

Quiz 5

Name:

2017/02/23

This quiz has 4 questions worth 9 points on 1 pages. Try to do as many questions as possible. You can use your calculator.

Suppose you're given an function $y = x^x$

1. (1 point) Verify that $(2, 4)$ is a point in the curve

Solution: LHS=4 while RHS= $2^2 = 4$

2. (2 points) Rewrite the function as $\ln y = \ln(x^x)$, which simplifies to $\ln y = x \ln x$. Calculate the derivative $\frac{dy}{dx}$ using the theory of implicit functions.

Solution:

$$\begin{aligned}\frac{d}{dx}(\ln y) &= \frac{d}{dx}(x \ln x) \\ \left(\frac{1}{y}\right)\left(\frac{dy}{dx}\right) &= \ln(x) + (x)\left(\frac{1}{x}\right) \\ \frac{dy}{dx} &= y(1 + \ln x)\end{aligned}$$

3. (2 points) Rewrite the function as $y = (e^{\ln x})^x$, which simplifies to $y = e^{x \ln x}$. Calculate the derivative $\frac{dy}{dx}$ directly.

Solution:

$$\begin{aligned}\frac{d}{dx}(y) &= \frac{d}{dx}(e^{x \ln x}) \\ &= e^{x \ln x}(\ln(x) + (x)\left(\frac{1}{x}\right)) \\ &= e^{x \ln x}(1 + \ln x)\end{aligned}$$

4. (4 points) Find the equation of the tangent line at point $(1, 1)$.

Solution: The derivative is $\frac{dy}{dx} = x^x(1 + \ln x)$, so the slope of the tangent line is $2^2(2 + \ln 2) = 4(2 + \ln 2)$. Therefore the tangent line is $y = 4(2 + \ln 2)(x - 2) + 4$.