

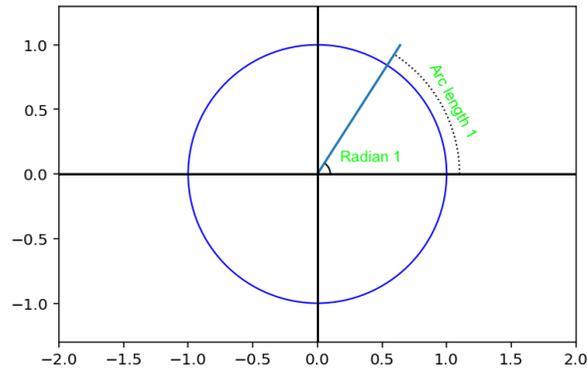
Trigonometric Functions

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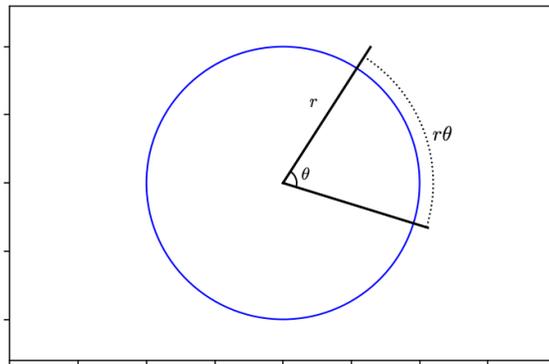
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1 Radians

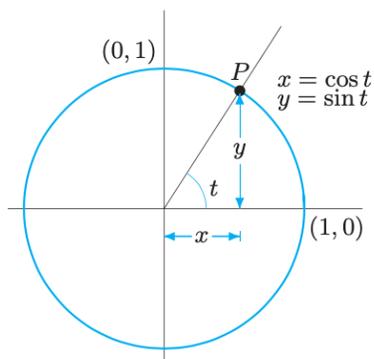
An angle of 1 *radian* is defined to be the angle at the center of a unit circle which cuts off an arc of length 1, measured counterclockwise.



The angle corresponding to an arc is θ in a circle of radius r , then the arc length is $r\theta$.



2 The sine and cosine functions

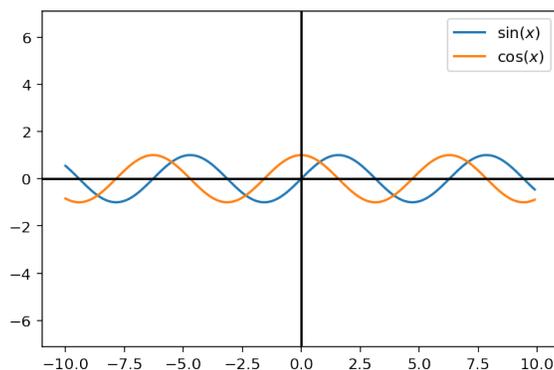


For a given angle t in radian, it determines a point (x, y) on the unit circle. Then the cosine value of t is defined to be the x -coordinates, and the sine value is defined to be the y -coordinates, i.e. $\cos(t) = x, \sin(t) = y$.

Pythagorean theorem $x^2 + y^2 = 1 \Rightarrow \cos^2(t) + \sin^2(t) = 1$.

2.1 Graph

Below are the graphs of $\sin(x)$ and $\cos(x)$.



There are several things to pay attention to.

- Both functions have domain _____ and range _____.
- sin is a _____ function while cos is a _____ function.

2.2 Amplitude, period, and phase

Definition 2.1. A function $f(x)$ is *periodic* if there is some T such that $f(x + T) = f(x)$ for any x .

Both sin and cos are periodic. Because $\sin(x + 2\pi) = \sin(x)$ and $\cos(x + 2\pi) = \cos(x)$.

For any periodic function, the

- *Amplitude* is half the distance between the maximum and minimum values (if it exists)
- *Period* is the smallest increasement in x needed for the function to execute one complete cycle.

Both sin and cos have amplitude _____ and period _____.

From the graph of sin and cos, we see that they are exactly the same, only shifted horizontally. The cosine graph is the sine graph shifted _____ to the left. We say that the *phase difference* or *phase shift* between $\sin(x)$ and $\cos(x)$ is _____.

What is the approximate period of the moon's revolution around the earth?

2.3 Sinusoidal functions

Functions whose graphs are the same shape of a sine (or cosine) curve are called *sinusoidal* functions. They are usually modeled by functions

$$f(t) = A \sin(Bt) \quad \text{or} \quad g(t) = A \cos(Bt).$$

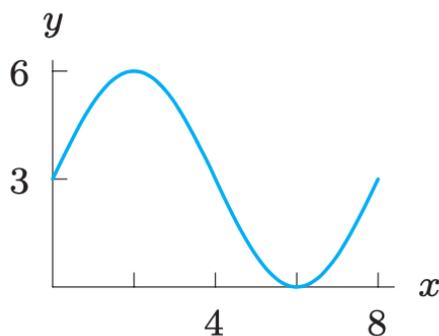
Say we have a sinusoidal function $f(t) = A \sin(Bt)$. What are the amplitude and the period of this function?

If we shift $f(t)$ to the left horizontally by h , then the new function is _____.

