3. [9 points] This problem concerns the function $f(x) = -x - 3e^{4x}$.

a. [3 points] Show that the function f is invertible.

Solution: We have $f'(x) = -1 - 12e^{-4x}$ which is negative for all values of x. This means that f is a strictly decreasing function. Since f is strictly decreasing, it never takes the same value twice so f is invertible.

b. [2 points] Find $f^{-1}(-3)$. You do not need to show any work.

Solution: $f^{-1}(-3) = 0$ because $f(0) = -3$.

c. [4 points] Evaluate $(f^{-1})'(-3)$. Show all of your work.

Solution: Using the formula for the derivative of an inverse function, we get

$$(f^{-1})'(-3) = \frac{1}{f'(f^{-1}(-3))} = \frac{1}{f'(0)}$$

Since $f'(x) = -1 - 12e^{-4x}$ we have $f'(0) = -13$ and so

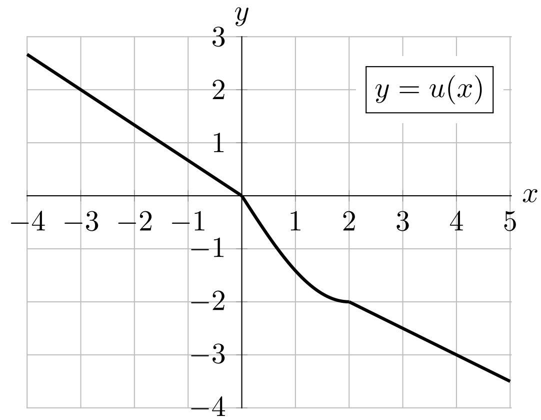
$$(f^{-1})'(-3) = -\frac{1}{13}$$

4. [13 points]

The function $u(x)$ is defined and invertible on $(-\infty, \infty)$. A portion of its graph is shown to the right.

Note that:

- $u(x) = -2\sin\left(\frac{\pi}{4}x\right)$ on $[0, 2]$, and
- $u(x)$ is linear on the intervals $(-4, 0)$ and $(2, 5)$.



a. [11 points] Evaluate each of the following quantities **exactly**, or write DNE if the value does not exist. You do not need to show work, but limited partial credit may be awarded for work shown. Your answers should not contain the letter u, but do not need to be fully simplified.

i. [2 points] Find $(u^{-1})'(-3)$.

Solution:
$$(u^{-1})'(-3) = \frac{1}{u'(u^{-1}(-3))} = \frac{1}{u'(4)} = \frac{1}{-1/2} = -2$$

Answer: $(u^{-1})'(-3) =$ -2

ii. [2 points] Let $v(x) = u(-1-x)$. Find $v'(-1)$.

Solution: The graph of $v(x) = u(-x-1)$ is the result of shifting the graph of $u(x)$ to the right 1 unit and then reflecting it across the y-axis. The point at $x = -1$ on the graph of $v(x)$ comes from the point $(0, 0)$ on the graph of $y = u(x)$, so there is a sharp corner at this point and $v'(-1)$ therefore does not exist.

Answer: $v'(-1) =$ **DNE**

iii. [3 points] Let $w(x) = \dfrac{x}{2^{u(x)}}$. Find $w'(-3)$.

Solution: Note that $u(-3) = 2$ and $u'(-3) = -2/3$.
$$w'(x) = \frac{2^{u(x)} - x\ln(2)\, 2^{u(x)}u'(x)}{\left(2^{u(x)}\right)^2} \quad \text{so} \quad w'(-3) = \frac{2^2 - (-3)\ln(2)\, 2^2(-\frac{2}{3})}{(2^2)^2}$$

Answer: $w'(-3) =$ $\dfrac{4 - 8\ln(2)}{16} = \dfrac{1 - 2\ln(2)}{4}$

iv. [4 points] Let $z(x) = \ln(2x+1)u(x)$. Find $z'(1)$.

Solution: $u'(x) = -2\frac{\pi}{4}\cos(\frac{\pi}{4}x)$ on $(0, 2)$, so $u(1) = -\sqrt{2}$ and $u'(1) = -\frac{\pi\sqrt{2}}{4}$
$$z'(x) = \ln(2x+1)u'(x) + u(x)\left(\frac{2}{2x+1}\right) \quad \text{so} \quad z'(1) = \ln(3)u'(1) + u(1)\left(\frac{2}{2+1}\right)$$

Answer: $z'(1) =$ $\ln(3)\left(\dfrac{-\pi\sqrt{2}}{4}\right) - \sqrt{2}\left(\dfrac{2}{3}\right)$

b. [2 points] At $x = 7$, the tangent line to $u(x)$ is given by $y = -5 - 2(x-7)$. Find an equation for the tangent line to $u^{-1}(x)$ at $x = -5$.

Solution: Since $(u^{-1})'(-5) = \frac{1}{u'(7)} = \frac{1}{-2}$, such an equation is $y = 7 - \frac{1}{2}(x+5)$.

Answer: $y = 7 - \dfrac{1}{2}(x+5)$