11. [5 points] A curve $\mathcal{C}$ gives y as an implicit function of x. The curve $\mathcal{C}$ passes through the point $(1, 2)$ and satisfies

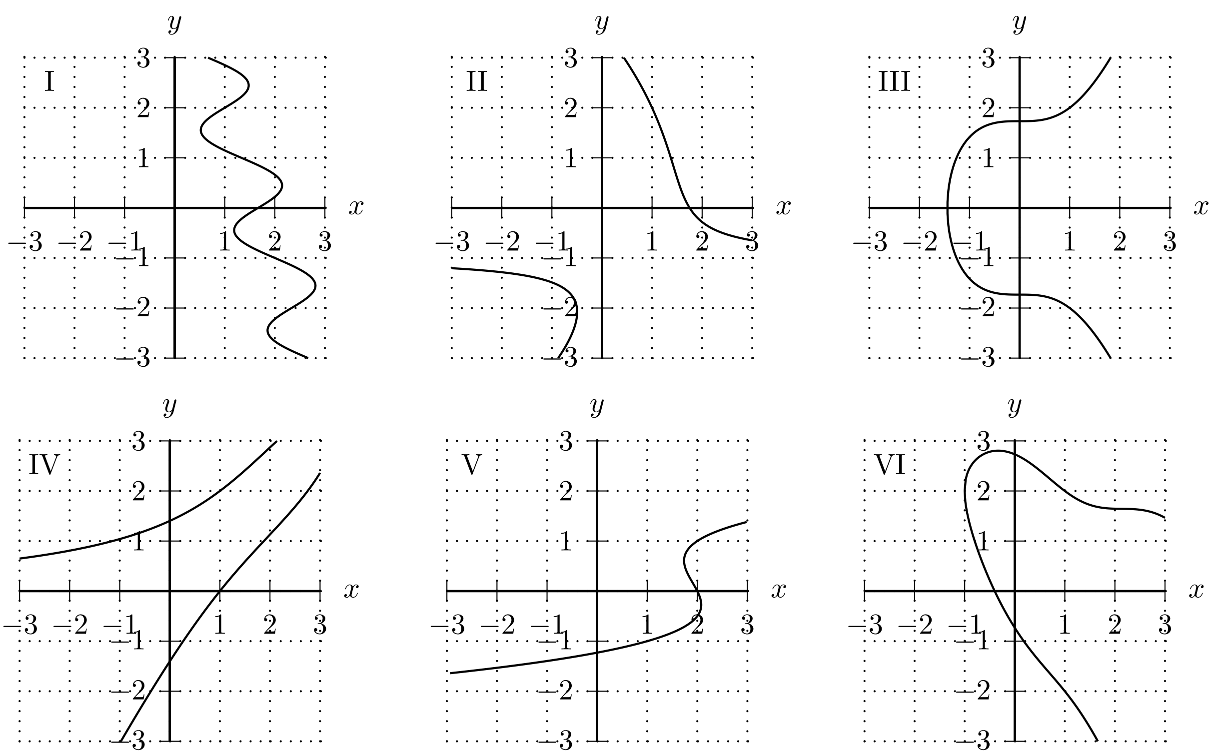
$$\frac{dy}{dx} = \frac{y^2 - 2xy + 4y - 5}{4(y - x)}.$$

a. [1 point] One of the values below is the slope of the curve $\mathcal{C}$ at the point $(1, 2)$. Circle that one value.

Solution: Plugging $x = 1$ and $y = 2$ into the given formula for $\frac{dy}{dx}$ yields $3/4$.

Answer: The slope at $(1, 2)$ is $\frac{1}{4}$ $\frac{1}{3}$ $\frac{1}{2}$ $\frac{5}{8}$ $\frac{2}{3}$ $\boxed{\frac{3}{4}}$ $\frac{4}{5}$

b. [4 points] One of the following graphs is the graph of the curve $\mathcal{C}$.
Which of the graphs I-VI is it? To receive any credit on this question, you must circle your answer next to the word "Answer" below.



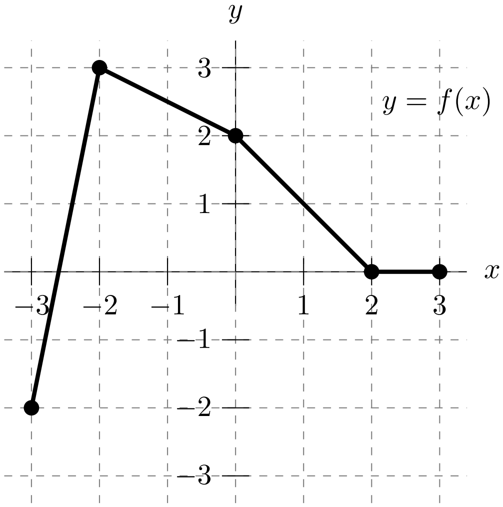
Solution: We know that the desired curve passes through the point $(1, 2)$ with slope $3/4$. This allows us to eliminate Graph V (which doesn't pass through $(1, 2)$) and Graphs II and VI (which have negative slope at $(1, 2)$).
To decide between Graphs I, III, and IV, we look at other points on the graphs.
Graph I passes through the point $(2, -1)$ with negative slope, but the above formula for $\frac{dy}{dx}$ says that it should have positive slope there, so Graph I is incorrect.
Graph III passes through the point $(1, -2)$ with negative slope, but the above formula for $\frac{dy}{dx}$ says that it too should have positive slope there, so Graph III is incorrect.
The only remaining possibility is Graph IV.
(Note that we could have also eliminated all but Graph IV by checking for vertical tangent lines at points (x, y) with $y = x$.)

