Quiz 6

Name:

2017/03/09

This quiz has 7 questions worth 12 points on 2 pages. Try to do as many questions as possible. You can use your calculator.

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1. (1 point) [**True** or **False**] If a continuous function f is defined on a closed interval $[a, b]$, then f must have a local maximaum

   Solution: True. By Extreme Value Theorem, f must have a global maximum, and this is necessarily a local maximum.

2. (1 point) [**True** or **False**] If a continuous function f is defined on an open interval (a, b), then both a and b are critical points of f.

   Solution: False, because a and b are not in the domain of f.

3. (1 point) [**True** or **False**] If p is a critical point of the function f, then $f'(p) = 0$

   Solution: False, it could be the case that $f'(p)$ doesn't exist.

4. (1 point) [**True** or **False**] If a continuous function f has only one critical point in its domain, then that point has to be a global extremum.

   Solution: False, the critical point doesn't necessarily to be a local extremum. For example, if $f(x) = x^3$ is defined on $(-1, 1)$. Then it has only one critical point 0, which is not an extremum.

5. (1 point) [**True** or **False**] If a point p in the domain of f satisfies $f'(p) = 0$ and $f''(p) < 0$, then p must be a local maximum of f

   Solution: True. That's the second derivative test.

6. (1 point) [**True** or **False**] A point p in the domain of f is called a inflection point if $f''(p) = 0$

Solution: False. A **infletion** point is a point where f changes its concavity. A point whose second derivative is zero is a possible inflection point.

7. (6 points) Find the global maxima and minima of $f(x) = x^3 - 9x^2 - 48x + 52$ on the following intervals

- $-5 \leq x \leq 12$
- $-5 \leq x \leq 14$
- $-5 \leq x < \infty$

Solution: First we calculate $f'(x) = 3x^2 - 18x - 48 = 3(x + 2)(x - 8)$.

- The critical points are $-5, -2, 8, 12$. Compare $f(-5) = -58, f(-2) = 104, f(8) = -396$ and $f(12) = -92$ we see that maximum is 104 at -2 while minimum is -396 at $x = 8$
- The critical points are $-5, -2, 8, 14$. Compare $f(-5) = -58, f(-2) = 104, f(8) = -396$ and $f(14) = 360$ we see that maximum is 360 at 14 while minimum is -396 at $x = 8$
- The critical points are $-5, -2, 8$. Compare $f(-5) = -58, f(-2) = 104, f(8) = -396$ and $\lim_{x\to\infty} f(x) = \infty$ we see that there is no maximum while minimum is -396 at $x = 8$