Group Quiz - Level 1

Name:

02/20/2020

This quiz has 1 questions worth 10 points on 1 pages. Try to do as many questions as possible. You can use your calculator.

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1. (10 points) Find the derivative of following functions.

   (a) $f(x) = x^{100} + x$.

   Solution: $f'(x) = 100x^{99} + 1$.

   (b) $f(\theta) = \cos(\theta) - \sin(\theta)$.

   Solution: $f'(\theta) = -\sin(\theta) - \cos(\theta)$.

   (c) $g(t) = 100^t + 100^{\mathrm{e}}$.

   Solution: $g'(t) = 100^t \ln(100)$.

   (d) $h(z) = z^2 - 2^z$.

   Solution: $h'(z) = 2z - 2^z \ln(2)$.

   (e) $w(s) = s^{0.01} + s^{0.02} - 10s^{0.03}$.

   Solution: $w'(s) = 0.01s^{-0.99} + 0.02s^{-0.98} - 0.3s^{-0.97}$.

   (f) $k(x) = x\ln(x)$.

   Solution: $k'(x) = \ln(x) + \frac{x}{x} = 1 + \ln(x)$.

   (g) $j(u) = \dfrac{1}{\sqrt{u}+1}$.

   Solution: $j'(u) = \dfrac{-\frac{1}{2}u^{-1/2}}{(\sqrt{u}+1)^2}$.

   (h) $f(t) = \cos(t)\sin(t) + \cos(t)$.

   Solution: $f'(t) = -\sin^2(t) + \cos^2(t) - \sin(t)$.

   (i) $q(v) = \dfrac{\sqrt{v}}{1+v}$.

   Solution: $q'(v) = \dfrac{\frac{1}{2}v^{-1/2}(1+v) - \sqrt{v}}{(1+v)^2}$.

   (j) $p(r) = \dfrac{1}{r + 1/r}$.

   Solution: $p'(r) = \frac{-(1-1/r^2)}{(r+1/r)^2}$.

Group Quiz - Level 2

Name:

02/20/2020

This quiz has 1 questions worth 20 points on 1 pages. Try to do as many questions as possible. You can use your calculator.

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1. (20 points) Find the derivative of following functions.

   (a) $g(t) = \mathrm{e}^{-2t}$.

   Solution: $g'(t) = -2\mathrm{e}^{-2t}$.

   (b) $f(t) = 2^{t/\ln(3)}$.

   Solution: $f'(t) = (\ln(2)/\ln(3))2^{t/\ln(3)}$.

   (c) $g(t) = (\cos(t) - \sin(t))^5$.

   Solution: $g'(t) = 5(\cos(t) - \sin(t))^4(-\sin(t) - \cos(t))$.

   (d) $j(r) = \left(r^6 - 1\right)^3 \left(r^3 + r^2 + 1\right)^7$.

   Solution:

   $$j'(r) = 3\left(r^6 - 1\right)^2\left(6r^5\right)\left(r^3 + r^2 + 1\right)^7 + 7\left(r^6 - 1\right)^3\left(r^3 + r^2 + 1\right)^6\left(3r^2 + 2r\right).$$

   (e) $m(z) = \dfrac{z}{\sqrt{1+z}}$.

   Solution: $m'(z) = \dfrac{\sqrt{1+z} - \frac{z}{2\sqrt{1+z}}}{1+z}$.

   (f) $h(s) = \mathrm{e}^{2\mathrm{e}^{3s}}$.

   Solution: $h'(s) = (\mathrm{e}^{2\mathrm{e}^{3s}})(2\mathrm{e}^{3s})(3)$.

   (g) $k(x) = \ln(\cos(x))$.

   Solution: $k'(x) = \frac{-\sin(x)}{\cos(x)} = -\tan(x)$.

   (h) $p(u) = \sqrt{u}\mathrm{e}^{\sqrt{u}}$.

   Solution: $p'(u) = \frac{1}{2}u^{-1/2}\mathrm{e}^{\sqrt{u}}(1 + \sqrt{u})$.

   (i) $q(t) = \dfrac{\sin(\mathrm{e}^t)}{t^{3/2}}$.

   Solution:

   $$q'(t) = \frac{\cos(\mathrm{e}^t)\mathrm{e}^t t^{3/2} - \frac{3}{2}t^{1/2}\sin(\mathrm{e}^t)}{t^3}.$$

   (j) $f(v) = \dfrac{v}{\sqrt{1 - (v/v_0)^2}}$.

   Solution:

   $$f'(v) = \frac{\sqrt{1 - (v/v_0)^2} - \frac{v}{v_0}(1 - (v/v_0)^2)^{-1/2}}{1 - (v/v_0)^2}$$

Group Quiz - Level 3

Name:

02/20/2020

This quiz has 1 questions worth 12 points on 1 pages. Try to do as many questions as possible. You can use your calculator.

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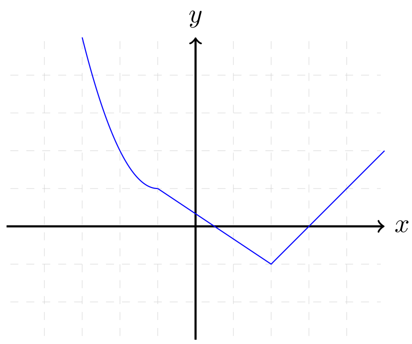
1. Part of the graph of a piecewise defined function $f(x)$ is shown below. Note that $f(x)$ is defined on all real numbers and this graph is large enough to show all features of f.



