Part I. Horizontal mergers

1) Mergers and economies of scale

When marginal cost is constant and there are no fixed costs, no two firms have a profit incentive to merge unless it is a merger from duopoly to monopoly. The situation is different when marginal costs are increasing: when two firms merge, the new firm can achieve better cost minimization by allocating its output across multiple plants. Then a multi-plant firm has a cost advantage over its one-plant competitors, and this may provide a profit incentive to merge.

The example illustrates this motive for merger: a merged firm will operate multiple plants and will be able produce output at lower average cost than a one-plant firm.

An industry consists of \( N = 3 \) firms with identical costs. Market demand is

\[
p = A - BQ = 150 - Q
\]

a) Show that if the marginal cost function is linear (marginal cost is constant), then it will never pay for two firms to merge if the resulting two firms are again Cournot competitors.

b) Now let the cost function be

\[
C(q) = cq + q^2 = 18q + q^2.
\]

Suppose that two firms (2 and 3) merge, assume the name of firm 2 and play Cournot against the remaining firm 1. Will there be a profit incentive to merge? Will the merger benefit consumers? (Hint: carefully consider if the merged firm would produce using both original firms’ plants of just those of one firm.)

Winter 2002 final, question 2

2) Mergers, cost synergies and welfare

Let the industry initially consist of 4 firms that operate on a market with linear demand given by

\[
p = 25 - Q.
\]

In order to operate, each firm has to pay a fixed cost \( F = 15 \) Marginal cost is constant and equals \( c = 5 \) for every firm Assume that the firms engage in Cournot competition.

a) Show that any two firms in this industry will have a profit incentive to merge.
b) Compute welfare (the sum of all profits net of fixed costs plus consumer surplus) pre-merger (when $N = 4$) and post-merger (when $N = 3$). Does this merger make consumers better off? Does this merger make firms and consumers jointly better off? Why?

3) **Capacity motive for merger and market price**

Let the industry initially consist of 4 firms that operate on a market with linear demand given by

\[ p = 25 - Q. \]

There are no fixed costs. Marginal cost is constant and equals $c = 5$ for every firm. Assume that initially the firms engage in Cournot competition.

a) Suppose that if two firms merge, they become a Stackelberg leader, and the other two firms are followers. Show that this merger leads to a price drop. Also show that the two firms have a profit incentive to merge.

b) Demonstrate that further mergers in this industry are not in the public interest.

**Part II Vertical mergers**
Winter 2002 final, question 3

**Part III Vertical relations**
Winter 2002 final, question 4, Fall 2003 final, question 3
Part I. Horizontal mergers

1) Mergers and economies of scale

When marginal cost is constant and there are no fixed costs, no two firms have a profit incentive to merge unless it is a merger from duopoly to monopoly. The situation is different when marginal costs are increasing: when two firms merge, the new firm can achieve better cost minimization by allocating its output across multiple plants. Then a multi-plant firm has a cost advantage over its one-plant competitors, and this may provide a profit incentive to merge.

The example illustrates this motive for merger: a merged firm will operate multiple plants and will be able produce output at lower average cost than a one-plant firm.

An industry consists of $N = 3$ firms with identical costs. Market demand is

$$p = A - BQ = 150 - Q$$

a) Show that if the marginal cost function is linear (marginal cost is constant), then it will never pay for two firms to merge if the resulting two firms are again Cournot competitors.

b) Now let the cost function be

$$C(q) = cq + q^2 = 18q + q^2.$$ 

Suppose that two firms (2 and 3) merge, assume the name of firm 2 and play Cournot against the remaining firm 1. Will there be a profit incentive to merge? Will the merger benefit consumers? (Hint: carefully consider if the merged firm would produce using both original firms’ plants of just those of one firm.)

a) Suppose that we have a three industry firm with linear demand

$$p = A - BQ$$

$$MC(q) = c$$

and identical linear cost functions across firms:

$$C(q) = cq$$

If three firms play a Cournot equilibrium

$$\pi = \frac{(A - c)^2}{B(N + 1)^2} = \frac{1}{16} \frac{(A - c)^2}{B}$$
If two of the firms merge, they will get the profit of \( \frac{1}{9} \) between themselves. This does not exceed the sum of their pre-merger profits

\[
\frac{1}{9} < \frac{1}{16} + \frac{1}{16}
\]

The two firms will not choose to merge.

