Review 4.1-4.3

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1 Some Useful Materials

All materials here are from http://www.math.lsa.umich.edu/courses/115/Exams/Exam_2/index.html.

- Handout with the material of Section 4.1: http://www.math.lsa.umich.edu/courses/115/Exams/Exam_2/Materials/Section4.1Material.pdf
- Justifications required for local extrema: http://www.math.lsa.umich.edu/courses/115/Exams/Exam_2/Materials/Math115LocalExtremaJustification.pdf
- Justifications required for global extrema: http://www.math.lsa.umich.edu/courses/115/Exams/Exam_2/Materials/Math115GlobalExtremaJustification.pdf
- Formulas that you will be expected to know: http://www.math.lsa.umich.edu/courses/115/Exams/Exam_2/Materials/Math115GeometryFormulasW20.pdf

2 Examples

(Local Extrema) Find the x-coordinate(s) of all local maxima and minima of $f(x) = x^5 - 10x^3 - 8$. You must use calculus to find your answers, and be sure to show enough evidence to fully justify your answers.

(Inflection Points) Suppose that $h(x)$ is a continuous function defined for all real numbers whose second derivative is given by $h''(x) = \dfrac{(6x-9)(x-2)^2}{(x+1)^{1/3}}$. Find the x-coordinate(s) of all inflection points of h.

(Global Extrema) Let

$$f(x) = \begin{cases} \frac{2-x}{e} & x < 1 \\ x^2 e^{-x} & x \geqslant 1 \end{cases}$$

Find the x-coordinate(s) of all global extrema of $f(x)$ on $[-3, 3]$. You must use calculus to find your answers, and be sure to show enough evidence to fully justify your answers.

9. [12 points] Suppose $w(x)$ is an everywhere differentiable function which satisfies the following conditions:

- $w'(0) = 0$.
- $w'(x) > 0$ for $x > 0$.
- $w'(x) < 0$ for $x < 0$.

Let $f(t) = t^2 + bt + c$ where b and c are positive constants with $b^2 > 4c$. Define $L(t) = w(f(t))$.

a. [2 points] Compute $L'(t)$. Your answer may involve w and/or w' and constants b and c.

b. [4 points] Using your answer from (a), find the critical points of $L(t)$ in terms of the constants b and c.

c. [6 points] Classify each critical point you found in (b). Be sure to fully justify your answer.

5. [10 points] As a software engineer, Tendai spends many hours every day writing code. Let $w(t)$ be a function that models the number of lines of code that Tendai writes in a day if he works t hours that day. Tendai works at least one hour and at most 18 hours each day. A formula for $w(t)$ is given by

$$w(t) = \begin{cases} -2t^2 + 28t & \text{if } 1 \le t \le 3 \\ -0.5t^2 + 9t + 43.5 & \text{if } 3 < t \le 18. \end{cases}$$

a. [8 points] Find the values of t that minimize and maximize $w(t)$ on the interval $[1, 18]$. Use calculus to find your answers, and be sure to show enough evidence that the points you find are indeed global extrema. For each answer blank, write NONE if appropriate.

Answer: global max(es) at $t =$ ____________________

Answer: global min(s) at $t =$ ____________________

b. [2 points] What is the largest number of lines of code that Tendai can expect to write in a day according to this model?

Answer: ____________________

10. [8 points] Consider the family of functions $g(x) = e^x - kx$, where k is a positive constant.

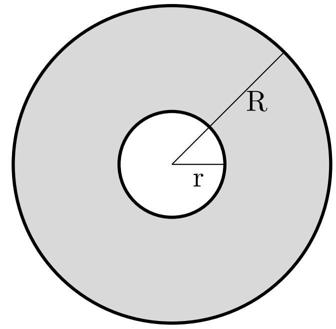
a. [2 points] Show that the point $\Big(\ln(k)\ ,\ k - k\ln(k) \Big)$ is the only critical point of $g(x)$ for all positive k. Show all your work to receive full credit.

b. [2 points] Show that $g(x)$ has a global minimum on $(-\infty, \infty)$ at $x = \ln(k)$. Use calculus to justify your answer.

c. [4 points] Find all values of $0.5 \leq k \leq 2$ that maximize the y-value of the global minimum of $g(x)$ on $(-\infty, \infty)$. Use calculus to justify your answer. Write NONE if no such value exists.

Answer: $k =$ ____________________

8. [8 points]



Kristen is machining a metal washer to fix her broken down motorcycle. A washer is a flat, circular piece of metal with a hole in the middle. Kristen's washer is depicted by the shaded region in the figure to the right. The washer has an inner radius of r centimeters and an outer radius of R centimeters. The area of the washer must be exactly 5 square centimeters, and r must be at least 1 centimeter.

a. [3 points] Find a formula for r in terms of R.

Answer: $r =$ ____________________

b. [2 points] The structural integrity of the washer depends on both its inner radius and its outer radius. Specifically, the structural integrity is given by the equation

$$S = 32R\left(\ln(rR + 1) + 7\right).$$

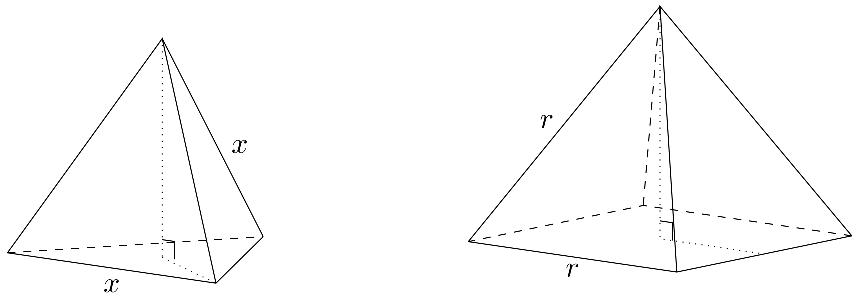
Express S as a function of R. *Your answer should not include* r.

Answer: $S(R) =$ ____________________

c. [3 points] What is the domain of $S(R)$ in the context of this problem? You may give your answer as an interval or using inequalities.

Answer: ____________________

5. [7 points] An alien is building the wire frames of two pyramids. One has a base that is an equilateral triangle with side length x meters, and the other has a base that is a square with side length r meters. These shapes are shown below. For both, all triangular faces are equilateral.



Triangular Pyramid Square Pyramid

The alien has 2 meters of wire available to build the frames, and **will use all of it**.

a. [2 points] Find a formula for r in terms of x.

Answer: $r =$ ____________________

b. [3 points] Find a formula for $A(x)$, the combined surface area of the two pyramids (i.e. the total area of all sides and bases of both shapes). Your formula should be in terms of x only. Recall that the area of an equilateral triangle with side length L is $\frac{\sqrt{3}}{4}L^2$.

Answer: $A(x) =$ ____________________

c. [2 points] The alien wants to actually build one of each type of pyramid. In the context of the problem, what is the domain of the function $A(x)$ from part **b.**? You may give your answer as an interval or using inequalities.

Answer: ____________________