitment\textsubscript{R}.

§5 delineated two dimensions along which modals can differ: strength, and tendencies for (non-)endorsing use, i.e. the extent to which uses of ‘MODAL \(\phi\)’ convey the speaker’s endorsement of the considerations that would verify the modal claim. Delineating these dimensions brings out problems with diagnosing the weak/strong necessity modal distinction in terms of collective commitment\textsubscript{R}. First, there are uses of weak necessity modals expressing collective endorsement of the relevant priority. In (49) we are both publicly committed\textsubscript{R} to the value of family; my helping my mother is “publicly endorsed as desirable” by every conversational participant. Yet ‘ought’ is felicitous, indeed preferred.
(49)  
Me:   Family is very important. I think I would rather stay here.
You:  I agree. You ought to tend to your mother.

Second, there are uses of strong necessity modals where the interlocutors expressly reject commitments to the relevant priorities, as in (43) and (45) (and possibly (42)). The lack of endorsement can even be common ground:

(50)  
[Context: Our parents are asleep. We’ve settled on staying out to go to a party.]
You:  When is curfew, again? We need to make sure that we tell Mom we got back before then if she asks.
Me:   We have to be home by 11. Aren’t her rules stupid? This party is going to be great.

Counterexamples such as these aren’t atypical. Corpus studies uniformly attest to variation among modals in each class (weak/strong) vis-à-vis tendencies to express collective commitment. Indeed collective commitment has been appealed to in distinguishing among weak necessity modals, and among strong necessity modals.\(^\text{39}\) Summarizing his corpus analyses of ‘ought’ and ‘should’ in contemporary American English, Myhill concludes, “using ought suggests that people have the same feelings about the specific obligation in question and there is agreement about it, while should does not suggest the same feelings or agreement” (1997: 8). Far from being exceptional, expressing collective commitment with ‘ought’ is the norm, as in the naturally occurring example in (51) (slightly modified).

(51)  
A:   I won’t tell anyone… but the Dean, of course.
B:   And Mrs. Reynolds.
A:   Yes. She ought to know. \textit{(Myhill 1997: 10)}

Conversely, though ‘must’ tends to express collective commitment, other strong necessity modals, such as ‘(have) got to’ do not. ‘(Have) got to’ differs in being “associated with conflicts between the speaker and the listener” \textit{(Myhill 1996: 365)}, as in the naturally occurring example in (52).

(52)  
Edie, you’ve got to stop bothering me when I’m working. \textit{(Myhill 1996: 369)}

Collective commitment to the contents of premise sets may affect the distribution of modals in discourse, but it isn’t what explains the distinction between weak and strong necessity modals.

A modal-past approach captures intuitions about negotiability and collective commitment which may be motivating Rubinstein’s account. Strong necessity modals “presuppose (collective) commitment” in the same boring sense as any other context-sensitive expression: one commits to the value of the context-dependent item being as one’s utterance assumes, and to the world being as one’s utterance says it is, given this assumed value. Such a commitment is compatible with not endorsing-as-desirable the premises \( p \in P(w) \), for any \( w \in c \). In (50) we can accept that the house rules require us to be home by 11 while denying that those rules are desirable or that they are to guide our plans. Conversely, uses of weak necessity modals express “negotiability” in the sense that they don’t commit one to being in a world where the relevant considerations verify the necessity of the prejacent. Lacking commitment to the prejacent’s being necessary is compatible with (collectively) endorsing the considerations encoded in the given premise frame or premises from which the prejacent would follow. In (5) we can endorse the value of family even if we prefer not to settle how it relates to other potentially competing values.

