The calculation on the previous page clearly shows that particles that were created by \( b \) contribute oppositely to those created by \( a \) to the total charge. We concluded in Homework 2 that this charge was electric charge.

2. a) We are asked to compute the general, K-type Bessel function solution of the Wightman propagator,

\[
D_W(x) \equiv \langle 0 | \phi(x) \phi(0) | 0 \rangle = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2E_p} e^{-ipx}.
\]

Because \( x \) is a space-like vector, there exists a reference frame such that \( x^0 = 0 \). This implies that \( x^2 = -x^0 \). And this implies that \( px = -p \cdot x = -|p||x| \cos(\theta) = -|p|\sqrt{-x^2} \cos(\theta) \). We can then write \( D_W(x) \) in polar coordinates as

\[
D_W(x) = \frac{1}{(2\pi)^2} \int_0^{2\pi} d\phi \int_{-\infty}^{\infty} e^{i|p|\sqrt{-x^2} \cos(\theta)} \int_0^{\infty} p^2 dp \frac{1}{2\sqrt{p^2 + m^2}},
\]

\[
= \frac{1}{(2\pi)^2} \int_0^{2\pi} d\theta \int_{-\infty}^{\infty} e^{i|p|\sqrt{-x^2} \cos(\theta)} \int_0^{\infty} p^2 dp \frac{1}{2\sqrt{p^2 + m^2}},
\]

(\text{where } \xi = \cos(\theta))

\[
= \frac{1}{4\pi^2} \int_{-\infty}^{\infty} p^2 dp \frac{1}{2\sqrt{p^2 + m^2}} \frac{1}{i|p|\sqrt{-x^2}} \left( e^{i|p|\sqrt{-x^2}} - e^{-i|p|\sqrt{-x^2}} \right),
\]

\[
= \frac{1}{4\pi^2 \sqrt{-x^2}} \int_{-\infty}^{\infty} dp \frac{p \sin(|p|\sqrt{-x^2})}{\sqrt{p^2 + m^2}}.
\]

Gradstein and Ryzhik’s equation (3.754.2) states that for a K Bessel function,

\[
\int_0^\infty dx \frac{\cos(ax)}{\sqrt{b^2 + x^2}} = K_0(a\beta).
\]

By differentiating both sides with respect to \( a \), it is shown that

\[
- \int_0^\infty dx \frac{a \sin(ax)}{\sqrt{b^2 + x^2}} = -\beta K_0'(a\beta) = \beta K_1(a\beta).
\]

We can use this identity to write a more concise equation for \( D_W(x) \). We may conclude

\[
D_W(x) = \frac{m}{4\pi^2 \sqrt{-x^2}} K_1(m\sqrt{-x^2}).
\]

b) We may compute directly,

\[
iD(x) = \langle 0 | \{\phi(x), \phi(0)\} | 0 \rangle,
\]

\[
= \langle 0 | \phi(x), \phi(0) | 0 \rangle - \langle 0 | \phi(0), \phi(x) | 0 \rangle,
\]

\[
= D_W(x) - D_W(-x),
\]

\[
\implies D(x) = iD_W(-x) - D_W(x).
\]

Similarly,

\[
D_1(x) = \langle 0 | \{\phi(x), \phi(0)\} | 0 \rangle = D_W(x) + D_W(-x).
\]

It is clear that both function ‘die off’ very rapidly at large distances. I was not able to conclude that they were truly vanishing, but they are certainly nearly-so at even moderately small distances.