Math 115 — Practice for Exam 3

Generated April 5, 2017

Name: SOLUTIONS

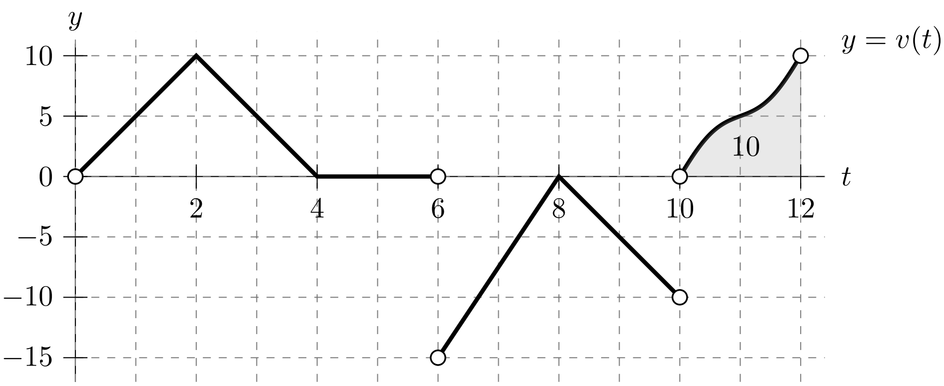
Instructor: ______________________ Section Number: __________

1. This exam has 7 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Winter 2016	3	4	Demon Drop	15	
Fall 2013	3	3	water tank	12	
Fall 2007	3	6	model rocket	10	
Winter 2004	3	9	mosquitos	8	
Winter 2003	3	5	reaction	9	
Fall 2008	3	1	Galleon	12	
Winter 2009	3	1		14	
Total				80	

Recommended time (based on points): 98 minutes

4. [15 points] Elana goes on an amusement park ride that moves straight up and down. Let $v(t)$ model Elana's velocity (in meters/second) t seconds after the ride begins (where $v(t)$ is positive when the ride is moving upwards, and negative when the ride is moving downwards). A graph of $v(t)$ for $0 < t < 12$ is shown below. Assume that $v(t)$ is piecewise linear for $0 < t < 6$ and $6 < t < 10$, and that the area of the shaded region is 10, as indicated on the graph.



a. [4 points] Write an integral that gives Elana's average velocity, in meters/second, from 2 seconds into the ride until 4 seconds into the ride. Then compute the exact value of this integral.

Answer: $\frac{1}{4-2}\int_2^4 v(t)dt$ = 5

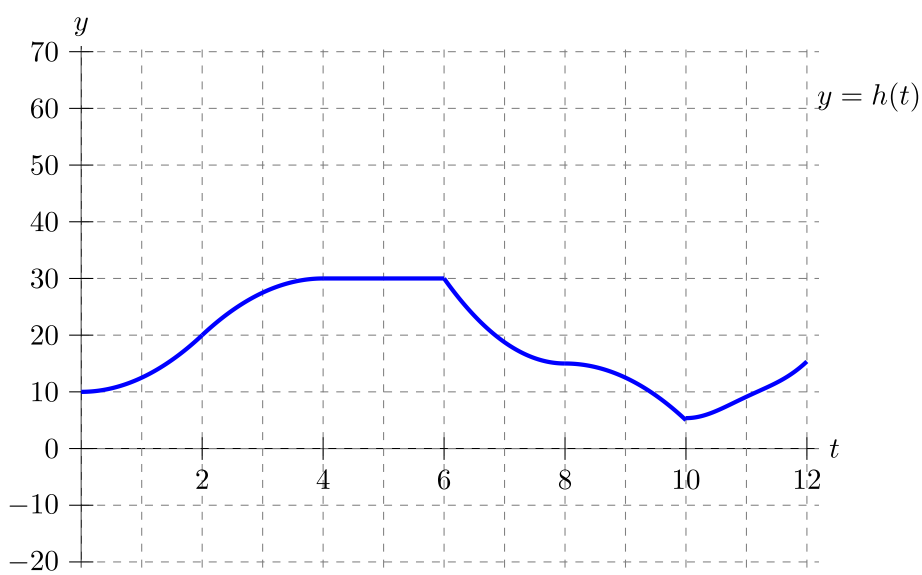
Let $h(t)$ be Elana's height (in meters) above the ground t seconds after the ride begins. Assume that h is continuous, and suppose Elana is at a height of 10 meters above the ground when the ride begins.

b. [6 points] Fill in the exact values of $h(t)$ in the table below.

