Optimizations and Modeling

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1 General strategy

Step 1 Identify what quantity to be optimized

Step 2 Find a formula for the function of the quantity. If necessary, eliminate from this formula all but one variable. Identify the domain over which this variable varies.

Step 3 Optimization – Find critical points and evaluate the function at these points and the endpoints (if relevant) to find the global maxima and/or minima.

2 Geometrical Facts

http://www.math.lsa.umich.edu/courses/115/Exams/Exam_2/Materials/Math115GeometryFormulasW20.pdf

Geometry facts/formulas students are expected to know for exams

If you are wondering about something that does not appear on either list below, please ask.

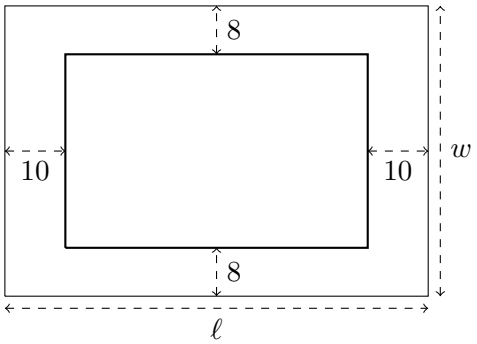
Geometry Facts Students Should Know (or have on their notecard)

- Perimeter of
  - polygon (sum of side lengths)
  - circle ($C = 2\pi r$)
- Area of
  - rectangle ($A = bh$)
  - triangle ($A = bh/2$)
  - circle ($A = \pi r^2$)
  - trapezoid ($A = \frac{(b_1+b_2)}{2} \cdot h$)
- Volume of
  - box ($V = \ell wh$)
  - cylinder ($V = \pi r^2 h$)
- Surface area of
  - box ($A = 2\ell w + 2\ell h + 2wh$)
  - cylinder ($A = 2\pi r^2 + 2\pi rh$)
- Pythagorean Theorem and Distance Formula
- Trigonometry
  - Triangle Trigonometry (e.g. $\sin(\theta) =$"opposite/hypotenuse")
  - Pythagorean Identity ($\sin^2 t + \cos^2 t = 1$)
  - $\tan t = (\sin t)/(\cos t)$
- Similar Triangles

Formulas That Would Be Provided If Needed (by no means a complete list!)

- Area
  - of quadrilaterals other than rectangles and trapezoids
  - of sector of a circle
  - Hero's formula
- Volume of
  - sphere
  - cone
- Surface area of
  - sphere
  - cone
- Trigonometry
  - double angle identities
- Arclength in a circle

1. [7 points] Liam wants to build a rectangular swimming pool behind his new house. The pool will have an area of 1600 square feet. He will have 8-foot wide decks on two sides of the pool and 10-foot wide decks on the other two sides of the pool (see the diagram below).



a. [4 points] Let ℓ and w be the length and width (in feet) of the pool area including the decks as shown in the diagram. Write a formula for ℓ in terms of w.

$\ell =$ _____

b. [3 points] Write a formula for the function $A(w)$ which gives the total area (in square feet) of the pool **and** the decks in terms of only the width w. Your formula should not include the variable ℓ. (This is the function Liam would minimize in order to find the minimum area that his pool and deck will take up in his yard. You do not need to do the optimization in this case.)

$A(w) =$ _____