# How do we measure speed?

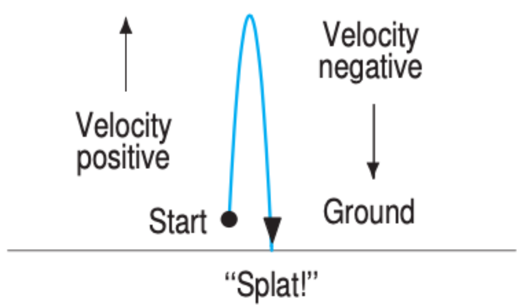
Zhan Jiang

January 28, 2020

## 1 Velocity

### 1.1 Speed and velocity

Suppose that a small object (say, a grapefruit) is thrown straight upward into the air at $t = 0$ seconds. The grapefruit leaves the thrower's hand at high speed, slows down until it reaches its maximum height, and then speeds up in the downward direction and finally, "Splat!".




We want to distinguish between velocity and speed. Suppose an object moves along a line. We pick one direction to be positive and say that the *velocity* is positive if it is in that direction and negative if it is in the opposite direction. *Speed* is the magnitude of the velocity and so is always positive or zero.

### 1.2 Average velocity

If $s(t)$ is the position of an object at time $t$, then the *average velocity* of the object over the interval $a \leqslant t \leqslant b$ is

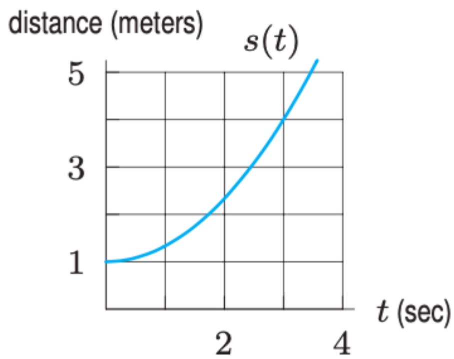
$$\text{Average velocity} = \frac{\text{Change in position}}{\text{Change in time}} = \frac{s(b) - s(a)}{b - a}$$

So the average velocity of an object over an interval is the net change in position during the interval divided by the change in time.

1. The table below gives the position of a particle moving along the $x$-axis as a function of time in seconds, where $x$ is in angstroms. What is the average velocity of the particle from $t = 2$ to $t = 8$?

| $t$ | 0 | 2 | 4 | 6 | 8 |
|---|---|---|---|---|---|
| $x(t)$ | 0 | 14 | -6 | -18 | -4 |

2. Figure below shows a particle's distance from a point. What is the particle's average velocity from $t = 0$ to $t = 3$?




3. At time $t$ in seconds, a particle's distance $s(t)$, in micrometers ($\mu$m), from a point is given by $s(t) = e^t - 1$. What is the average velocity of the particle from $t = 2$ to $t = 4$?

### 1.3 Instantaneous velocity

Let $s(t)$ be the position at tiem $t$. Then the *instantaneous velocity* at $t = a$ is defined as

$$\text{Instantaneous velocity at } (t = a) = \lim_{h \to 0} \frac{s(a+h) - s(a)}{h}$$

So the instantaneous velocity of an object at time $t = a$ is given by the limit of the average velocity over an interval, as the interval shrinks around $a$.

1. In a time of $t$ seconds, a particle moves a distance of $s$ meters from its starting point, where $s = 4t^3$.
   (a) Find the average velocity between $t = 0$ and $t = h$ if:

      (i) $h = 0.1$ (ii) $h = 0.01$ (iii) $h = 0.001$

   (b) Use your answers to part (a) to estimate the instantaneous velocity of the particle at time $t = 0$.

2. In a time of $t$ seconds, a particle moves a distance of $s$ meters from its starting point, where $s = \sin(2t)$.
   (a) Find the average velocity between $t = 1$ and $t = 1 + h$ if:

