

5. (12 points) At woot.com the staff has become quite good at predicting the number of items that will be sold based on the brand name, reliability reports, the price, and the predicted popularity of the item. The maximum number of items, N , that they expect to sell during the entire sale period on a given day is a function of what they call the Max Sales Index, i , so $N = f(i)$, where the units of i are referred to as "points."

(a) In the context of this problem, give a practical interpretation of $f(10)$.

The expression $f(10)$ represents the maximum number of items that woot.com expects to sell during the entire sale period on a given day when the Max Sales Index is 10 points.

(b) In the context of this problem, what is the practical interpretation of $f'(5) = 2500$?

The practical interpretation of $f'(5) = 2500$ is that woot.com expects a product with Max Sales Index of 6 will sell approximately 2500 more items than a product with Max Sales Index of 5.

(c) The number of Wooters (registered members of Woot.com) is currently over 500,000. Since there is not a mechanism for "un-registering," and the membership has grown very quickly, assume that the number of Wooters, W in thousands, is an invertible function of time, t , in hours, $W = g(t)$. In this context, give a practical interpretation of $(g^{-1})'(200) = .05$

$(g^{-1})'(200) = 0.05$ means that when the number of Wooters is 200,000, it takes approximately 0.05 hours (or three minutes) for the next 1000 Wooters to register.

(d) Sometimes woot.com sells bags of junk, "like shopping blindfolded at the Dollar Store." We can't say the exact name here, so we'll call them BoCs. Even these bags sell quickly on woot.com—typically in minutes. A recent BoC sale recorded the following data, where $s(t)$ gives the total number of BoC sales t minutes after the sale began. Use the data to estimate the $s'(10)$. Show your work.

time (minutes)	6	8	10	12	14	16	18
$s(t)$ (number of BoCs)	46	88	136	184	243	313	436

$$s'(10) \approx \frac{s(12) - s(8)}{12 - 8} = \frac{184 - 88}{4} = 24 \text{ BoCs per minute}$$

8. (12 points) The potential energy E , in joules, of an object above the Earth's surface is a function of the distance, h , in meters, of the object from the surface of the Earth. That is, $E = f(h)$.

(a) In the context of this problem, explain the meaning of $f(20) = 1000$?

The potential energy of an object 20 meters above the Earth's surface is 1000 Joules.

(b) In the context of this problem, explain the meaning of $f'(9) = 50$?

When an object is 9 meters above the Earth's surface, increasing the height of the object one meter, the potential energy will increase by approximately 50 Joules.

(c) In the context of this problem, explain the meaning of $f^{-1}(150) = 3$?

An object with 150 Joules of potential energy is 3 meters above the surface of the Earth.

(d) In the context of this problem, explain the meaning of $(f^{-1})'(400) = \frac{1}{50}$?

When an object has 400 Joules of potential energy, its height above the surface of the Earth must be increased by approximately $\frac{1}{50}$ of a meter to increase the potential energy by 1 Joule.

7. (11 points) Suppose $P = m(t)$ is the population of Mexico in millions, where t is the number of years since 1980. Explain the meaning of the statements below. You should only use terms and phrases understandable to someone who has never taken calculus. (Assume that the population function is invertible.)

(a) (3 pts.) $\left. \frac{dP}{dt} \right|_{t=0} > 0$

- In 1980, Mexico's population was increasing.

(b) (4 pts.) $m^{-1}(97.5) = 18$

- In 1998 (18 years after 1980) Mexico's population was 97.5 million people.

(c) (4 pts.) $(m^{-1})'(97.5) = 0.46$

- When 97.5 million people lived in Mexico, it took about half a year for the population to increase by another million people.

2. [10 points] Louis owns a small soda company and is experimenting with new flavors. Let $b(p)$ model the number of thousands of bottles of bacon-flavored soda sold by his company per month if he charges p cents per bottle. You may assume $b(p)$ is differentiable and invertible.

a. [2 points] Give a practical interpretation of the statement $b^{-1}(8) = 150$.

