4. [9 points] Let $P(v) = \begin{cases} v^2 \sin\left(\frac{1}{v}\right) - v\sin(2) & \text{if } v \neq 0 \\ 0 & \text{if } v = 0. \end{cases}$

a. [5 points]
Use the limit definition of the derivative to write down an explicit expression for $P'(0)$.
Your answer should not include the letter P.
Do not attempt to evaluate or simplify the limit.

$$P'(0) = \lim_{h\to 0} \frac{\left((0+h)^2 \sin\left(\frac{1}{0+h}\right) - (0+h)\sin(2)\right) - 0}{h}$$

b. [4 points] Use your answer to (a) to estimate $P'(0)$ to the nearest hundredth.
Be sure to include enough clear graphical or numerical evidence to justify your answer.

Solution: We plug in small values of h approaching 0. Since the difference quotient is an even function of h, we need only check positive values of h (as evenness implies that negative h give precisely the same results).

$h = 0.1$:
$$\frac{0.1^2 \sin(1/0.1) - 0.1\sin(2) - 0}{0.1} \approx -0.964$$

$h = 0.01$:
$$\frac{0.01^2 \sin(1/0.01) - 0.01\sin(2) - 0}{0.01} \approx -0.914$$

$h = 0.001$:
$$\frac{0.001^2 \sin(1/0.001) - 0.001\sin(2) - 0}{0.001} \approx -0.908$$

$h = 0.0001$:
$$\frac{0.0001^2 \sin(1/0.0001) - 0.0001\sin(2) - 0}{0.0001} \approx -0.909$$

We see at this point that the numbers seem to have stabilized to the nearest hundredth at -0.91.

Answer: $P'(0) \approx$ -0.91

2. [5 points] Let

$$K(p) = (1 + \cos(p))^{1+2p}.$$

Use the limit definition of the derivative to write an explicit expression for $K'(4)$. *Your answer should not involve the letter* K*. Do not attempt to evaluate or simplify the limit. Please write your final answer in the answer box provided below.*

Answer: $K'(4) =$ $\lim_{h \to 0} \dfrac{(1+\cos(4+h))^{1+2(4+h)} - (1+\cos(4))^{1+2(4)}}{h}$

3. [5 points] Let

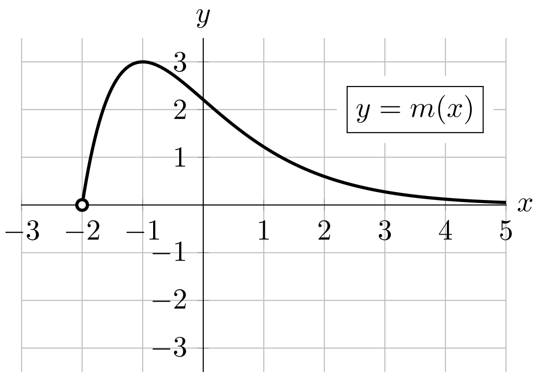
$$B(k) = e^{-4k^2}\tan(k+3).$$

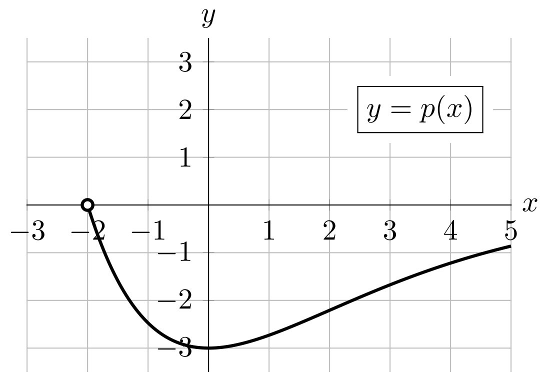
Use the limit definition of the derivative to write an explicit expression for $B'(5)$. *Your answer should not involve the letter B. Do not attempt to evaluate or simplify the limit.* Please write your final answer in the answer box provided below.

Solution:

$$B'(5) = \lim_{h\to 0} \frac{e^{-4(5+h)^2}\tan(h+8) - e^{-100}\tan(8)}{h}.$$

6. [4 points] Shown below at left is a portion of the graph of a function $m(x)$. Shown below at right is a portion of the graph of a function $p(x)$, which can be obtained from $m(x)$ through one or more graph transformations. Find a formula for $p(x)$ in terms of $m(x)$.





Answer: $p(x) = -m\left(\frac{1}{2}x - 1\right) = -m\left(\frac{1}{2}(x-2)\right)$

7. [9 points] For a constant c, let

$$K(x) = \frac{2^{cx}}{e^{x-c}}.$$

a. [5 points] Use the limit definition of the derivative to write an explicit expression for $K'(3)$. *Your answer may include the constant c but should not involve the letter K. Do not attempt to evaluate or simplify the limit.* Write your final answer in the answer box provided below.

Answer: $K'(3) = \lim_{h \to 0} \frac{\frac{2^{c(3+h)}}{e^{(3+h)-c}} - \frac{2^{c(3)}}{e^{3-c}}}{h}$

b. [4 points] Find the value of c so that $K(1) = 5$. Give your answer in **exact form** and show all your work.

Solution: We want c such that

$$\frac{2^{c(1)}}{e^{1-c}} = 5, \text{ or}$$

$$2^c = 5e^{1-c}.$$

Solving, we find that

$$\ln(2^c) = \ln\left(5e^{1-c}\right)$$
$$\ln(2^c) = \ln(5) + \ln\left(e^{1-c}\right)$$
$$c\ln(2) = \ln(5) + 1 - c$$
$$c\ln(2) + c = \ln(5) + 1$$
$$c(\ln(2) + 1) = \ln(5) + 1$$
$$c = \frac{\ln(5) + 1}{\ln(2) + 1}.$$

Answer: $c = \frac{\ln(5) + 1}{\ln(2) + 1}$