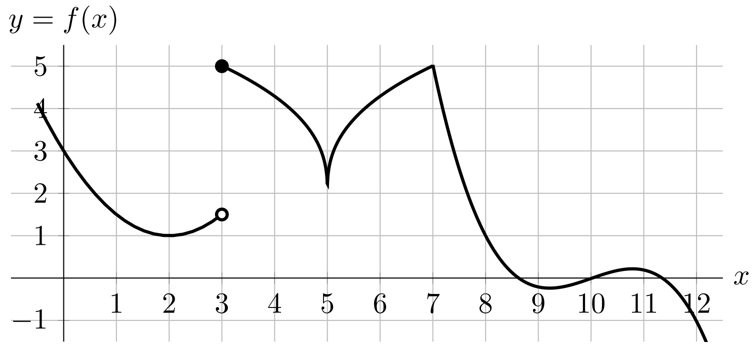
6. [4 points] The graph of the function $f(x)$ is shown below. Note that $f(x)$ has a vertical tangent line at $x = 5$.



a. [2 points] On which of the following intervals does the function $f(x)$ satisfy the hypotheses of the Mean Value Theorem? Circle the correct answer(s).

[0,2] [1,3] [2,4] [3,5] NONE OF THESE

b. [2 points] On the interval $[8, 12]$ the hypotheses of the Mean Value Theorem are true for the function $f(x)$. What does the conclusion of this theorem say in this interval?

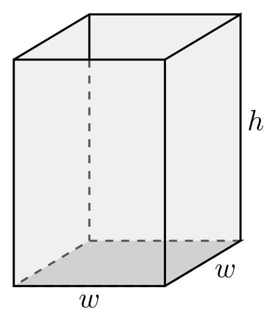
Answer:

Solution: There is some c on the interval $(8, 12)$ such that $f'(c) = \frac{f(12) - f(8)}{12 - 8} = -\frac{1}{2}$.

7. [5 points]

Yi is constructing a cardboard box. The base of the box will be a square of width w inches. The height of the box will be h inches. Yi will use gray cardboard for the sides of the box and brown cardboard for the bottom (the box does not have a top). Gray cardboard costs \$0.05 per square inch, while brown cardboard costs \$0.03 per square inch. Yi wants to spend \$20 on the cardboard for his box.

Write a formula for h in terms of w.



Solution: The area covered by the gray and the brown cardboard are $A_g = 4wh$ and $A_b = w^2$ respectively. Then the cost of the cardboard, in dollars, used in the cardboard is $C = 0.05A_g + 0,03A_b$. Hence w and h satisfy

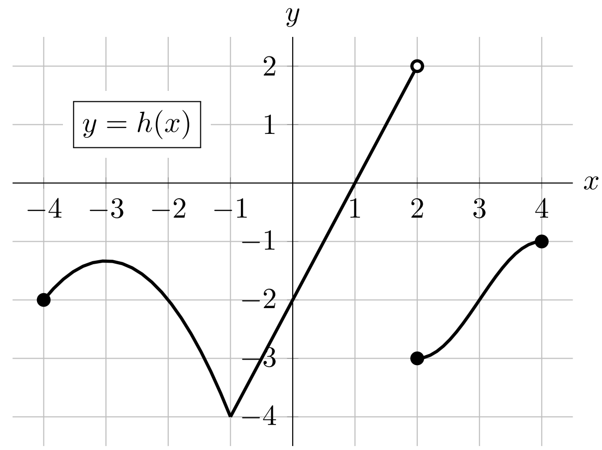
$$C = 20 = 0.05(4wh) + 0.03w^2 = 0.2wh + 0.03w^2.$$

Then

$$h = \frac{20 - 0.03w^2}{0.2w}.$$

Answer: $h = \frac{20 - 0.03w^2}{0.2w}$

7. [11 points]
Shown to the right is the graph of a function $h(x)$.



For parts **a.–c.**, circle **all** correct choices.

a. [2 points] Which of the following are critical points of $h(x)$?

$\boxed{x = -3}$ $\boxed{x = -1}$ $x = 1$ $\boxed{x = 2}$ $x = 3$ NONE OF THESE

b. [2 points] On which of the following interval(s) does $h(x)$ satisfy the hypotheses of the Mean Value Theorem?

$\boxed{[-4, -1]}$ $[-4, 0]$ $[0, 2]$ $\boxed{[3, 4]}$ NONE OF THESE

c. [2 points] On which of the following interval(s) does $h(x)$ satisfy the conclusion of the Mean Value Theorem?

$\boxed{[-4, -1]}$ $\boxed{[-4, 0]}$ $[0, 2]$ $\boxed{[3, 4]}$ NONE OF THESE

d. [5 points] Define the function $k(x)$ such that

$$k(x) = \begin{cases} h(x) & -4 \le x < 1 \\ A^2 \sin(Ax + B) & 1 \le x \le 4, \end{cases}$$

where A and B are constants. Find one pair of values for A and B that make $k(x)$ differentiable at $x = 1$. *Show your work.*

Solution: For k to be differentiable, it must be continuous. At $x = 1$, continuity implies that

$$0 = A^2 \sin(A + B), \quad \text{so} \quad A = 0 \text{ or } \sin(A + B) = 0.$$

We also need the slope of each piece to match at $x = 1$, that is,

$$h'(1) = A^2 (\cos(A + B) \cdot A), \quad \text{so} \quad 2 = A^3 \cos(A + B).$$

Notice that we can rule out the possibility that $A = 0$ (since $2 \neq 0$), which forces us to choose $\sin(A + B) = 0$. The problem only asks for *one* pair of values, and *one* way to get $\sin(A + B) = 0$ is to set $A + B = 0$ so $A = -B$. This means

$$2 = A^3 \cos(0) = A^3, \quad \text{which we can solve to find} \quad A = \sqrt[3]{2} \quad \text{and} \quad B = -A = -\sqrt[3]{2}.$$

Note: There are other possible values. We could have also chosen $A + B = n\pi$ where n is any integer. If n is even, this gives $(A, B) = (\sqrt[3]{2}, n\pi - \sqrt[3]{2})$. If n is odd, this gives $(A, B) = (-\sqrt[3]{2}, n\pi + \sqrt[3]{2})$.

Answer: $A =$ $\sqrt[3]{2}$ and $B =$ $-\sqrt[3]{2}$