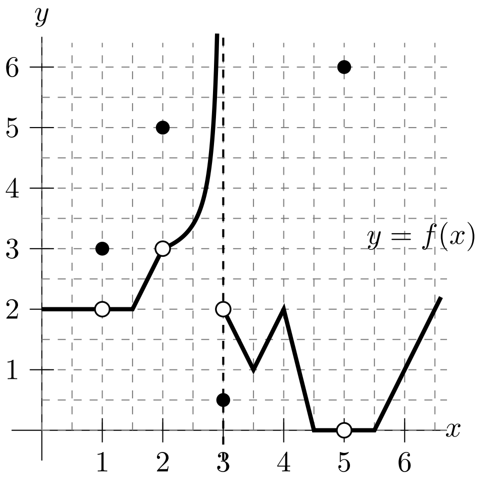
1. [10 points] A portion of the graph of a function f is shown below.



Note: You may assume that pieces of the function that appear linear are indeed linear.

Use the graph above to evaluate each of the expressions below, and write your answer on the answer blank provided. If any of the quantities do not exist (including the case of limits that diverge to ∞ or $-\infty$), write DNE.

a. [1 point] $f(1)$

Answer: 3

b. [1 point] $\lim_{x\to 5} f(x)$

Answer: 0

c. [1 point] $\lim_{q\to 3} f(q)$

Answer: **DNE**

d. [1 point] $\lim_{z\to 2} f(2)$

Answer: 5

e. [1 point] $\lim_{r\to 6^-} f(r)$

Answer: 1

f. [1 point] $\lim_{h\to 0} \frac{f(4.25+h)-f(4.25)}{h}$

Answer: -4

g. [1 point] $\lim_{p\to 0.5} \frac{f(p)}{p}$

Answer: 4

h. [1 point] $\lim_{t\to 3} f(t)f(t+2)$

Answer: 0

i. [1 point] $\lim_{x\to 3^+} f(f(x))$

Answer: 3

j. [1 point] $\lim_{s\to 1} f(f(s))$

Answer: 5

3. [9 points] Consider the function h defined by

$$h(x) = \begin{cases} \dfrac{60(x^2-x)}{(x^2+1)(3-x)} & \text{for } x < 2 \\ c & \text{for } x = 2 \\ 5e^{ax} - 1 & \text{for } x > 2 \end{cases}$$

where a and c are constants.

a. [5 points] Find values of a and c so that both of the following conditions hold.

- $\lim\limits_{x \to 2} h(x)$ exists.
- $h(x)$ is not continuous at $x = 2$.

Note that this problem may have more than one correct answer. You only need to find one value of a and one value of c so that both conditions above hold. Remember to show your work clearly.

Solution: In order for $\lim\limits_{x\to 2} h(x)$ to exist, it must be true that $\lim\limits_{x\to 2^-} h(x) = \lim\limits_{x \to 2^+} h(x)$. Now $\lim\limits_{x\to 2^-} h(x) = \dfrac{60(2^2-2)}{(2^2+1)(3-2)} = 24$ and $\lim\limits_{x\to 2^+} h(x) = 5e^{2a} - 1$. So it follows that $5e^{2a} - 1 = 24$. Solving for a, we have

$$\begin{aligned} 5e^{2a} - 1 &= 24 \\ e^{2a} &= 5 \\ 2a &= \ln(5) \\ a &= \ln(5)/2 \approx 0.804. \end{aligned}$$

When $a = \ln(5)/2$, $\lim_{x\to 2} h(x) = 5e^{(\ln(5)/2)*2} = 5e^{\ln(5)} - 1 = 24$. So, h is not continuous at $x = 2$ as long as $\lim_{x\to 2} h(x) \neq h(2)$. Since $h(2) = c$, we can choose c to be any number other than 24.

Answer: $a =$ $\ln(5)/2$ and $c =$ **7 (or any value other than 24)**

b. [2 points] Determine $\lim\limits_{x\to -\infty} h(x)$. If the limit does not exist, write DNE.

