# Logarithmic Functions

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# 1 Logarithmic Functions

## 1.1 General Logarithmic Functions

The notion $\log_a b$ represents the *power* of $a$ we need to get $b$, i.e.

$$\log_a b = c \quad \text{means} \quad a^c = b.$$

Here $a$ is called the *base*.

**Definition 1.1.** The base $a$ logarithmic function $f(x) = \log_a x$ is defined to be the inverse function of the exponential function $a^x$.

**In this course (Math 115), we only consider two kinds of logarithmic functions,**

- **base** $10$ **log function, called log function, written as** $\log$,
- **base** $\mathrm{e}$ **log function, called natural log function, written as** $\ln$.

First three rules about log functions:

1. $\log_a 1 = 0$.
2. $a^{\log_a x} = x$.
3. $\log_a(a^x) = x$.

**QUESTIONS**

1. Simplify following expressions
   (a) $\mathrm{e}^{\ln(1/2)}$
   (b) $\log(0.1)$
   (c) $\ln(\mathrm{e}^{2AB})$
   (d) $10^{\log(AB)}$

2. Solve following equations using logs.

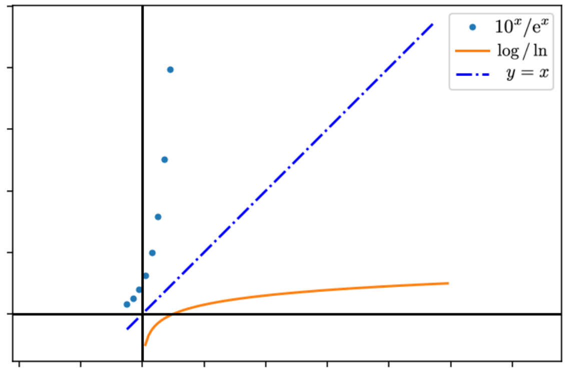
   (a) $10^x = 11$

   (b) $7 = 5e^{0.2x}$

## 1.2 Graph

Recall that the graphs of a function and its inverse function are symmetric about the line ________________

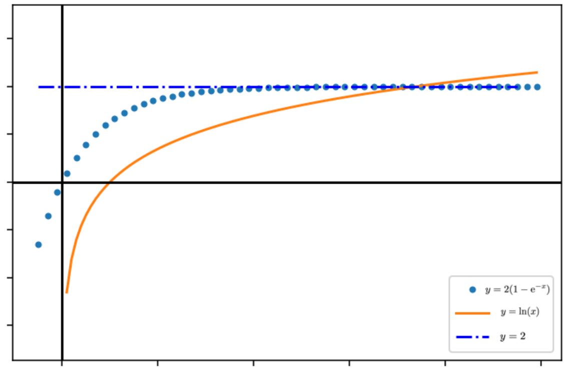
The graph for both ln and log are similar, just with different scales. See below




For both functions, the domain is ________________ and the range is ________________.

When $x$ approaches 0, the function value of both function are approaching $-\infty$. Therefore the vertical line $y$-axis is the *vertical asymptote* of the graph.

Don't confuse the graph of a log function with functions of the form $y = P_0(1 - e^{-x})$. See below



The key differences are

- $2(1-e^{-x})$ is defined for all real numbers, while $\ln(x)$ is only defined for all positive numbers.
- $2(1-e^{-x})$ has a horizontal asymptote $y = 2$ and no vertical asymptotes. $\ln(x)$ has a vertical asymptote $y$-axis and no horizontal asymptotes.

## 1.3 Arithmetic of Log Functions

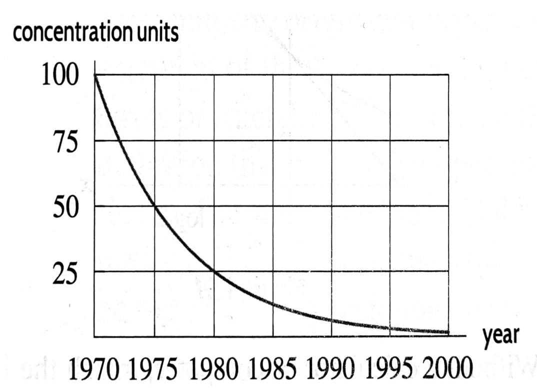
Three more rules

| Log function | Exponential Function |
|---|---|
| $\log(AB) = \log(A) + \log(B)$ | $10^a \cdot 10^b = 10^{a+b}$ |
| $\log(\frac{A}{B}) = \log(A) - \log(B)$ | $\frac{10^a}{10^b} = 10^{a-b}$ |
| $\log(A^n) = n\log(A)$ | $(10^a)^n = 10^{na}$ |

## 1.4 Questions

1. Solve following equations
   (a) $3^x = 11$
   (b) $2^x = e^{x+1}$
   (c) $7^{x+2} = e^{17x}$
   (d) $4e^{2x-3} - 5 = e$

2. Persistent organic pollutants (POPS) are a serious environment hazard. Figure below shows their natural decay over time in human fat.



(a) It takes ___________ years for the concentration to decrease from 100 units to 50 units, and ___________ years for the concentration to decrease from 50 units to 25 units. Explain why your answers suggest that the decay may be exponential.

(b) Find an exponential function that models concentration $C$ as a function $t$, the number of years since 1970.

3. At time $t$ hours after taking the cough suppressant hydrocodone bitartrate, the amount $A$ in mg, remaining in the body is given by $A = 10(0.82)^t$.

   (a) How much of the drug is left in the body 6 hours after the dose is administered?

   (b) How long is it until only 1 mg of the drug remains in the body?

4. The size of an exponentially growing bacteria colony doubles in 5 hours. How long will it take for the number of bacteria to triple?

5. Cyanide is used in solution to isolate gold in a mine. This may result in contaminated groundwater near the mine, requiring the poison be removed, as in the following table, where $t$ is in years since 2012.

| $t$ (years) | 0 | 1 | 2 |
|---|---|---|---|
| $c(t)$ (ppm) | 25.0 | 21.8 | 19.01 |

   (a) Find an exponential model for $c(t)$, the concentration, in parts per million, of cyanide in teh groundwater.

   (b) Use the model in part (a) to find the number of years it takes for the cyanide concentration to fall to 10 ppm.

   (c) The filtering process removing the cyanide is sped up so that the new model is $D(t) = c(2t)$. Find $D(t)$.

(d) If the cyanide removal was started three years earlier, but run at the speed of part (a), find a new model $E(t)$.