# Indefinite Integral

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## 1 Indefinite Integral

**Proposition 1.1.** *If $F$ and $G$ are both antiderivatives of $f$ on an interval, then $G(x) = F(x) + C$.*

So if $F(x)$ is an antiderivaive of $f(x)$, then $F(x) + C$, with different $C$ values, are the possiblity of all antiderivatives of $f$. Therefore we use following *indefinite integral* notion to represent a general element in the family of the antiderivatives of $f(x)$, i.e.,

$$\int f(x)\ \mathrm{d}x = F(x) + C$$

## 2 Formulas for indefinite integral

We have following formulas

$$\begin{aligned}
\int 0\ \mathrm{d}x &= C \\
\int k\ \mathrm{d}x &= kx + C \\
\int x^n\ \mathrm{d}x &= \frac{x^{n+1}}{n+1} + C, \quad n \neq -1 \\
\int \frac{1}{x}\ \mathrm{d}x &= \ln(|x|) + C \\
\int \mathrm{e}^x\ \mathrm{d}x &= \mathrm{e}^x + C \\
\int \cos(x)\ \mathrm{d}x &= \sin(x) + C \\
\int \sin(x)\ \mathrm{d}x &= -\cos(x) + C
\end{aligned}$$

**Proposition 2.1** (Properties of Antiderivatives: Sums and Constant Multiples)**.** *In indefinite integral notation,*

1. $\int (f(x) \pm g(x))\mathrm{d}x = \int f(x)\mathrm{d}x \pm \int g(x)\mathrm{d}x$
2. $\int cf(x)\mathrm{d}x = c\int f(x)\mathrm{d}x$

*In words,*

1. *An antiderivative of the sum (or difference) of two functions is the sum (or difference) of their antiderivatives.*
2. *An antiderivative of a constant times a function is the constant times an antiderivative of the function.*