Solution: By looking at the rational function

$$\frac{60(x^2-x)}{(x^2+1)(3-x)} = \frac{60(x^2-x)}{-x^3+3x^2-x+3},$$

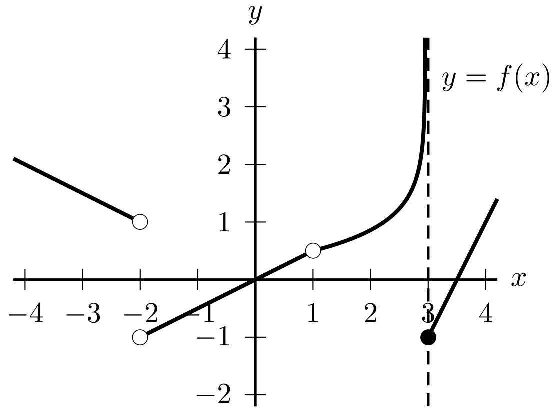
(the relevant piece of the function here) we see that as $x \to -\infty$, $h(x)$ approaches 0.

Answer: $\lim\limits_{x\to -\infty} h(x) =$ 0

c. [2 points] Find all vertical asymptotes of the graph of $h(x)$. If there are none, write NONE.

Answer: Vertical asymptote(s): NONE

6. [11 points] Below is the graph of a portion of a function $f(x)$.



a. [2 points] Give all values of a in the interval $-4 < a < 4$ that are not in the domain of $f(x)$. If there are none, write NONE.

Answer: $-2,\ 1$

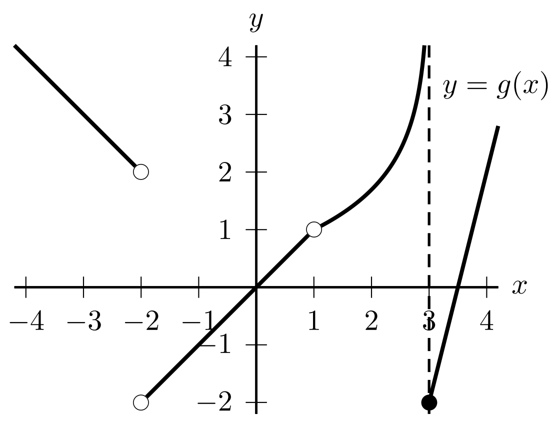
b. [2 points] Give all values of a in the interval $-4 < a < 4$ where $f(x)$ is not continuous at $x = a$. If there are none, write NONE.

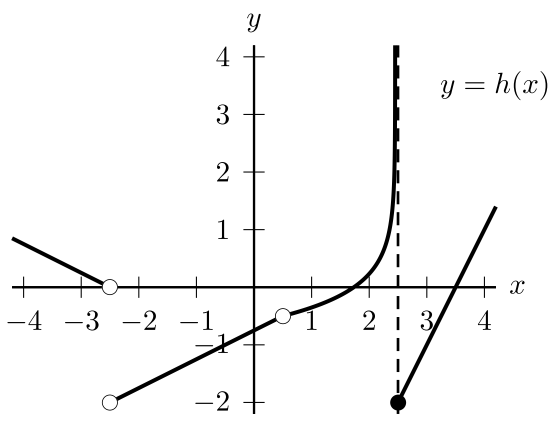
Answer: $-2,\ 1,\ 3$

c. [2 points] Give all values of a in the interval $-4 < a < 4$ where $\lim_{x\to a} f(x)$ does not exist. If there are none, write NONE.

Answer: $-2,\ 3$

d. [5 points] The graphs below show portions of two other functions $g(x)$ and $h(x)$ which are transformations of $f(x)$. Express $g(x)$ and $h(x)$ as transformations of $f(x)$.

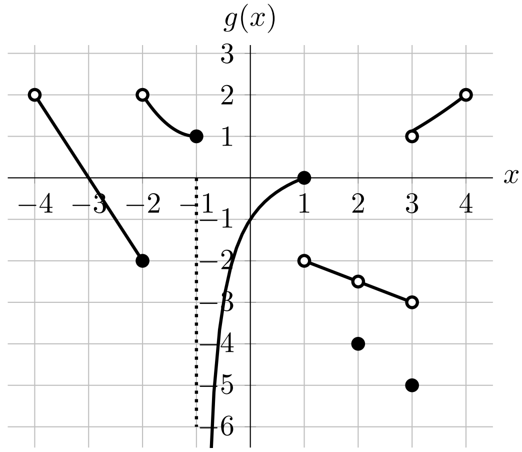




Answer: $g(x) =$ $2f(x)$ and $h(x) =$ $f(x + 0.5) - 1$

8. [15 points] Consider the functions $f(x)$ and $g(x)$ given by the formula and graph below.

$$f(x) = \begin{cases} 2x^3 - 2x^2 & \text{for } x \leq 1, \\ x^3 + 1 & \text{for } x > 1. \end{cases}$$



a. [5 points] Circle the correct answer(s) in each of the following questions.