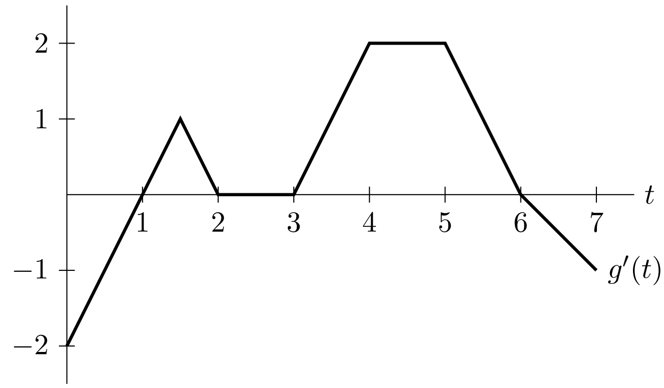
t	0	2	4	6	8	10	12
$h(t)$	10	20	30	30	15	5	15

c. [5 points] Using your work from part **b.**, sketch a detailed graph of $h(t)$ for $0 < t < 12$. In your sketch, be sure that you pay close attention to each of the following:

- where h is increasing, decreasing, or constant
- the values of $h(t)$ you found in part **b.** above
- where h is/is not differentiable
- the concavity of the graph of $y = h(t)$



3. [12 points] The function $g(t)$ is the volume of water in the town water tank, in thousands of gallons, t hours after 8 A.M. A graph of $g'(t)$, the **derivative** of $g(t)$, is shown below. Note that $g'(t)$ is a piecewise-linear function.



a. [4 points] Write an integral which represents the average rate of change, in thousands of gallons per hour, of the volume of water in the tank between 9 A.M. and 1 P.M. Compute the exact value of this integral.

Solution:

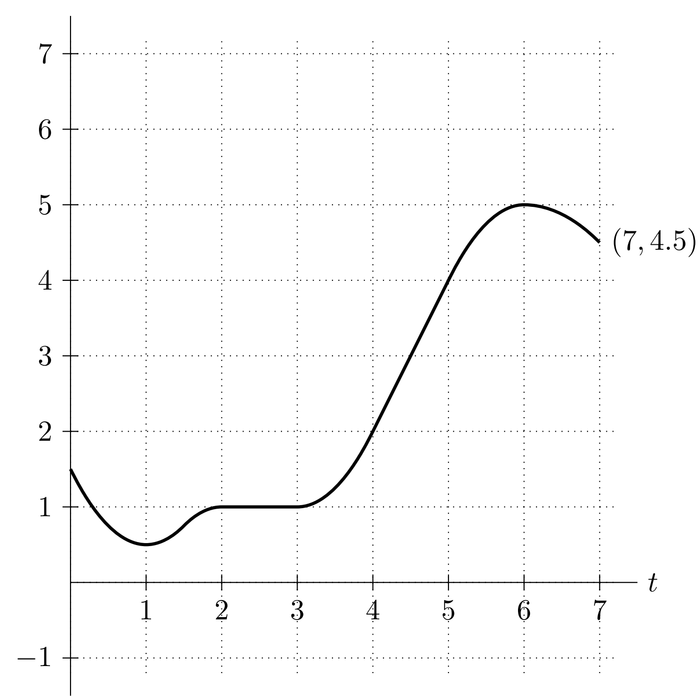
$$\frac{1}{4}\int_1^5 g'(t)\,dt = \frac{1}{4}\left(\frac{7}{2}\right) = \frac{7}{8}$$

b. [2 points] At what time does the tank have the most water in it? At what time does it have the least water?

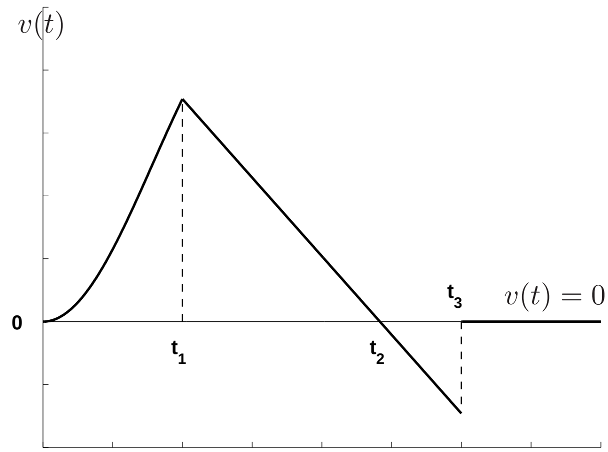
Answer: The tank has the most water in it at **2 P.M.**.

