Effectively Using Applied Writing Projects in Undergraduate Mathematics Classes
(Especially Calculus)

Project NExT Workshop
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the University of Michigan
0. Overview

- **Definitions**: what are “Applied Writing Projects”?

- **The Logic**: why use them?

  - exemplary plunge

- **Logistical** and **Practical** matters

- **The Elephant** in the room...

  - (... grading)

- **Where** do we go from here?
1. An Applied Writing Project is . . .

- A *Big Story Problem* (. . .Gary Larson)
  - An *Applied*, (mostly) *Real-World*,
  - *Multi-step*, often *Multi-part* problem.

- Presented as a *Contract Letter* (or, *letter requesting help*)
  - *Dear IMC*: The Spate Gallery has recently been approached by an antiques dealer . . .
  - *Dear Calculus Students*, I have a problem . . .

- Worked on by *Groups* of 2+ students

- Requiring a *Written*, *multi-page* (3–7 pages) solution paper,

- Produced over 2–3 weeks.

  *In many undergraduate courses* . . .
2. Why might we use them?

You have too much time on weekends?

- Applications are *fun* and *motivational*: they show mathematics beyond the boundary of the textbook, *important* and *relevant* in its own right.
  - And there are applications *all over the undergraduate curriculum* (even Calculus).
  - They *Can* be as *Applied* as desired.

- They require *conceptual understanding* and *problem solving*.

- And *writing* is in itself a worthy goal!

  ...and you can grade them without instructor fatalities
Dear IMC:

The Spate Gallery has recently been approached by an antiques dealer with headquarters in the outskirts of Athens who has offered us first purchase rights to a so-called ancient tapestry. However, we are concerned with its authenticity and so are contacting you to ascertain its age.

The dealer asserts that the tapestry is 2000 years old, having been loomed sometime in the first century B.C. In order to determine the veracity of this claim, our technical analysis department was able to obtain a small portion of the tapestry and has done an analysis of Carbon-14 present therein. They report the presence of 190 nanograms (ng) of Carbon-14 in the 1 gram sample they took.

As you will of course know from your own references on the subject, the amount of C14 present in the environment has varied with time. This amount is constant in living creatures, and then decays (with a half-life of 5,780 years) after they die. The figure below shows the amount (in ng) of C14 (per gram) present in the environment for the past 2,100 years. This was compiled by our consulting scientist, Dr. LaRose, so we have every reason to believe its accuracy.

We need from you as accurate an estimate of the age of this tapestry as possible, so as to determine both whether we should pay the dealer’s usurious price and whether we should continue dealing with the company he represents. Unfortunately, we are under some time constraints, as the dealer requires an answer soon; we therefore need your report by the 22nd of this month.

In addition, as it is in both of our interests to assure the successful and timely completion of your project, our consulting scientist, Dr. Gavin LaRose, is available to answer any and all additional questions that you may have regarding the technicalities or requirements related to your effort. You are urged to contact him as a group with any questions you may have—and, as is also indicated in your contract, you should see him (also as a group) to give a progress report on the project by the 15th.

Sincerely,

M. Ike Langelo, President
Art Inc.
4. Some Observations

we’ll look at solutions & grading in a bit…

• I like “really” applied projects.
  ○ But “Reality” is in the eye of the beholder.
  ○ Applied letters provide a Context for students’ responses.
    (couldn’t they be multiple choice?)

• Writing is fundamental to these projects.
  ○ Writing to explain mathematics is a learning process.
  ○ Writing (about mathematics) is a useful skill.
  ○ Reading (mathematical) detailed writing is required by this format, and is also a useful skill.
5. Some Logistical Matters

- These are *Non-Trivial*: 1–3 (or 2–3)/semester, in one class at a time, is probably good,
- As 10–30% of course grade(?)
- I don’t use classtime for project work,
  - in which case, *2+ weeks* of work time is reasonable.
- *Small Classes* (*≤30 students*?) may be essential,
- and *Groups* are an excellent thing
  - $\frac{30}{2} = 15$, and $\frac{30}{3} = 10$. *(grading)*
  - Problems can be... *sufficiently difficult.*
6. And Some Philosophical Ones

- *Propaganda* is a key part of these (and many other) teaching efforts:
  - Groups *are good*. Problem solving *is good*. Math *is fun*. These are *real-life applications*.

- Students *can* Write!
  - *...but sample papers or writing tips may be good.*

- Students *can* Procrastinate!
  - An *early meeting* with groups may be good (*do students know their group members?!)*
  - A *cut off* on aid (e.g., for the last weekend) may decrease procrastination.
    - *...or it might not.*

- *How should we manage groups?*
7. A Group HowTo

- Propaganda: Groups are pedagogically sound. Groups make hard problems easier. Groups are loved and appreciated by all.

- Composition:
  - Self-selected, or instructor assigned
    mix-up groups, reduce class tension
  - Heterogeneous, or match ability
    pedagogy vs. group dynamic
  - Four, three or two students
    pedagogy vs. freeloaders
8. Grading

- **Holistic Grading**

  *I am one with the infinity of papers I have read.*
  
  *Do not question the master.*

  C-

  ... or, point-based grading:

- **Rubrics**

  - Emphasize the *mathematical solution* to the problem.
  - *Structure (, quality,) and format* of the solution paper are graded *implicitly*.