Solution:

i) At which of the following values of x is the function $g(x)$ <u>not</u> continuous?

$x = -3$ $\boxed{x = -1}$ $x = 0$ $\boxed{x = 2}$ $x = 3.5$

ii) At which of the following values of x is the function $f(x) + g(x)$ continuous?

$x = -2$ $x = -1$ $\boxed{x = 0}$ $\boxed{x = 1}$ $x = 2$

Note that $g(x)$ is linear on the interval $(-4, -2)$,$(1, 2)$ and $(2, 3)$. All your answers below should be *exact*. If any of the quantities do not exist, write DNE.

b. [2 points] Find $\lim_{x \to 2} (2f(x) + g(x))$.

Solution: $2(2^3 + 1) - 2.5 = 15.5$ **Answer:** 15.5

c. [2 points] Find $\lim_{x \to \infty} \frac{f(2x)}{x^3}$.

Solution: $\lim_{x \to \infty} \frac{f(2x)}{x^3} = \lim_{x \to \infty} \frac{8x^3 + 1}{x^3} = 8$ **Answer:** 8

d. [2 points] Find $\lim_{x \to \infty} g\left(x^2 e^{-x} + 3\right)$.

Solution: $\lim_{x \to \infty} g\left(x^2 e^{-x} + 3\right) = \lim_{u \to 3^+} g(u) = 1$ **Answer:** 1

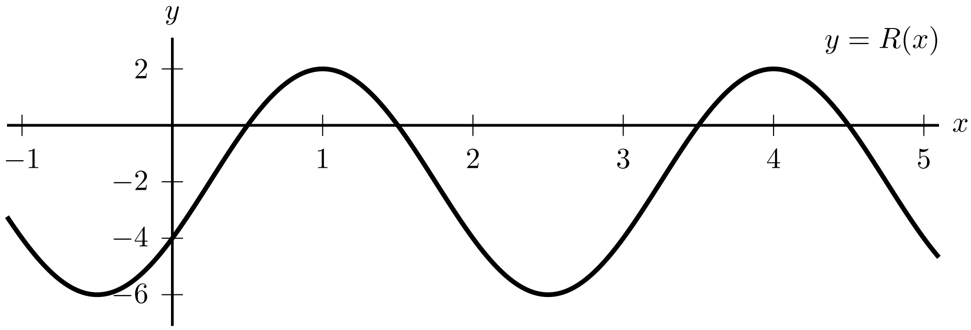
e. [2 points] For which value(s) of p does $\lim_{x \to p^+} g(x) = 1$?

Solution: **Answer:** $p = -3.5, 3$

f. [2 points] Find $\lim_{x \to -1^-} f(-x)$.

Solution: **Answer:** 2

8. [6 points] Given below is the graph of a sinusoidal function $R(x)$.



Find a possible formula for $R(x)$.

Solution: The graph shown above is of a sinusoidal function with amplitude 4, period 3, and midline $y = -2$. We first consider the graph of

$$y = 4\cos\left(\frac{2\pi}{3}x\right) - 2\,.$$

This graph has the proper amplitude, period, and midline. We shift this graph over to the right 1 unit to obtain the graph of $y = R(x)$. Thus, one possible formula for $R(x)$ is given by

$$R(x) = 4\cos\left(\frac{2\pi}{3}(x-1)\right) - 2.$$

Answer: $R(x) = $ $4\cos\left(\frac{2\pi}{3}(x-1)\right) - 2$

9. [4 points] The table below gives several values of a function $w(x)$.

x	4.5	4.9	4.99	5	5.01	5.1	5.5
$w(x)$	-0.879	-0.154	-0.015	0	0.060	0.630	3.750

Use the information in the table above to estimate the following limit.

$$\lim_{h\to 0^-} \frac{w(5+h)}{h}$$

Clearly show any computations that you use to make this estimate.

Solution: The left limit can be approximated by $w(5+h)/h$ for small negative values of h. The table of values provided for w allows us to compute this when $h = -0.1$ and when $h = -0.01$. The results are shown in the table below.

h	$\frac{w(5+h)}{h}$
-0.1	$\frac{w(4.9)}{-0.1} = 1.54$
-0.01	$\frac{w(4.99)}{-.01} = 1.5$

Using these values, we estimate that the desired left-hand limit is approximately 1.5.

Answer: $\lim_{h\to 0^-} \frac{w(5+h)}{h} \approx$ 1.5