

Functions and change

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1 Functions

1.1 Terminologies

Definition 1.1. A *function* is a rule that takes certain numbers as inputs and assigns to each a definite output number.

- The set of all input numbers is called the *domain* of the function.
- The set of resulting output numbers is called the *range* of the function.
- The input is called the *independent variable*.
- The output is called the *dependent variable*.

Notationally, we write

$$y = f(x)$$

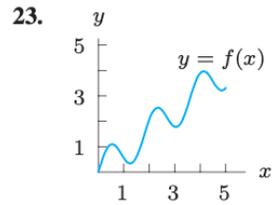
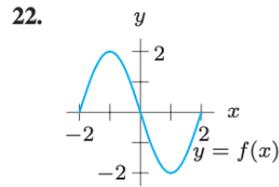
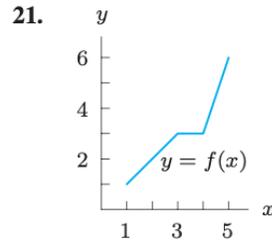
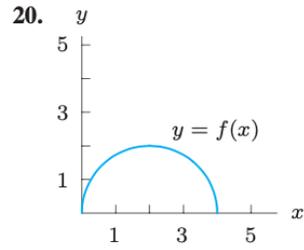
where f is the rule, x is the independent variable and y is the dependent variable.

- The set of numbers x such that $a \leq x \leq b$ is called a *closed interval* and written $[a, b]$
- The set of numbers x such that $a < x < b$ is called an *open interval* and written (a, b) .

QUESTIONS

1. What is the domain and range of following functions
 - (a) $y = 1/x$
 - (b) $y = e^x$
 - (c) $y = \ln(x)$

2. Give the approximate domain and range of each function.



3. Which of the following functions has its domain identical with its range

- (a) $f(x) = x^2$
- (b) $g(x) = \sqrt{x}$
- (c) $h(x) = x^3$
- (d) $i(x) = |x|$

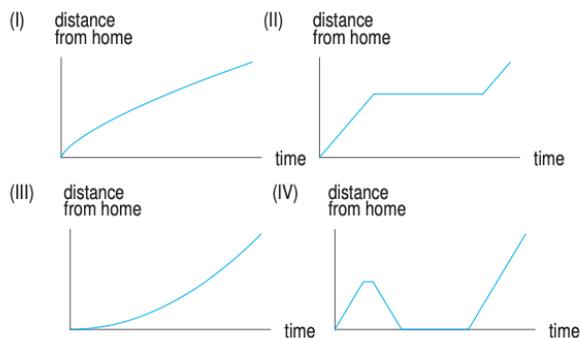
4. The function $S = f(t)$ gives the average annual sea level, S , in meters, in Aberdeen, Scotland, as a function of t , the number of years before 2008. Write a mathematical expression that represents the statement “the average annual sea level in Aberdeen was the same in 1865 and 1911”.

1.2 The Rule of Four: Tables, Graphs, Formulas, and Words

Functions can be represented by tables, graphs, formulas, and descriptions in words. Key point is that function \neq formula.

QUESTIONS

1. Which graph below best matches each of the following stories?



- (a) I had just left home when I realized I had forgotten my books, and so I went back to pick them up.
- (b) Things went fine until I had a flat tire.
- (c) I started out calmly but sped up when I realized I was going to be late.

2 Linear Functions

2.1 Definition

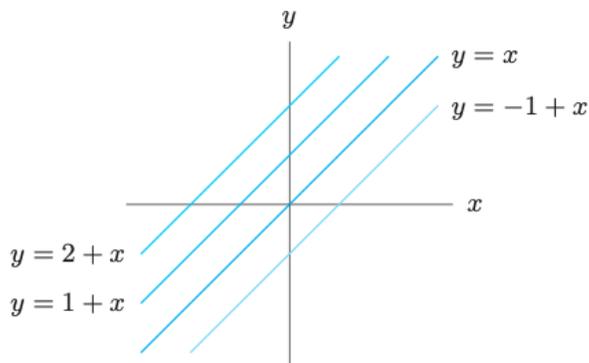
A linear function has the form

$$y = f(x) = mx + b$$

where

- m is the *slope*, or rate of change of y with respect to x ,
- b is the *vertical intercept*, or value of y when x is zero.