- **Checklists**

  - Emphasize *paper structure* and general mathematical *process*.
  - The *specific mathematical solution* to the problem is graded *implicitly*.
9. Rubrics (...and how to create them)

- **Identify steps** in the solution to the problem.
  - *e.g.*, Find exponential decay model; Verify approximate age of tapestry; Calculate actual age using model and data

- **Assign each 1–4 points**
  - *e.g.*, for exponential model:
    - 0 pts = no sensible model
    - 1 pt = model, error in all params
    - 2 pts = one of base, initial value, exponent correct, or two with poor derivation.
    - 3 pts = two of base, initial value, exponent correct, or all three with poor derivation
    - 4 pts = correct and well-explained model

- Assign some points for *clarity/quality* and *deadlines* (total $\approx$ 15–20 points)

- Read each paper once + spot-check.
## 10. A Sample Rubric

for project *Into the World of Art*

<table>
<thead>
<tr>
<th>Objective</th>
<th>0 points</th>
<th>1 point</th>
<th>2 points</th>
<th>3 points</th>
<th>4 points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential model</td>
<td>no sensible model</td>
<td>exponential model, errors in base, initial value and exponent</td>
<td>one of base, initial value, exponent correct, or two correct with questionable derivation</td>
<td>two of base, initial value, exponent correct, or all three but questionable or unclear derivation</td>
<td>correct and well explained model</td>
</tr>
<tr>
<td>Correctness of claim</td>
<td>no determination that age is lt 2000 years</td>
<td>statement that age is lt 2000 years, no or poor argument</td>
<td>statement that age is lt 2000 years, good intuitive or mathematical argument</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Age of tapestry</td>
<td>no determination of actual age</td>
<td>estimate of age, little or no logical support</td>
<td>estimate of age, referring to data, but with logical gaps or errors or unclear argument</td>
<td>accurate estimate of age, generally correct argument from data, not fully supported or not entirely clear</td>
<td>accurate estimate of age, clearly supported mathematically</td>
</tr>
<tr>
<td>Clarity and Organization</td>
<td>no paper, or utter disaster</td>
<td>multiple unclear sections and limited organization</td>
<td>some unclear section(s) or spotty organization</td>
<td>good paper with few flaws</td>
<td></td>
</tr>
<tr>
<td>Deadlines</td>
<td>all deadlines missed, no paper</td>
<td>one or more deadlines missed</td>
<td>all deadlines met</td>
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glr / NExTWrite 12(5)
11. Checklists (...and where to find them)

- A checklist indicates what is expected of a student paper, e.g.,
  
  - *required components* (problem statement, problem solution, acknowledgment...)
  
  - *content* (explanation of solution, definitions, labeled diagrams...)
  
  - *format* (scientific paper or business letter, spelling and grammar...)

- Each is worth 1–2 *points*

- *Total = 10–15 points*

- Read each paper once + spot-check.

  *sources: Elyn Rykken, Annalisa Crannell*
### 12. Sample Checklist (from Elyn Rykken, Muhlenberg College)

Instructions given to students with the checklist:
- Please attach this page with a paper-clip to your writing assignment when you turn it in.
- This list will be used to grade your assignment and will be returned to you with comments. Keep a copy of your paper for your own reference.
- Use this checklist as a guide for yourself while writing the assignment.

<table>
<thead>
<tr>
<th>Form:</th>
<th>5 points</th>
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</thead>
<tbody>
<tr>
<td>Does this paper:</td>
<td>1. clearly (re)state the problem to be solved (including on the essential details)?</td>
</tr>
<tr>
<td></td>
<td>2. explain what level and types of math will be used?</td>
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<tr>
<td></td>
<td>3. solve the question that was originally asked? (2 pts)</td>
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<td></td>
<td>4. give acknowledgment where it is due (did anyone work with you on the math)?</td>
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<table>
<thead>
<tr>
<th>Content:</th>
<th>7 points</th>
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<tbody>
<tr>
<td>Does this paper:</td>
<td>5. give a precise and well-organized explanation of how the answer was found? (2 pts)</td>
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<td></td>
<td>6. define all variables, terms, and notation used?</td>
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<td>7. explain how each formula is derived or where it can be found?</td>
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<td>8. clearly label diagrams, tables, graphs or other visual representations of the math?</td>
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<td></td>
<td>9. contain correct mathematics? (2 pts)</td>
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<table>
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<tr>
<th>Presentation:</th>
<th>3 points</th>
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<tr>
<td>Does this paper:</td>
<td>10. use standard business letter form?</td>
</tr>
<tr>
<td></td>
<td>11. use correct spelling, grammar, and punctuation?</td>
</tr>
<tr>
<td></td>
<td>12. look neat? (typing helps with this)</td>
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</table>

| Comments: |
13. Some Other Thoughts

- *Solving projects yourself* is a good idea.
- In general, students *Rise to Challenges* (both *writing*, and *problem-solving*).
- Cross-group *collaboration* is probably ok.
- How are *Project Topics* picked? Syllabus driven: syllabus → topic → project
  e.g., Calc 1, Fall 08

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<td>8</td>
<td>10</td>
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- *Functions Review*
- *Project Due Date*
- *First Midterm*

  so the first project has to do with functions...

- Where are *Project Problems* found? *text, project books, divine inspiration*...
14. Concluding Thoughts

- There are varying levels of *realism* in these projects. I like “*real applications*.”

- Grading the projects does *not* have to take the better part of a lifetime. But it *does* take some time.

- Students might (will?) *complain* about projects. Even if they *like the applications* and *rise to the problem solving*.

- I’ve never dwelt on teaching writing. Do they *rise to challenges*? These are college students…

- I think projects are *fun and productive*.

- It is *not* necessary to write all your own projects: *adapt, reuse, recycle*
  - Others’ projects and ideas.
  - Your projects and ideas.
15. Resources

- Others’ projects on-line (Annalisa, me, Tommy, Elyn…).
- Textbooks and modeling texts, esp. Hughes-Hallett et al. (Wiley); also *Mathematical Modeling*…, Hadlock (MAA, ’98); *Mathematical Biology*, Murray (Springer ’90); Differential Equations texts (esp. *Borrelli & Coleman*, Wiley ’04);...