The tank has the least water in it at **9 A.M.**.

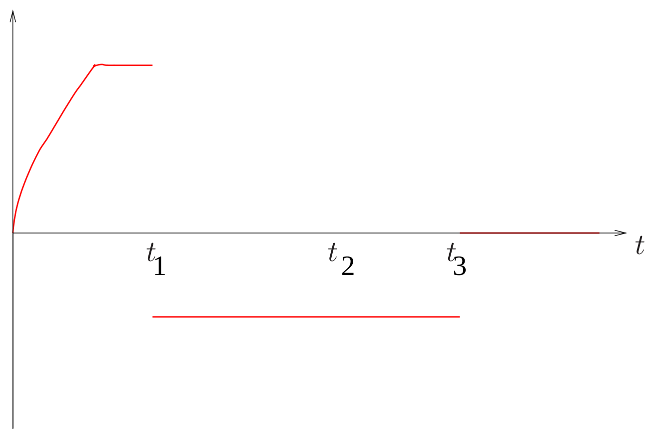
c. [6 points] Suppose that $g(3) = 1$. Sketch a detailed graph of $g(t)$ and give both coordinates of the point on the graph at $t = 7$.



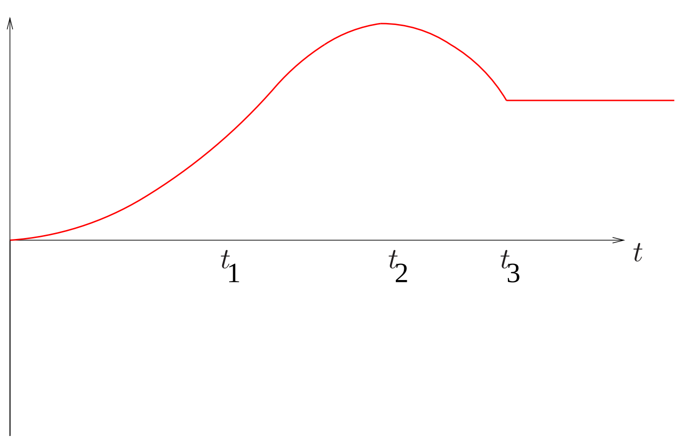
6. (10 points) A budding rocket scientist, Seema, has launched her model rocket from the ground at time $t = 0$. The velocity profile for Seema's rocket is given in the graph below. (Note: Since the vertical scale is not given, we are interested in the "shapes" and general behavior on the graphs below.)



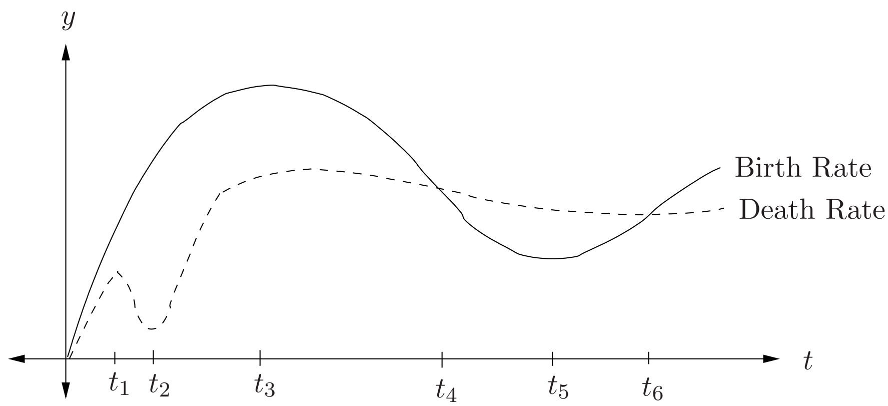
(a) Sketch a graph of the acceleration of the rocket as a function of time on the axes below.



(b) Sketch a graph of the height of the rocket as a function of time.



9. (8 points) Last year a local entomologist studied the birth and death rate of mosquitos in the Ann Arbor area during the month of May. His research yielded the following graph.