If we shift $f(t)$ up vertically by C , then the new function is _____.

2.4 Questions

1. Find a possible formula for the graph below



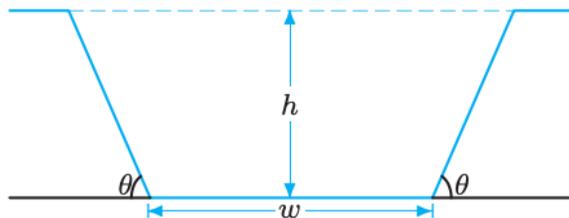
2. The voltage V of an electrical outlet in a home as a function of time t (in seconds) is $V = V_0 \cos(120\pi t)$.

- (a) What is the period of the oscillation?
- (b) What does V_0 represent?

3. The depth of water in a tank oscillates sinusoidally once every 6 hours. If the smallest depth is 5.5 feet and the largest depth is 8.5 feet, find a possible formula for the depth in terms of time in hours.

- 4. (a) Use a graphing calculator or computer to estimate the period of $2\sin(\theta) + 3\cos(2\theta)$.
- (b) Explain your answer, given that the period of $\sin(\theta)$ is 2π and the period of $\cos(2\theta)$ is π .

5. Find the area of the trapezoidal cross-section of the irrigation canal shown below.

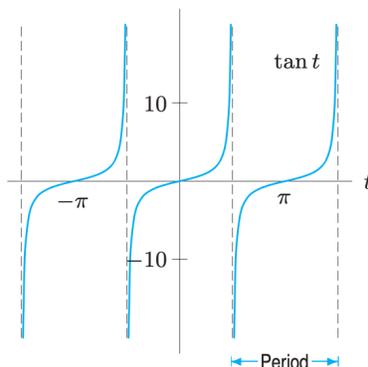


3 The tangent functions

If t is any number with $\cos(t) \neq 0$, we define the tangent function as follows

$$\tan(t) = \frac{\sin(t)}{\cos(t)}$$

The graph is below



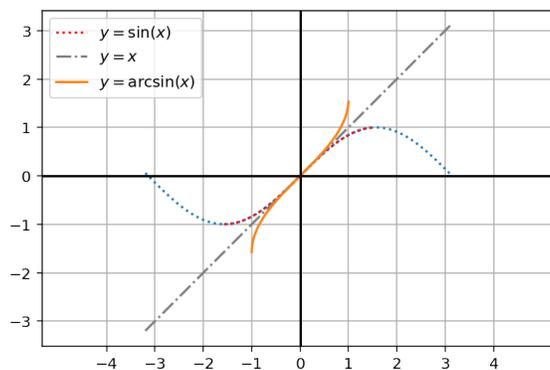
So the period is _____ and the amplitude is _____.

4 The inverse trigonometric functions

For $-1 \leq y \leq 1$,

$$\arcsin(y) = x$$

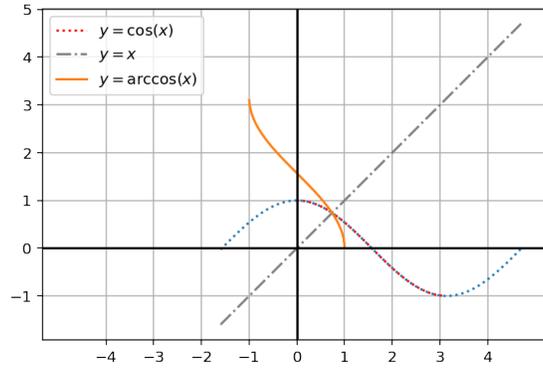
means $\sin(x) = y$ with $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.



Similarly,

$$\arccos(y) = x$$

means $\cos(x) = y$ with $0 \leq x \leq \pi$.



For any y ,

$$\arctan(y) = x$$

means $\tan(x) = y$ with $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

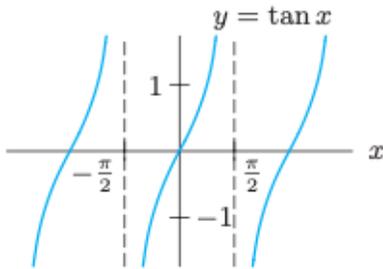


Figure 1.59: The tangent function

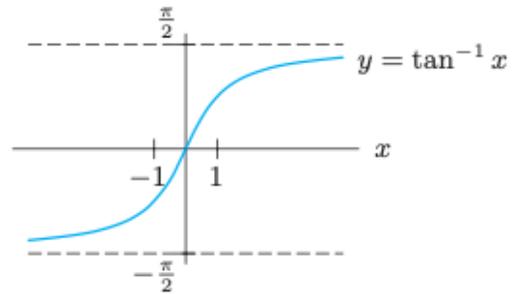


Figure 1.60: The arctangent function

People also write \sin^{-1} / \cos^{-1} / \tan^{-1} for arcsin / arccos / arctan.

4.1 Questions

1. Find a solution to the equation below

- (a) $2 = 5 \sin(3x)$.
- (b) $1 = 8 \cos(2x + 1) - 3$.
- (c) $8 = 4 \tan(5x)$.
- (d) $1 = 8 \tan(2x + 1) - 3$.