Quiz 2

Name:

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This quiz has 3 questions worth 23 points on 3 pages. Try to do as many questions as possible. You can use your calculator.

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1. The function $r(x)$ is given by the following formula, where c is a positive constant:

$$r(x) = \begin{cases} \dfrac{3x+3}{(x+5)(x-2)} & x < 0 \\ \\ \dfrac{c}{x^3-1} & 0 \leqslant x < 4 \\ \\ \sqrt{2 - \dfrac{8}{x}} & x \geqslant 4 \end{cases}$$

It is not necessary to show work in this problem

(a) (2 points) Find $\lim_{x \to -\infty} r(x)$. If the limit does not exist, write DNE.

$\lim_{x \to -\infty} r(x) =$ **0**

(b) (2 points) For what value(s) of x does $r(x)$ have a vertical asymptote? Write NONE if there are no such values.

$x =$ **-5,1**

(c) (2 points) For what value(s) of x is $r(x) = 0$? Write NONE if there are no such values.

$x =$ **-1,4**

(d) (2 points) For what value(s) of c is the function $r(x)$ continuous at $x = 0$? Write NONE if there are no such values.

$c =$ $\frac{3}{10}$

Solution: We have

$$\lim_{x \to 0^-} r(x) = \lim_{x \to 0^+} r(x)$$
$$\lim_{x \to 0^-} \frac{3x+3}{(x+5)(x-2)} = \lim_{x \to 0^+} \frac{c}{x^3-1}$$
$$\frac{3}{-10} = \frac{c}{-1}$$
$$c = \frac{3}{10}$$

2. (7 points) Consider the function $f(x)$ defined by

$$f(x) = \begin{cases} x\mathrm{e}^{Ax} + B & x < 3 \\ C(x-3)^2 & 3 \leqslant x \leqslant 5 \\ \frac{130}{x} & x > 5 \end{cases}$$

Suppose $f(x)$ satisfies all of the following:

- $f(x)$ is continuous at $x = 3$.
- $\lim\limits_{x \to 5^+} f(x) = 2 + \lim\limits_{x \to 5^-} f(x)$.
- $\lim\limits_{x \to -\infty} f(x) = -4$.

Find the values of A, B and C. Show your work. Your answer must be in *exact form*. DO NOT USE decimal approximations.

Solution: For each condition, we have

- Beause $f(x)$ is continuous at $x = 3$, $\lim\limits_{x \to 3^+} f(x) = \lim\limits_{x \to 3^-} f(x)$. So $3\mathrm{e}^{3A} + B = 0$.
- Because $\lim\limits_{x \to 5^+} f(x) = 2 + \lim\limits_{x \to 5^-} f(x)$, we have $\frac{130}{5} = 2 + 4C \Rightarrow C = 6$.
- Since $\lim\limits_{x \to -\infty} f(x) = -4$ exists. We must have $A > 0$ and $\lim\limits_{x \to -\infty} x\mathrm{e}^{Ax} = 0$. So $B = -4$ and $A = \frac{1}{3} \ln(\frac{4}{3})$.

3. (8 points) On the axes provided below, sketch the graph of a single function f satisfying all of the following:

(1) The graph of f is concave up for $x < -2$.

(2) The graph of f has a vertical asymptote at $x = -2$.

(3) $f'(-1) = -3$.

(4) $\lim_{x \to 0} f(x) = 2$.

(5) $f(0) = -2$.

(6) $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$.

(7) f is not continuous at $x = 1$.

(8) $f'(x) > 0$ for $x > 3$.

(9) $\lim_{x \to \infty} f(x)$=4