Remember: To receive any credit on this question, you must circle your answer next to the word "Answer" below.

Answer: I II III $\boxed{\text{IV}}$ V VI

2. [12 points]

Let f be the piecewise linear function with graph shown below.



The table below gives several values of a differentiable function g and its derivative g'.
Assume that both $g(x)$ and $g'(x)$ are invertible.

x	-2	-1	0	2	5
$g(x)$	21	11	5	-1	-3
$g'(x)$	-12	-8	-4	-2	-0.4

You are not required to show your work on this problem. However, limited partial credit may be awarded based on work shown.

For each of parts **a.-f.** below, find the value of the given quantity. If there is not enough information provided to find the value, write "NOT ENOUGH INFO". If the value does not exist, write "DOES NOT EXIST".

a. [2 points] Let $j(x) = e^{g(x)}$. Find $j'(2)$.

Answer: $-\frac{2}{e} \approx -0.736$

b. [2 points] Let $k(x) = f(x)f(x+2)$. Find $k'(-1)$.

Answer: -3

c. [2 points] Let $h(x) = 3f(x) + g(x)$. Find $h'(-2)$.

Answer: **DOES NOT EXIST**

d. [2 points] Find $(g^{-1})'(2)$.

Answer: **NOT ENOUGH INFO**

e. [2 points] Let $m(x) = g(f(g(x)))$. Find $m'(2)$.

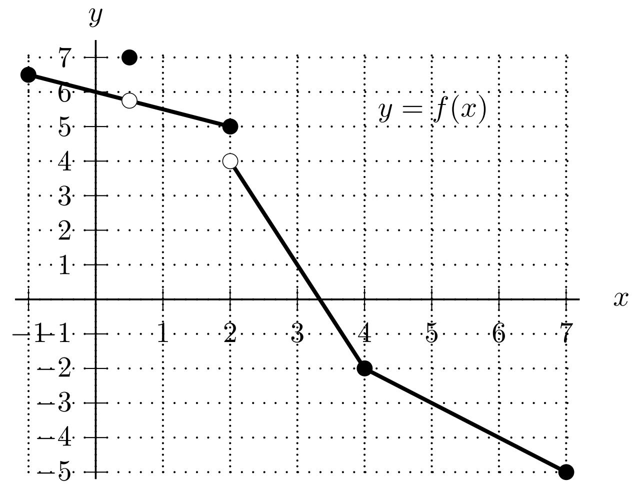
Answer: **NOT ENOUGH INFO**

f. [2 points] Let $\ell(x) = \dfrac{f(x)}{g(2x)}$. Find $\ell'(-1)$.

Answer: $\dfrac{21(-.5) - 2(2.5)(-12)}{21^2} \approx 0.1122$

2. [11 points]

Shown to the right is the graph of a function $f(x)$.



Note that you are not required to show your work on this problem. However, limited partial credit may be awarded based on work shown.

Find each of the following values. If the value does not exist, write DOES NOT EXIST.

a. [3 points] Let $h(x) = f(3x+1)$. Find $h'(1)$.

Solution: Since the graph $y = h(x)$ corresponds to the graph of $y = f(x)$ shifted left 1 unit and then horizontally compressed by a factor of 1/3, $h(x)$ has a "sharp corner" at $x = 1$ so is not differentiable there.

Answer: $h'(1) =$ DOES NOT EXIST

b. [3 points] Let $k(x) = e^{f'(x)}$. Find $k'(6)$.

Solution: By the chain rule, $k'(x) = e^{f'(x)} f''(x)$. So $k'(6) = e^{f'(6)} f''(6) = e^{-1}(0) = 0$.

Answer: $k'(6) =$ 0

c. [2 points] Find $(f^{-1})'(0)$.

Solution: By the formula for the derivative of an inverse,

$$(f^{-1})'(0) = \frac{1}{f'(f^{-1}(0))} = \frac{1}{f'(10/3)} = \frac{1}{-3}.$$

Answer: $(f^{-1})'(0) =$ $-1/3$

d. [3 points] Let $j(x) = \dfrac{f(2x+1)}{x+1}$. Find $j'(1)$.

Solution: Applying the quotient and chain rules, we find that

$$j'(x) = \frac{2f'(2x+1)(x+1) - f(2x+1)(1)}{(x+1)^2}.$$

Thus,

$$j'(1) = \frac{2f'(3)(2) - f(3)}{2^2} = \frac{(2)(-3)(2) - (1)}{4} = \frac{-13}{4}.$$

Answer: $j'(1) =$ $-13/4$