7 Conclusion

This paper developed a modal-past approach to ‘ought’ and the distinction between so-called weak necessity modals (‘ought’, ‘should’) and strong necessity modals (‘must’, ‘have to’). There is nothing specially “strong” about the necessity expressed by strong necessity modals: Strong necessity modals are given their ordinary semantics/pragmatics of necessity; uses of ‘Must \( \phi \)’ predicate the (deontic/epistemic/etc.) necessity of the prejacent \( \phi \) of every candidate for the actual world. The apparent “weakness” of weak necessity modals derives from their bracketing whether the prejacent is necessary in the actual world: uses of ‘Ought \( \phi \)’ fail to presuppose that the topic worlds in which \( \phi \) is necessary are included in the context set. ‘Ought \( \phi \)’ can be accepted without needing to settle that the relevant considerations (norms, goals, etc.) which actually apply verify the necessity of \( \phi \). This analysis carves out important roles for weak necessity modals in conversation and deliberation. As emphasized throughout Gibbard’s (1990, 2003, 2012) developments of expressivism, ‘ought’ affords a means of coordinating our conditional attitudes and plans. Weak necessity modals allow us to entertain and plan for hypothetical continuations or minimal revisions of the current context; they afford conventional devices for coordinating our norms, values, expectations, without having to settle precisely how the relevant considerations apply and compare. The proposed account systematizes a spectrum of semantic and pragmatic data — e.g., concerning the relative felicity of weak and strong necessity modals, relations between uses of weak and strong necessity modals and standing contextual assumptions, the morphosyntactic properties of expressions of ought cross-linguistically, and contrasting logical and illocutionary properties of weak
and strong necessity modals. The range of linguistic phenomena that are unified under and explained by the account lend it a robust base of support.

The data considered here aren’t the only data to be explained by an overall theory of weak and strong necessity modals. For instance, there are additional linguistic contrasts between the classes of weak and strong necessity modals, such as in data with incomparabilities, comparatives, quantifiers, conditionals, modifiers, and neg-raising.⁴⁰Second, we briefly examined one further dimension of difference among modals in tendencies for endorsing/non-endorsing use. It’s worth investigating interactions between weak/strong necessity modals and other such differences in modal meanings, e.g. with implicatures (cf. Verstraete 2005a). Third, although I argued against treating ‘ought’ and ‘must’/‘may’ as ordered by logical or quantificational strength, more thorough comparisons of inference patterns with different types of readings are needed. Fourth, our discussion highlighted interactions between weak and strong necessity modals and general issues such as context-sensitivity, counterfactuality, attitude expression, and performativity. These interactions afford rich avenues for future research. Further investigation of such issues would be helpful in adjudicating among alternative formal implementations of a modal-past approach.

In closing I would like to raise two potential avenues for future work, one more narrowly linguistic, one more philosophical.

7.1 Counterfactuality?

In §4 we noted that it’s common in many languages to express a notion of ought by using the form of a strong necessity modal used in counterfactuals. Such languages use the same string to express ought and counterfactual necessity (would have to). Other things equal, it would seem preferable to take the surface forms at face value. Yet as von Fintel & Iatridou (2008: 128–131) observe, ‘ought’ cannot generally be replaced by ‘would have to’:

(53) a. I ought to help the poor, but I don’t have to.
   b. ?I would have to help the poor, but I don’t have to.

Open questions include (i) how non-lexicalized expressions of ought are diachronically related to the dedicated lexical items, (ii) how interpretations of certain uses of STRONG+CF might become conventionalized into expressions of ought, (iii) how counterfactual interpretations of STRONG+CF are synchronically related to meanings of grammaticalized forms like ‘ought’, and (iv) how meanings of the grammaticalized forms compare across languages. The modal-past analysis in Definition 4 avoids giving ‘Ought ϕ’ the semantics of an implicit counterfactual necessity claim and conflating ‘ought’ with ‘would have to’: the necessity of

isn’t evaluated at the closest \( \psi \)-worlds, for any implicit condition \( \psi \), but at the relevant minimal/“preferred” (desirable, normal, etc.) worlds, determined by \( h \). Although the focus of this paper has been on lexicalized English weak necessity modals, it’s worth considering how such an interpretation might become associated with certain uses of \( \text{STRONG+CF} \) in other languages. Channeling my inner Gibbard, “What I can suggest will have to be quite speculative, but the speculations, I hope, may prompt more solid investigation” (1990: 61).

Using a clause that lacks the standard indicative presupposition places a burden on the interpreter’s task of inferring which possibilities are being talked about. A condition specifying which possibilities are relevant typically must be salient, either implicitly or in the linguistic context (cf. Schueler 2011):

\[
(54) \quad \text{[Context: We’re trying out guitars in a music store. Looking at an expensive vintage Les Paul, I say:]} \\
(\text{If I bought it,}) \text{ my wife would kill me.}
\]

So, if no condition is retrievable, using ‘would have to’ is anomalous:

\[
(55) \quad \text{(If I wanted to buy it,) I would have to check with my wife first.}
\]

\[
(56) \quad \text{[Context: We’re strangers in a hotel lobby. I notice you fumbling with your bags. I say:]} \\
?\text{Here, I would have to help you.}
\]

Uses of ‘ought’ lack this salience/retrievability requirement:

\[
(57) \quad \text{[Context: Same as (56)]} \\
\text{Here, I ought to help you.}
\]

A hypothesis, then, is that distinctive discourse effects of weak necessity modals are products of using an expression that is neither marked as being about the context set (as via indicative mood) nor indicated as being about some other topical possibility (via an implicit/explicit condition).