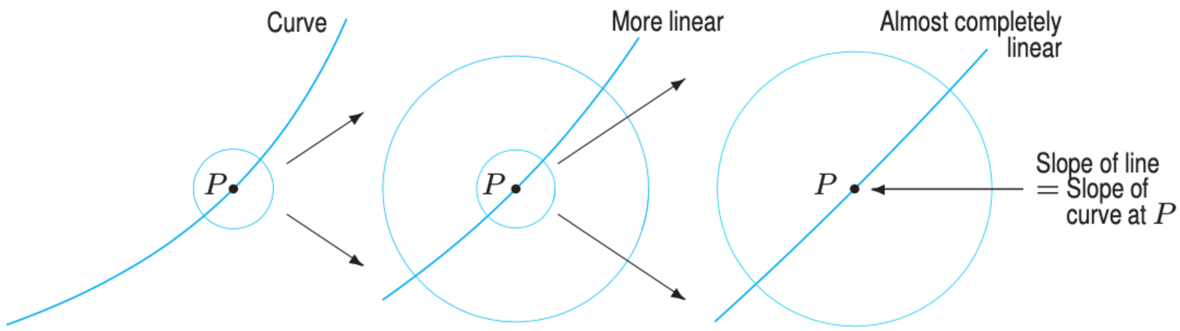
      (i) $h = 0.1$ (ii) $h = 0.01$ (iii) $h = 0.001$

   (b) Use your answers to part (a) to estimate the instantaneous velocity of the particle at time $t = 1$.

# 2 Graph

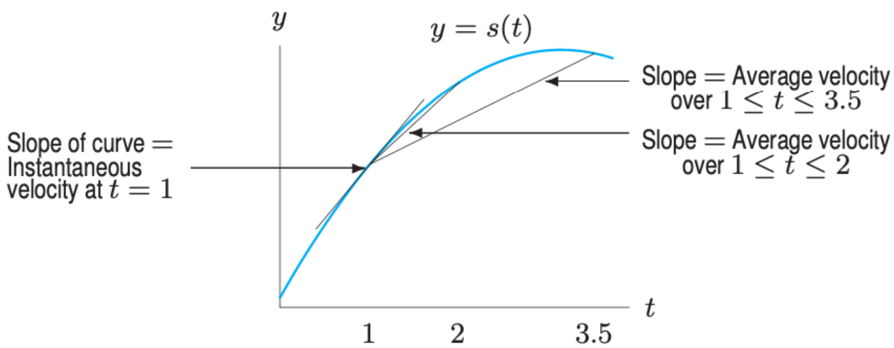
### 2.1 Slope of curve

If we zoom in close enough, then the curve will appear to be a line. We call the slope of this line the *slope of the curve* at the point.

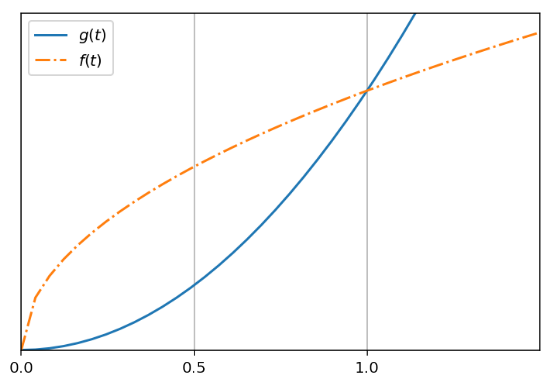



## 2.2 Velocity and slope

If we graph the position function $s(t)$, the instantaneous velocity is the slope of the curve at a point. The average velocity over any time interval $a \leqslant t \leqslant b$ is the slope of the line joining the points on the graph of $s(t)$ corresponding to $t = a$ and $t = b$.




1. Graph below shows $f(t)$ and $g(t)$, the positions $y$ of two cars in feet with respect to time $t$ in minutes.




   (a) Describe how the velocity of each car is changing during the time shown.

(b) Find an interval over which the cars have the same average velocity.

(c) Which of the following statements are true?

(i) Sometime in the first half minute, the two cars are traveling at the same instantaneous velocity.

(ii) During the second half minute (from $t = 1/2$ to $t = 1$), there is a time that the cars are traveling at the same instantaneous velocity.

(iii) The cars are traveling at the same velocity at $t = 1$ minute.

(iv) There is no time during the period shown that the cars are traveling at the same velocity.

## 3 Use limits to compute the instantaneous velocity

1. Estimate following limits using your calculator. Give answers to one decimal place.

(a) $\lim_{h\to 0} \frac{(3+h)^3 - 27}{h}$

(b) $\lim_{h\to 0} \frac{\cos(h) - 1}{h}$

(c) $\lim_{h\to 0} \frac{7^h - 1}{h}$

(d) $\lim_{h\to 0} \frac{e^{1+h} - e}{h}$

2. Use algebra to evaluate the limit.

(a) $\lim_{h\to 0} \frac{3(2+h)^2 - 12}{h}$

(b) $\lim_{h\to 0} \frac{(3+h)^2 - (3-h)^2}{2h}$