The graph of $x \leqslant -1$ is given by $f(x) = x^2 + 2x + 2$, and the graphs for $x > -1$ are linear. The graph goes through $(-1, 1), (2, -1)$ and $(5, 2)$. For each of the following, write answers in *exact form* if exist, otherwise write DNE.

(a) (4 points) Let $h(x) = \dfrac{f(2x)}{f(-x)}$. Find $h'(x)$ and $h'(2)$.

Solution: We have
$$h'(x) = \frac{2f'(2x)f(-x) + f'(-x)f(2x)}{(f(-x))^2}$$
Then $h'(2) = \dfrac{2f'(4)f(-2) + f'(-2)f(4)}{(f(-2))^2} = 1/2.$

(b) (4 points) Let $g(x) = f(f(f(x)))$. Find $g'(x)$ and $g'(2)$.

Solution: Since $g'(x) = f'(f(f(x))) \cdot f'(f(x)) \cdot f'(x)$. Then $g'(2)$ DNE because $f'(2)$ DNE.

(c) (4 points) Let $j(x) = f\left(\dfrac{1+f(x)}{1-f(x)}\right)$. Find $j'(x)$ and $j'(0)$.

Solution: Since $j'(x) = f'\left(\dfrac{1+f(x)}{1-f(x)}\right) \cdot \dfrac{2f'(x)}{(1-f(x))^2}$. Since $f(0) = \frac{1}{3}$, we see that $\dfrac{1+f(0)}{1-f(0)} = 2$. But $f'(2)$ DNE. Hence $j'(0)$ DNE.

Group Quiz - Level 4

Name:

02/20/2020

This quiz has 2 questions worth 14 points on 1 pages. Try to do as many questions as possible. You can use your calculator.

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1. (4 points) Let $f(x) = \sin^2(x)$. Find $4f(x) + f''(x)$.

   Solution: Note that $f'(x) = 2\sin(x)\cos(x)$ and $f''(x) = 2\cos^2(x) - 2\sin^2(x)$. So
   $$4f(x) + f''(x) = 4\sin^2(x) + 2\cos^2(x) - 2\sin^2(x) = 2\sin^2(x) + 2\cos^2(x) = 2$$

2. The *hyperbolic trigonometric function* $\cosh(x)$ and $\sinh(x)$ is defined by
   $$\cosh(x) = \frac{\mathrm{e}^x + \mathrm{e}^{-x}}{2} \qquad \sinh(x) = \frac{\mathrm{e}^x - \mathrm{e}^{-x}}{2}$$
   Find following derivatives

   (a) (4 points) $(\cosh(x))''$ and $(\sinh(x))''$.

   Solution: Since $(\cosh(x))' = \sinh(x)$ and $(\sinh(x))' = \cosh(x)$, $(\cosh(x))'' = \cosh(x)$ and $(\sinh(x))'' = \sinh(x)$.

   (b) (4 points) $(\cosh^2(x))'$ and $(\sinh^2(x))'$.

   Solution: $(\cosh^2(x))' = 2\cosh(x)\sinh(x)$ and $(\sinh^2(x)) = 2\sinh(x)\cosh(x)$.

   (c) (2 points) If we write $f(x) = \cosh^2(x) - \sinh^2(x)$, then what is $f'(x)$? What does it tell you about $f(x)$? What is $f(x)$?

   Solution: From (b), we see that $f'(x) = 0$. Hence $f(x)$ must be a constant. Therefore
   $$f(x) = f(0) = \left(\frac{1+1}{2}\right)^2 - \left(\frac{1-1}{2}\right)^2 = 1$$

Group Quiz - Level 5

Name:

02/20/2020

This quiz has 2 questions worth 20 points on 1 pages. Try to do as many questions as possible. You can use your calculator.

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1. (4 points) Let the function $j(x) = x^x$. Find $j'(2)$ in *exact form.*

Solution: Rewrite $j(x) = \mathrm{e}^{x\ln(x)}$, then $j'(x) = \mathrm{e}^{x\ln(x)}(x/x + \ln(x)) = (1+\ln(x))x^x$. Therefore $j'(2) = 4(1+\ln(2))$.

2. (16 points) The function $f(x)$ and $g(x)$, and their derivatives are given by following table.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	1	2	π	$\ln(\pi)$
2	2	$\mathrm{e} = 2.71828...$	1	$\ln(2)$
3	3	8/9	3	0

Let $h(x) = f(x)^{g(x)}$. Find $h'(1), h'(2)$ and $h'(3)$ in *exact form.*

Solution: Rewrite $h(x)$ as $(\mathrm{e}^{\ln(f(x))})^{g(x)} = \mathrm{e}^{g(x)\ln(f(x))}$, then

$$h'(x) = \mathrm{e}^{g(x)\ln(f(x))} \cdot \left(g'(x)\ln(f(x)) + g(x)\frac{f'(x)}{f(x)}\right)$$
$$= f(x)^{g(x)} \cdot \left(g'(x)\ln(f(x)) + g(x)\frac{f'(x)}{f(x)}\right)$$

Hence

$$h'(1) = f(1)^{g(1)} \cdot \left(g'(1)\ln(f(1)) + g(1)\frac{f'(1)}{f(1)}\right) = 2\pi$$
$$h'(2) = f(2)^{g(2)} \cdot \left(g'(2)\ln(f(2)) + g(2)\frac{f'(2)}{f(2)}\right) = 2\ln(2) + \mathrm{e}$$
$$h'(3) = f(3)^{g(3)} \cdot \left(g'(3)\ln(f(3)) + g(3)\frac{f'(3)}{f(3)}\right) = 24$$