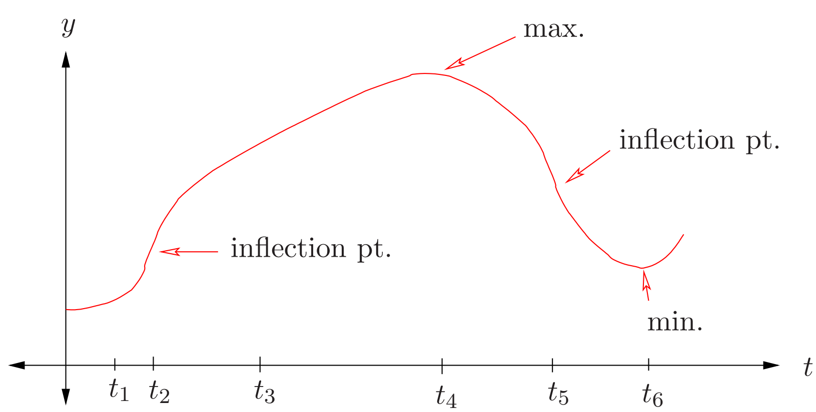
(a) Which of the labelled times t_1 through t_6 is the time when there were the largest number of mosquitos in Ann Arbor during May?

The largest number of mosquitos in Ann Arbor during May occurred at $t = t_4$. Up to this point more mosquitos are being born then die off, so our number of mosquitos is increasing. Between t_4 and t_6 more are dying then being born, so we are losing mosquitos. After t_6 we are gaining mosquitos again, but since the area between the Death Rate and Birth Rate is greater from t_4 to t_6 then after t_6, we still have less mosquitos then we did at $t = t_4$.

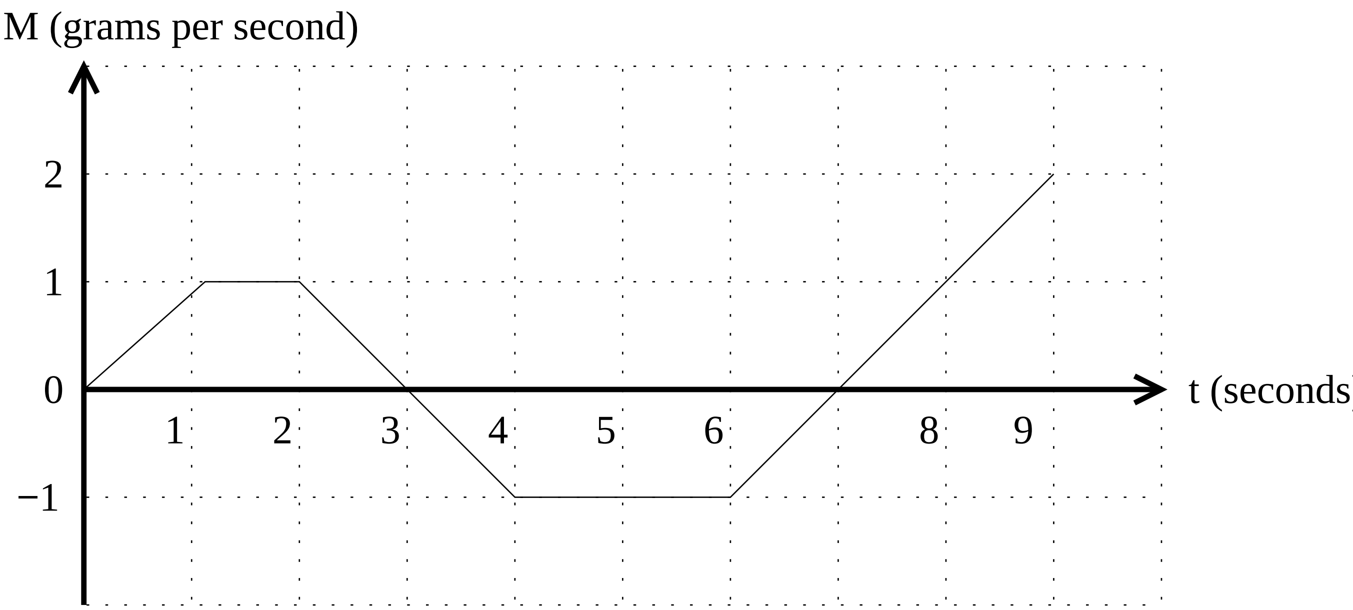
(b) Which of the labelled times t_1 through t_6 is the time when the quantity of mosquitos in Ann Arbor was increasing most rapidly during May?

The quantity of mosquitos is increasing most rapidly when there is the greatest difference between the Birth Rate and the Death Rate. This occurs when $t = t_2$.

(c) Sketch a possible graph of the number of mosquitos alive during the month of May on the axes below. Make sure to clearly indicate any maxima, minima, or inflection points.