Now suppose the cost function is

\[ C(q) = cq + q^2 \]

The best response function of a Cournot competitor

\[
p = A - BQ_i - Bq_i
\]

\[
\max_q (A - BQ_i - Bq) q - cq - q^2
\]

\[
A - BQ_i - 2Bq - c - 2q = 0
\]

\[
A - c - BQ = q(B + 2)
\]

\[
A - c - BQ = \frac{Q}{N}(B + 2)
\]

\[
Q = \frac{N(A - c)}{BN + B + 2}, \quad q = \frac{(A - c)}{BN + B + 2}
\]

\[
p = A - BQ = A - \frac{N(A - c)}{N + 1 + 2/B} = \frac{A(N + 1 + 2/B) - N(A - c)}{N + 1 + 2/B} = \frac{A + cN + \frac{2A}{B}}{N + 1 + 2/B}
\]

\[
\pi = q^2(B + 2) - q^2 = q^2(B + 1)
\]

\[
\pi = \frac{(A - c)^2(B + 1)}{(BN + B + 2)^2}
\]

Now \( A = 150, B = 1, c = 18 \). If \( N = 3 \)

\[
q = \frac{(A - c)}{BN + B + 2} = \frac{132}{6} = 22
\]

\[
\pi = q^2(B + 1) = 968
\]

Suppose now that two firms merge. The new firm has two plants, \( A \) and \( B \), each of which has the cost function

\[ C(q) = cq + q^2 \]

We must now figure out whether production will happen at one or two plants. With two-plant production, output

\[
q = q_A + q_B
\]
will be allocated to equate marginal costs at each plant:

\[ MC_A(q_A) = MC_B(q_B) \]
\[ c + 2q_A = c + 2q_B \]
\[ q_A = q_B = \frac{q}{2} \]

The two-plant cost function is

\[ C_2(q) = C_A \left( \frac{q}{2} \right) + C_B \left( \frac{q}{2} \right) = c \frac{q}{2} + \left( \frac{q}{2} \right)^2 + c \frac{q}{2} + \left( \frac{q}{2} \right)^2 = cq + \frac{q^2}{2} \]

For any quantity to be produced, \( q \),

\[ C_2(q) = cq + \frac{q^2}{2} < C(q) = cq + q^2 \]

It is always cheaper to produce at two plants. The merged firm has a cost advantage over the remaining firm.

Let us now solve for the resulting Cournot equilibrium. Let us call the merged firm 2, and the remaining firm 1. Best response of firm 2 to the quantity produced by firm 1 is the quantity that maximizes firm 2’s profit given \( q_1 \)

\[ \max_{q_2} (A - Bq_1 - Bq_2)q_2 - cq_2 - \frac{q_2^3}{2} \]

(note that firm 2 uses the two plant cost function \( C_2(q) \))

\[ A - Bq_1 - 2Bq_2 = c + q_2 \]
\[ q_2 = BR_2(q_1) = \frac{A - c - Bq_1}{2B + 1} \]

We have found the best response of firm 2 to the quantity produced by firm 1.

Now let us find firm 1’s best response function.

\[ \max_{q_1} (A - Bq_2 - Bq_1)q_1 - cq_1 - q_1^2 \]

(note that firm 2 uses the one-plant cost function \( C(q) \))

\[ A - Bq_2 - 2Bq_1 = c + 2q_2 \]
\[ q_1 = BR_1(q_2) = \frac{A - c - Bq_2}{2B + 2} \]

Now remembering that \( A = 150, B = 1, c = 18 \), let us find the Nash equilibrium pair of quantities \( (q_1^*, q_2^*) \) (they are where the two best response functions intersect).

\[ q_2^* = \frac{132 - q_1^*}{3} = 44 - \frac{q_1^*}{3} \]
\[ q_1^* = \frac{132 - q_2^*}{4} = 33 - \frac{q_2^*}{4} \]
\[ 3q_2^* = 132 - q_1^* \]
\[ 4q_1^* = 132 - q_2^* \]
\[ 4(132 - 3q_2^*) = 132 - q_2^* \]
\[ 396 = 11q_2^* \]
\[ q_2^* = 36 \]
\[ q_1^* = 132 - 3q_2^* = 24 \]

Market price
\[ p = A - Bq_1^* - Bq_2^* = 150 - 24 - 36 = 90 \]
\[ \pi_1 = pq_1^* - C(q_1^*) = 90 \cdot 24 - \left[ 18 \cdot 24 + 24^2 \right] = 1152 \]
\[ \pi_2 = pq_2^* - C_2(q_2^*) = 90 \cdot 36 - \left[ 18 \cdot 36 + \frac{36^2}{2} \right] = 1944 \]

Will the two firms want to merge into one? The post-merger profit of the whole firm is greater than the pre-merger sum of profits of the parts:
\[ 968 + 968 = 1936 < 1944 \]

Note that the remaining firm also gains from merger, because price goes up. Consumers, therefore, lose.