The graph is a straight line

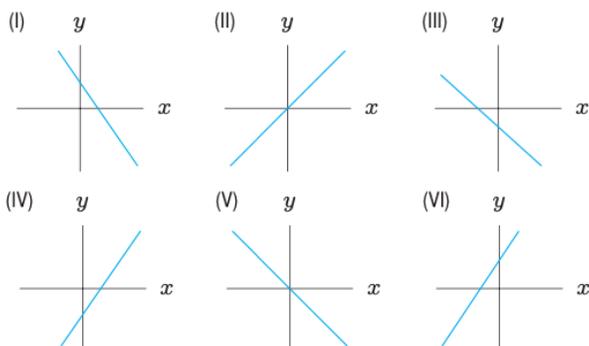


Two lines are parallel to each if and only if their slopes are the same. Two lines are perpendicular to each other if and only if _____

To recognize that a table of x and y values comes from a linear function, $y = mx + b$, look for differences in y -values that are constant for equally spaced x -values.

QUESTIONS

1. Match the graphs with the following equations



- (a) $y = -2.72x$
- (b) $y = 0.01 + 0.001x$
- (c) $y = 27.9 - 0.1x$
- (d) $y = 0.1x - 27.9$
- (e) $y = -5.7 - 200x$
- (f) $y = x/3.14$

2. Find a linear function that generates the values in table below

x	5.2	5.3	5.4	5.5	5.6
y	27.8	29.2	30.6	32.0	33.4

3. The table gives the average weight, w , in pounds, of American men in their sixties for height, h , in inches.

$h(\text{inches})$	68	69	70	71	72	73	74	75
$w(\text{pounds})$	166	171	176	181	186	191	196	201

- (a) How do you know that the data in this table could represent a linear function?
- (b) Find weight w as a linear function of height h . What is the slope of the line? What are the units for the slope?
- (c) Find height h as a linear function of weight w . What is the slope of the line? What are the units for the slope?
4. Find an equation for the line through the point $(2, 1)$ which is perpendicular to the line $y = 5x - 3$.
5. A company rents cars at \$40 a day and 15 cents a mile. Its competitor's car are \$50 a day and 10 cents a mile.
- (a) For each company, give a formula for the cost of renting a car for a day as a function of the distance traveled.
- (b) On the same axes, graph both functions.
- (c) How should you decide which company is cheaper?

3 More Terminologies

3.1 Proportionality

We say y is (*directly*) *proportional* to x if there is a nonzero constant k such that $y = kx$. This k is called the constant of proportionality.

We also say that one quantity is *inversely proportional* to another if one is proportional to the reciprocal of the other. For example, if $y = \frac{2}{x}$, then we can say that y is inversely proportional to x .

3.2 Difference Quotients and Delta Notation

We use the symbol Δ to mean “change in”. So Δx means change in x and Δy means change in y .

For any function $y = f(x)$, we can calculate the difference quotient between two points x_1 and x_2 using following formula

$$m = \frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

What is the difference quotient when $f(x) = mx + b$ is a linear function?

QUESTIONS

1. Find an equation for the line that passes through $(-2, 1)$ and $(2, 3)$.

3.3 Increasing and Decreasing Functions

A function f is *increasing* if the values of $f(x)$ increases as x increases

A function f is *decreasing* if the values of $f(x)$ decreases as x increases

The graph of an increasing function climbs as we move from left to right.

The graph of a decreasing function falls as we move from left to right.

A function $f(x)$ is monotonic if it increases for all x or decreases for all x .



QUESTIONS

1. (a) A function f is _____ if the values of $f(x)$ increases as x *decreases*.
(b) A function f is _____ if the values of $f(x)$ decreases as x *decreases*.
2. Given a linear function $f(x) = mx + b$. What is the relation between its monotonicity, its slope m and its intercept b ?