

## 1. Teichmüller Theory

- Basics
  - Equivalent constructions of Teichmüller space: As a quotient of the space of Beltrami differentials, as a component of the  $\mathrm{PSL}(2, \mathbb{R})$  character variety (the algebraic topology)
  - Fenchel-Nielsen coordinates
  - Fricke's theorem; moduli space is an orbifold, but is not a manifold (though is finitely covered by one)
  - Quasiconformal maps, Teichmüller metric, Grötzsch's problem, Teichmüller's theorems
- The complex structure on Teichmüller space
  - The Schwarzian parametrization of the space of marked projective structures
  - The Bers projection and the Bers embedding, the latter providing an atlas of charts for Teichmüller space as a complex manifold
  - The derivative of the Bers projection, the characterization of the tangent/cotangent space of Teichmüller space via Beltrami/quadratic differentials
  - Quasifuchsian reciprocity
- The WP form and Wolpert's Magic Formula
  - McMullen's proof that the WP form is symplectic
  - Gardiner's formula
  - Proof of Wolpert's formula

## 2. Translation surfaces

- Basics
  - Equivalent definitions of translation surfaces: As surfaces with singular flat metrics with changes of coordinates that are translations, as abelian differentials, as disjoint unions of polygons with parallel side-identifications
  - The Teichmüller and moduli spaces of abelian differentials
  - Strata and period coordinates
  - Masur-Veech measure
- Further structure and dynamics
  - Affine invariant submanifolds
  - Square-tiled surfaces (density in the strata, Veech groups, etc.)
  - The  $\mathrm{GL}^+(2, \mathbb{R})$  and  $\mathrm{SL}(2, \mathbb{R})$  actions: definitions, unboundedness of  $\mathrm{SL}(2, \mathbb{R})$  action,  $\mathrm{GL}^+(2, \mathbb{R})$ -invariance of AISs
  - Teichmüller geodesic flow and unipotent flow: definitions, having a periodic orbit under unipotent flow means you decompose into horizontal cylinders
  - The Veech dichotomy
  - Theorem statements without proof: Masur's criterion for unique ergodicity, Magic Wand