Transformations

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1 Shifts and Stretches

Shifts:

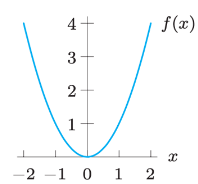
Vertically ($k > 0$)	$f(x) + k$ up by k	$f(x) - k$ down by k
Horizontally ($h > 0$)	$f(x+h)$ left by h	$f(x-h)$ right by h

Stretches or shrinks:

Vertically ($cf(x)$)	$c > 0$: stretches if $c > 1$ or shrinks if $c < 1$	$c < 0$: it also reflects the graph along y-axis
Horizontally ($f(cx)$)	$c > 0$: stretches if $c < 1$ or shrinks if $c > 1$	$c < 0$: it also reflects the graph along x-axis

QUESTIONS

1. The graph of $f(x)$ is shown below. Graph following functions.



   (a) $f(x+2)$
   (b) $f(x) - 4$
   (c) $f(x+1) + 3$
   (d) $3f(x)$
   (e) $-f(x) + 1$

2. For $g(x) = x^2 + 2x + 3$, find and simplfy:
   (a) $g(2 + h)$
   (b) $g(2)$
   (c) $g(2 + h) - g(2)$

2 Symmetry: Odd and Even Functions

For any function f,

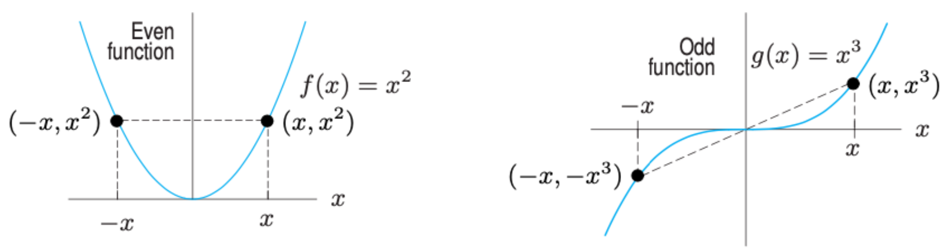
- f is an *even* function if $f(-x) = f(x)$ for all x.
- f is an *odd* function if $f(-x) = -f(x)$ for all x.

Many functions do not have any symmetry and are neither even nor odd.

2.1 Graphs

The graph of an even function is symmetric about the y-axis.

The graph of an odd function is symmetric about the origin.



Why?

QUESTIONS:

1. Are following functions even, odd, or neither?
   (a) $x^6 + x^3 + 1$
   (b) $x^3 + x^2 + x$
   (c) $x^4 - x^2 + 3$

(d) $x^3 + 1$

(e) $x(x^2 - 1)$

(f) e^{x^2-1}

3 Composite Functions

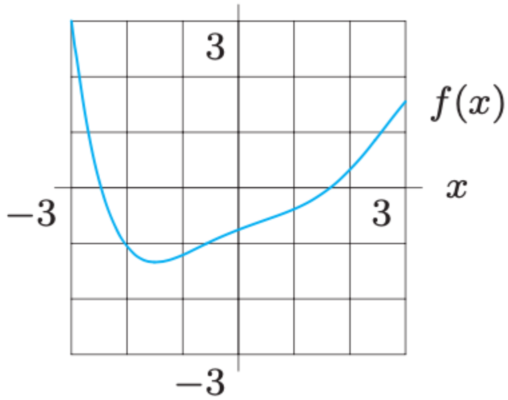
Given two functions $y = f(x)$ and $z = g(y)$, the new function

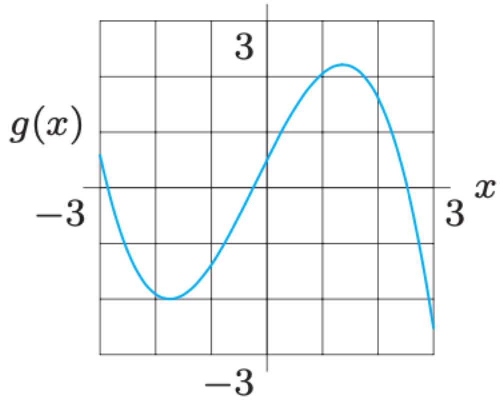
$$x \overset{f}{\mapsto} y \overset{g}{\mapsto} z$$

is called the *composite function* of f and g, denoted by $g(f(x))$ (or $g \circ f$).

1. For $f(x) = \sqrt{x+4}$ and $g(x) = x^2$, calculate following
   (a) $f(g(1))$
   (b) $g(f(1))$
   (c) $f(g(x))$
   (d) $g(f(x))$
   (e) $f(t)g(t)$

2. Estimate $f(g(1)), g(f(2))$ and $f(f(1))$.





3.1 Inverse Functions

A function has an inverse if and only if its graph intersects any horizontal line at most once. We call a function which has an inverse *invertible*. If $y = f(x)$ is invertible, its inverse is denoted by

$$x = f^{-1}(y)$$

Caution: don't confuse inverse function $f^{-1}(y)$ with the reciprocal of the outputs, which is denoted by $(f(y))^{-1} = \frac{1}{f(y)}$.

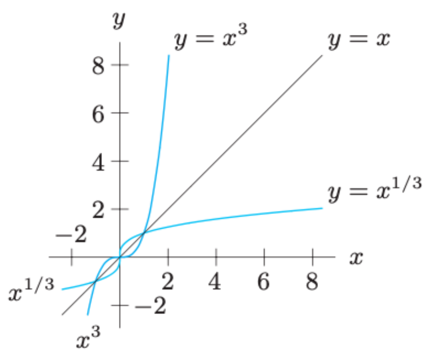
Why it is called an inverse? Because if f is invertible and we write $g(y) = f^{-1}(y)$, then $g(f(x)) = x$ and $f(g(y)) = y$.

3.1.1 Formulas

Given the formula $y = f(x)$, we solve for y directly to get the formula $x = f^{-1}(y)$.

3.1.2 Graphs

If the x- and y-axes have the same scales, the graph of f^{-1} is the reflection of the graph of f across the line $y = x$.



3.2 Questions

1. Use a graph of the function $f(x) = x^3 - 5x + 10$ to decide whether or not it is invertible.

2. Suppose that $y = f(x) = \frac{x}{4} + 10$. Find $x = f^{-1}(y)$.

3. Write a table of values for f^{-1}, where f is as given below. The domain of f is the integers from 1 to 7. State the domain of f^{-1}.

x	1	2	3	4	5	6	7
$f(x)$	3	-7	19	4	178	2	1

4. The functions $r = f(t)$ and $V = g(r)$ give the radius and the volume of a commercial hot air balloon being inflated for testing. The variable t is in minutes, r is in feet, and V is in cubic feet. The inflation begins at $t = 0$. In each case, give a mathematical expression that represents the given statement
   (a) The volume of the ballon t minutes after inflation began.
   (b) The volume of the ballon if its radius were twice as big.
   (c) The time that has elapsed when the radius of the balloon is 30 feet.
   (d) The time that has elapsed when the volume of the balloon is 10,000 cubic feet.