Suppose we’re inquiring about the necessity of \( \phi \). Uttering ‘\( \text{STRONG}\phi \)’ requires settling which considerations apply and aren’t defeated. We might not be prepared to restrict the future course of the conversation in this way. Nevertheless each of us takes some ways of extending the conversation and addressing the question under discussion to be more likely or better than others. Using the modal-past form allows us to consider the necessity of \( \phi \) as holding, not necessarily in the current context, as with a strong necessity modal, but in a preferred (likely/normal/desirable) continuation or minimal revision of the current context, whatever that might turn out to be (§§2–3). This predicts that one’s judgments about replacing ‘would have to’ with ‘ought’ should improve to the extent that the antecedent for
'would have to' describes what one regards as “preferred” conditions relevant for evaluating questions about the necessity of $\phi$. Indeed the discourse effects of ‘would have to’ and ‘ought’ are strikingly closer in (59) than in (58).

(58)  
a. (If I was a mobster, which I’m not,) I would have to kill you.  
b. I ought to kill you.

(59)  
[Context: Alice is considering with her mother whether to take the A- or C-train. The A is quicker; the C is safer. Her mother says:]

a. If safety was most important, you would have to take the C-train. In fact, safety is more important, as we can agree. So you must/have to take the C.  
b. You ought to take the C-train. In fact, safety is most important, as we can agree. So you must/have to take the C.

As often occurs in grammaticalization and pragmatization, the original (counterfactual) meaning may be semantically bleached, and the reinterpreted (weakness) meaning becomes more abstract; the attitude expressed (here, about the necessity claim) can thus appear vague or nonspecific.⁴¹ Such an interpretation is essentially what is delivered by the lexically unspecified generalized notion of minimality associated with $h$ in Definition 4.⁴² This grammaticalization path would predict the observed variations in how ought is expressed cross-

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⁴²The main-text hypothesis approaches weakness uses of $\text{STRONG+CF}$ on the model of a conditional consequent (one lacking a particular retrievable antecedent). An alternative would be to think of the uses on the model of the antecedent. Interestingly, there is a precedent for associating an interpretation like the one given to $h$ with a certain type of ‘if’-clause. Grosz 2012 argues that optatives/exclamatives should be treated on the model of non-logical ‘if’-clauses under a covert exclamation operator $\text{EX}$ (“non-logical” because the embedded clause provides the subject matter of the attitude).

(i) Oh, that it would snow!  
   a. $\text{EX} \ [\text{it snows}]$  
   b. $\approx \text{It would be good if it snowed.}$

(ii) Oh, that it snowed!  
   a. $\text{EX} \ [\text{it snowed}]$  
   b. $\approx \text{It’s surprising that it snowed.}$

Grosz’s $\text{EX}$ operator is essentially a scalar analogue of $h$, also generalized to express a lexically unspecified attitude toward the embedded proposition. A Grosz-style scalar version of Definition 4 — that ‘Ought $\phi$’ is true iff the proposition that $\phi$ is necessary is at least as $h$-“preferred” (desirable/normal/etc.) as a contextually relevant standard/threshold — might help capture connections between weak necessity modals and gradability/comparatives (Finlay 2014, Rubinstein 2014, Portner & Rubinstein 2016). Cross-linguistic comparisons among morphological/lexical expressions of ought, non-logical ‘if’-clauses, and optatives/exclamatives may provide fruitful avenues to explore.
linguistically — i.e. with some languages using only a conventionalized weakness interpretation of STRONG+CF (French); some languages using only a grammaticalized form (English); and some languages using both (German).

It's important to remember that everyone needs an account of how weakness/tentativeness interpretations of past forms are derived across modals (e.g. (17)–(19)), and how weakness interpretations versus implicit-counterfactual interpretations of past forms are determined across contexts (see also n. 14). To my knowledge, no such derivation has been given (or, in the case of ought-interpretations of STRONG+CF in other languages, even attempted). The above hypothesis provides a potential basis for weakness (tentativeness, politeness) effects associated with modal-past forms generally. Thorough synchronic/diachronic cross-linguistic work on the interpretation of modal-past forms, and how ought-interpretations of STRONG+CF become conventionalized and in some cases grammaticalized, is called for.

