1. [11 points] At a recent UM football game, a football scientist was measuring the excitement density, $E(x)$, in cheers per foot, in a one hundred foot row of the football stadium where x is the distance in feet from the beginning of the row. He took measurements every twenty feet and the data is recorded in this table.

x	0	20	40	60	80	100
$E(x)$	30	24	19	16	13	7

Assume for this problem that $E(x)$ is a decreasing function for $0 \leq x \leq 100$.

a. [6 points] Write a right sum and a left sum which approximate the total cheers in the row. Be sure to write all of the terms for each sum.

Solution:
LEFT $= 20(30) + 20(24) + 20(19) + 20(16) + 20(13) = 2040$
RIGHT $= 20(24) + 20(19) + 20(16) + 20(13) + 20(7) = 1580$

b. [2 points] Indicate whether the right and left sums are overestimates or underestimates for the total number of cheers in the row.

The right sum is an **overestimate** **[underestimate]**

The left sum is an **[overestimate]** **underestimate**

2. [8 points] Due to an accident, an oil pipeline is leaking.
Let $p(t)$ be the rate (in gallons/hour) at which the pipeline leaks oil t hours after the accident.
Assume that $p(t)$ is a strictly decreasing, differentiable function for $0 \leq t \leq 24$.
Engineers make the following measurements of $p(t)$.

t	0	6	12	18	24
$p(t)$	97	86	79	61	49

b. [3 points] Based on the data provided, write the right Riemann sum that best approximates the total amount of oil (in gallons) that leaked from the pipeline in the first 24 hours after the accident. *Be sure to carefully write out all of the terms in the sum.*

Solution: We want to estimate $\int_0^{24} p(t)\,dt$. Based on the limited data provided, the best we can do is to use 4 equal subintervals ($n = 4$, $\Delta t = 6$). The resulting approximation of the total number of gallons of oil that leaked from the pipeline in the first 24 hours after the accident is then

$$\int_0^{24} p(t)\,dt \approx p(6)\cdot 6 + p(12)\cdot 6 + p(18)\cdot 6 + p(24)\cdot 6 = 86(6) + 79(6) + 61(6) + 49(6) = 1650.$$

c. [1 point] Indicate whether the right sum above is an overestimate or an underestimate for the total amount of oil leaked. If there is not enough information to make this determination, circle "not enough information". You do not need to explain your answer.

Answer: The right sum is an (circle one):

overestimate **[underestimate]** not enough information

4. [10 points] Gabe the mouse is swimming alone in a very large puddle of water. He keeps track of his swimming time by logging his velocity at various points in time. Gabe starts at a point on the edge of the puddle and swims in a straight line with increasing speed. A table of Gabe's velocity $V(t)$, in feet per second, t seconds after he begins swimming is given below.

t	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
$V(t)$	0	0.3	0.4	0.45	0.9	1.2	1.8	2.4	2.7	2.9	3	3.2	3.5

b. [3 points] Estimate $\int_1^{5.5} V(t)\,dt$ by using a right-hand Riemann sum with 3 equal subdivisions. Make sure to write down all terms in your sum.

Solution: If we divide the interval $[1, 5.5]$ in three, we obtain $\Delta t = \frac{5.5-1}{3} = 1.5$. Then

$$Right(3) = (V(2.5) + V(4) + V(5.5))\Delta t = (1.2 + 2.7 + 3.2)(1.5) = (7.1)(1.5) = 10.65.$$

Answer=10.65 feet.

c. [1 point] Is your estimate from above an overestimate or an underestimate of the exact value of $\int_1^{5.5} V(t)\,dt$? Circle your answer.

Solution: **[OVERESTIMATE]** UNDERESTIMATE NOT ENOUGH INFORMATION