4. [13 points] One of the ways Captain Christina likes to relax in her retirement is to go for long walks around her neighborhood. She has noticed that early every Tuesday morning, a truck delivers butter to a local bakery famous for its cookie dough. Consider the following functions:

- Let $C(b)$ be the bakery's cost, in dollars, to buy b pounds of butter.
- Let $K(b)$ be the amount of cookie dough, in cups, the bakery makes from b pounds of butter.
- Let $u(t)$ be the instantaneous rate, in pounds per hour, at which butter is being unloaded t hours after 4 am.

Assume that C, K, and u are invertible and differentiable.

a. [2 points] Interpret $K(C^{-1}(10)) = 20$ in the context of this problem.
Use a complete sentence and include units.

Solution: If the bakery spends \$10 on butter, then it can make 20 cups of cookie dough.

b. [3 points] Interpret $\int_5^{12} K'(b)\,db = 40$ in the context of this problem.
Use a complete sentence and include units.

Solution: 12 pounds of butter makes 40 cups more cookie dough than 5 pounds of butter does.

c. [2 points] Give a single mathematical equality involving the <u>derivative</u> of C which supports the following claim:
It costs the bakery approximately \$0.70 less to buy 14.8 pounds of butter than to buy 15 pounds of butter.

Answer: $C'(15) = 3.5$

d. [3 points] Give a single mathematical equality which expresses the following claim:
The number of pounds of butter unloaded between 5 and 8 am is twice as many as the bakery needs to make 5000 cups of cookie dough.

Answer: $\int_1^4 u(t)\,dt = 2K^{-1}(5000)$

e. [3 points] Assume that $u(t) > 0$ and $u'(t) < 0$ for $0 \le t \le 4$ and that $u(2) = 800$.
Rank the following quantities in order from least to greatest by filling in the blanks below with the options I-IV.

I. 0 II. 800 III. $\int_1^2 u(t)\,dt$ IV. $\int_2^3 u(t)\,dt$

Solution: Since $u(t) > 0$, both integrals are greater than 0. Since $u'(t) < 0$, $u(t)$ is a decreasing function. Estimating $\int_1^2 u(t)\,dt$ with a right sum with one subdivision yields an underestimate of 800, and likewise, estimating $\int_2^3 u(t)\,dt$ with a left sum with one subdivision yields an overestimate of 800.

$0 < \int_2^3 u(t)\,dt < 800 < \int_1^2 u(t)\,dt$

8. [11 points] The energy, in megajoules (MJ), produced by a wind turbine depends on the speed of the wind. In particular, suppose $P(s)$ is the power, in megajoules per hour (MJ/h), produced by the turbine when the speed of the wind is s kilometers per hour (km/h). Also suppose that $W(t)$ gives the wind speed, in km/h, at the turbine's location t hours after noon on a typical day.

Assume that $P(s)$ is invertible, and that both $P(s)$ and $W(t)$ are differentiable.

a. [2 points] Give a practical interpretation of the equation $P(W(0)) = 8$.

Solution: At noon on a typical day, the turbine produces 8 MJ/h of power.

b. [3 points] Give a practical interpretation of the equation $\int_0^5 P(W(t))\,dt = 46$.

Solution: From noon to 5 p.m. on a typical day, the turbine generates 46 MJ of energy.

c. [3 points] Complete the following sentence to give a practical interpretation of the equation

$$W'(4) = 21$$

From 4 pm to 4:10 pm, ...

Solution: the wind speed at the turbine's location increases by approximately 3.5 km/h.

d. [3 points] Circle the one statement below that is best supported by the equation

$$(P^{-1})'(13) = 2.9.$$

i. *If the turbine is producing 13 MJ/h of power, the wind speed must increase by approximately 2.9 km/h to produce an additional MJ/h of power.*

ii. If the wind is blowing at 13 km/h and increases to 14 km/h, the power produced by the turbine will increase by about 2.9 MJ/h.

iii. If the wind speed is 13 km/h, the power generation of the turbine will increase by one MJ/h if the wind speed increases to about 15.9 km/h.

iv. When the turbine is generating 13 MJ/h of power, an increase of one km/h in wind speed will produce approximately 2.9 MJ/h more power.