7.2 Philosophical therapy?

I would be remiss if I concluded without at least asking how, if at all, our linguistic inquiry might inform theorizing about a “primitive concept ought” (Gibbard 2012: 204). How might investigating a word ‘ought’ teach us something about “concepts fraught with” a “primitive ought” (Gibbard 2003: x, 2012: 14)?

Consider our discussion of the practicality of deontic ‘ought’ and ‘must’ judgments. It’s common in discourse and deliberation to investigate what we are actually required to do. We wish to guide our planning and influence one another’s behavior in light of the norms we accept. An endorsing strong necessity modal like ‘must’ is well-suited to the task. However, using ‘must’ is often awkward. We may want to talk about obligations which held in the past, or which may go unfulfilled or be overridden in the future. Or we may want to communicate information about a body of norms without necessarily registering commitment to them or enjoining others to share in such commitment. A modal like ‘ought’ or ‘be required to’ can thus be more suitable. There is a range of expressive resources at our disposal for coordinating our actions and attitudes. This is for the better given the variety of our purposes. But it also raises a philosophical risk. Bracketing differences among necessity modals might turn out to be harmless for the purposes of (meta)normative inquiry. But it might not.

A core innovation in Gibbard’s expressivism is the thesis of normative concepts as essentially plan-laden:

(60) To believe that one ought to do X is to plan to do X.

Thinking what I ought to do is thinking what to do…

Ought claims… are claims about what to do. (Gibbard 2012: 204; 2003: ix–x, 10)

Generalizing:
The clear distinctive feature of normative concepts, I now think, lies in their conceptual ties. Oughts of action tie in conceptually with acting. (Gibbard 2011: 36)

Many take it as obvious that some form of internalism is true. After all, normative judgments are constitutive of deliberation, and deliberation is essentially practical; its aim is action. But many find clear counterexamples. What about the psychopath, or someone who is tired or depressed (such as perhaps the poor reader who has made it to this point in the paper)?

Attending to the dimensions of difference among necessity modals examined in this paper may shed light on conflicting intuitions about the ostensible “plan-ladenness” of “concepts fraught with ought.” Gibbard hedges: the concepts “fraught with ought” are so-fraught “not for every sense of the term ['ought'],” but for the “crucial sense” explained by the pattern in (60) (2003: x). It’s revealing that although Gibbard couches the thesis about the practicality of the normative using ‘ought’, he pumps the intuition using ‘must’:

…Oughts of action tie in conceptually with acting. Take, for example, the belief that the building is on fire and the one and only way to keep from being burned to a crisp is to leave forthwith. If that’s the case, we'd better leave forthwith, but it isn't strictly incoherent, conceptually, to have this belief and not to leave. Contrast this with the normative belief that one must leave forthwith. It is, I maintain, conceptually incoherent to hold this belief and not leave, if one can. (Gibbard 2011: 36)

As we’ve seen, it’s hard to hear a sincere utterance of ‘Must ϕ’ as consistent with the speaker’s being indifferent about whether ϕ; yet such judgments aren’t nearly as anomalous when expressed with weak necessity modals or (weak/strong) necessity modals that are more naturally used non-endorsingly (§§4–6).

(61) #I must (/I’ve got to) get home by 10, but forget that; I’m not going to.
(62) I [ought to, should, am supposed to, have to] get home by 10, but forget that; I’m not going to.

We should be wary of general claims about normative language and judgment. A thesis of the form in (60) may seem compelling when considering examples using terms that are paradigmatically entailing and endorsing. But when we consider cases using other terms, counterexamples can appear in the offing. A “primitive concept ought” may be expressed by ‘ought’ less often than initially supposed.

The skeptically inclined might be apt to wonder what is at-issue in debates such as about judgment internalism — that is, if not linguistic issues about directive/entailing/endorsing
uses of language, or empirical psychological issues about conditions under which different attitudes and judgments guide and motivate us. One might wonder how probative it is to couch metaethical inquiry in terms of a class of “normative” language/concepts/judgments at all. More fruitful, perhaps, to leave talk of the “normative” to the side, and ask directly about motivational states of mind and directive uses of language, and what type of person one wants to be and how to live.⁴³

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⁴³See Silk 2015b (esp. §§6–7) for extended discussion and additional applications.


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