2) Mergers, cost synergies and welfare

Let the industry initially consist of 4 firms that operate on a market with linear demand given by
\[ p = 25 - Q. \]

In order to operate, each firm has to pay a fixed cost \( F = 15 \) Marginal cost is constant and equals \( c = 5 \) for every firm Assume that the firms engage in Cournot competition.

a) Show that any two firms in this industry will have a profit incentive to merge.

For Cournot oligopoly,
\[ \pi (N) = \frac{(A - c)^2}{B (N + 1)^2} \]

For two firms to have an incentive to merge, the post-merger profit of the merged firm must exceed the sum of pre-merger profits of the parts.
\[ 2\pi (N) - 2F < \pi (N - 1) - F \]
\[ 15 = F > 2\pi (N) - \pi (N - 1) = \frac{(A - c)^2}{B} \left( \frac{2}{(N + 1)^2} - \frac{1}{N^2} \right) = \]
b) Compute welfare (the sum of all profits net of fixed costs plus consumer surplus) pre-merger (when \( N = 4 \)) and post-merger (when \( N = 3 \)). Does this merger make consumers better off? Does this merger make firms and consumers jointly better off? Why?

The merger does not make consumers better off because fewer firms on the market means higher price.

Total quantity produced with \( N \) firms on the market

\[
Q(N) = \frac{N (A - c)}{N + 1} B
\]

Price with \( N \) firms on the market

\[
p(N) = c + \frac{A - c}{N + 1}
\]

Welfare with \( N \) firms on the market

\[
W(N) = CS + \pi = \frac{1}{2} (A - c + p - c) \cdot Q
\]

\[
W(N) = CS + N \cdot \pi(N) - N \cdot F = \frac{1}{2} (A - c + (p(N) - c)) Q(N) - F \cdot N = \\
= \frac{1}{2} (A - c) \left( 1 + \frac{1}{N + 1} \right) \cdot \frac{N}{N + 1} \frac{(A - c)}{B} - F \cdot N = \\
= \frac{1}{2} \frac{(A - c)^2}{B} \frac{(N + 2) N}{(N + 1)^2} - F \cdot N
\]
Pre-merger welfare:
\[ W(4) = 200 \cdot \frac{6 \cdot 4}{5^2} - 15 \cdot 4 = 132 \]

Post-merger welfare:
\[ W(3) = 200 \cdot \frac{5 \cdot 3}{4^2} - 15 \cdot 3 = 142.5 \]

Welfare goes up as a result of merger, because the loss of consumer surplus resulting from higher price is outweighed by the gain in firms’ profits and saved fixed cost.

3) Capacity motive for merger and market price

Let the industry initially consist of 4 firms that operate on a market with linear demand given by
\[ p = 25 - Q. \]

There are no fixed costs. Marginal cost is constant and equals \( c = 5 \) for every firm.

Assume that initially the firms engage in Cournot competition.

a) Suppose that if two firms merge, they become a Stackelberg leader, and the other two firms are followers. Show that this merger leads to a price drop. Also show that the two firms have a profit incentive to merge.

\[ \pi_l(L, F) = (p - c) q_l = \frac{A - c}{(L + 1)(F + 1)} \cdot \frac{1}{L + 1} \cdot \frac{A - c}{B} = 16 \]

\[ \pi_f(L, F) = (p - c) q_f = \frac{A - c}{(L + 1)(F + 1)} \cdot \frac{1}{(L + 1)(F + 1)} \cdot \frac{A - c}{B} \]

Pre-merger every firm gets
\[ \pi_l(4, 0) = 16 \]

The sum of pre-merger profits is \( 16 + 16 = 32 \).

Post-merger, the leader gets
\[ \pi_l(1, 2) = \frac{10}{3} \cdot \frac{1}{2} \cdot 20 = \frac{33}{3} > 2 \pi_l(4, 0) = 32 \]

\[
\begin{array}{c|c}
\text{Pre-merger} & \text{Post merger} \\
\hline
4 \text{ leaders, 0 followers} & 1 \text{ leader, 2 followers} \\
p - c = \frac{A - c}{(L+1)(F+1)} = 4 & p - c = \frac{A - c}{(L+1)(F+1)} = \frac{20}{(1+1)(2+1)} = \frac{10}{3} < 4 \\
\end{array}
\]

b) Demonstrate that further mergers in this industry are not in the public interest.

\[
\begin{array}{c|c}
\text{Pre-merger} & \text{Post merger} \\
\hline
1 \text{ leader, 2 followers} & 2 \text{ leaders, 0 followers} \\
p - c = \frac{A - c}{(L+1)(F+1)} = \frac{10}{3} & p - c = \frac{A - c}{(L+1)(F+1)} = \frac{20}{3} \\
\end{array}
\]