Solution: In order to sell 8000 bottles of bacon-flavored soda per month, the company should charge 150 cents per bottle.

b. [3 points] Give a practical interpretation of the statement $(b^{-1})'(4) = -10$.

Solution: In order to increase the number of bottles sold per month from 4000 to 5000, the company should lower the price about 10 cents.
If the company is currently selling 4000 bottles per month, lowering the price by 10 cents will increase sales by about 1000 bottles per month.
(There are other possible answers.)

c. [3 points] Write an expression that is equal to the price (in cents) that the company would have to charge per bottle in order to sell twice as many bottles of bacon-flavored soda as it sells at a price of 125 cents per bottle.

Solution: $b^{-1}(2b(125))$

d. [2 points] Which of the following is a correct formula for a function $h(d)$ that gives the number of thousands of bottles sold per month at a price of d dollars per bottle? (Circle your answer.)

$$h(d) = 100b(d) \quad h(d) = \frac{b(d)}{100} \quad \boxed{h(d) = b(100d)} \quad h(d) = b\left(\frac{d}{100}\right)$$

3. [5 points] Use the limit definition of the derivative to write an explicit expression for $r'(3)$ where $r(t) = (t + 5)^{2t}$. Do not simplify or evaluate the limit. Your answer should not include the letter r .

Solution:

$$r'(3) = \lim_{h \rightarrow 0} \frac{(3 + h + 5)^{2(3+h)} - (3 + 5)^{2(3)}}{h}$$

5. [11 points] Oren, a Math 115 student, realizes that the more caffeine he consumes, the faster he completes his online homework assignments. Before starting tonight's assignment, he buys a cup of coffee containing a total of 100 milligrams of caffeine.

Let $T(c)$ be the number of minutes it will take Oren to complete tonight's assignment if he consumes c milligrams of caffeine. Suppose that T is continuous and differentiable.

- a. [2 points] Circle the ONE sentence below that is best supported by the statement "the more caffeine he consumes, the faster he completes his online homework assignments."

i. $T'(c) \geq 0$ for every value c in the domain of T .

ii. $T'(c) \leq 0$ for every value c in the domain of T .

iii. $T'(c) = 0$ for every value c in the domain of T .

- b. [1 point] Explain, in the context of this problem, why it is reasonable to assume that $T(c)$ is invertible.

Solution: Since the more caffeine Oren consumes the faster he is able to finish his homework, $T(c)$ is a decreasing function. Thus, $T(c)$ is invertible.

- c. [2 points] Interpret the equation $T^{-1}(100) = 45$ in the context of this problem. Use a complete sentence and include units.

Solution: In order for Oren to complete his homework assignment in 100 minutes, he must consume 45 milligrams of caffeine.

- d. [3 points] Suppose that p and k are constants. In the equation $T'(p) = k$, what are the units on p and k ?

Answer: Units on p are milligrams of caffeine

Answer: Units on k are minutes per milligram of caffeine

- e. [3 points] Which of the statements below is best supported by the equation $(T^{-1})'(20) = -10$? Circle the ONE best answer.

i. If Oren has consumed 20 milligrams of caffeine, then consuming an additional milligram of caffeine will save him about 10 minutes on tonight's assignment.

ii. The amount of caffeine that will result in Oren finishing his homework in 21 minutes is approximately 10 milligrams greater than the amount of caffeine that Oren will need in order to finish his homework in 20 minutes.

iii. The rate at which Oren is consuming caffeine 20 minutes into his homework assignment is decreasing by 10 milligrams per minute.

iv. In order to complete tonight's assignment in 19 rather than 20 minutes, Oren needs to consume about 10 milligrams of additional caffeine.

v. If Oren consumes 20 milligrams of caffeine, then he will finish tonight's assignment approximately 10 minutes faster than if he consumes no caffeine.