5. (9 points) A substance, B, is one of several substances involved in a complex chemical reaction. At certain times during this reaction, substance B is produced by the reaction while at other times it plays the role of a reactant and is consumed. Given that enough reactants are present, the rate M, of production of substance B is approximated by the function whose graph is given below.



(a) Over what interval(s) is the amount of substance B increasing?

Solution: *$M = dB/dt$ is positive for $0 < t < 3$ and for $7 < t < 9$ so the amount of substance B present is increasing on those two intervals.*

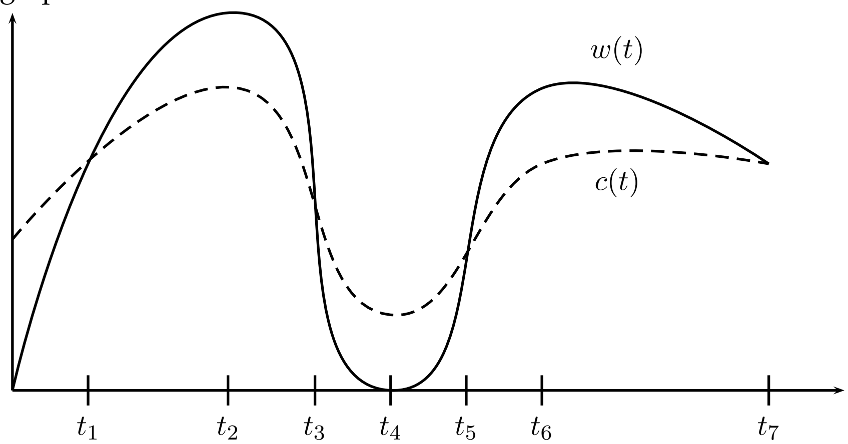
(b) At what time during the reaction is the least amount of substance B present? Explain.

Solution: *The least amount of substance B is present when $t = 7$. Because, if $B(t)$ denotes the amount of B present t seconds after the beginning of the reaction, then the change in B, $\Delta B(t) = B(t) - B(0)$ is equal to the integral of M, the rate of change of B over the interval from 0 to t. This shows the amount of B present increases for $0 \le t \le 3$ by 2 grams, the area under the graph of M over this interval, so $B(3) = B(0) + 2$ gms. For $3 \le t \le 7$, the amount of B present decreases by 3 grams, the area between the graph of M and the x-axis over this interval, so $B(7) = B(0) - 1$. And, the amount of B present then increases for $0 \le t \le 9$ (up to $B(9) = B(0) + 1$). So, the smallest amount occurs when $t = 7$.*

(c) The reaction takes 9 seconds to complete and will not proceed if there is no substance B present. There is a value, V, such that if the reaction begins with V or fewer grams of substance B, then the reaction will not proceed to completion. Find the value of V, and explain your answer.

Solution: *The value is $V = 1$. As explained in in part (**b**), the least amount of B is present at $t = 7$ and is $B(0) - 1$ gm, one gram less than at the beginning of the reaction. If there had been less than one gram of B at the beginning, the amount of B would have been exhausted before $t = 7$ so the reaction would not have completed.*

1. [12 points] In the 17th century, a ship's navigator would estimate the distance the ship has traveled using readings of the ship's velocity, $v(t)$, in knots (nautical miles per hour). Suppose that between noon and 3:00 pm a certain galleon is traveling with the wind and against the ocean current, and that its velocity is given as the difference between the wind velocity $w(t)$ and the velocity of the ocean current $c(t)$, so that $v(t) = w(t) - c(t)$, where t is in hours since noon. Consider the wind and ocean velocities for various times between noon and 3:00 p.m., given by the graphs below:



a. [1 point] Using *integral notation* write an expression giving the distance the ship traveled from noon to 3:00 pm. Give units.

Solution: $d = \int_0^3 v(t)dt$, with the distance in nautical miles.

b. [1 point] Using *integral notation* write an expression giving the average velocity of the ship between noon and 3:00 pm. Give units.

Solution: $v_{av} = \frac{1}{3}\int_0^3 v(t)dt$, with the distance in nautical miles/hour, or knots.

c. [2 points] For what intervals was the ship's velocity positive?

Solution: The ship's velocity is positive when $w(t) > c(t)$, which happens on the intervals (t_1, t_3) and (t_5, t_7).

d. [2 points] For what t values was the ship not moving towards its destination?

Solution: Since this happens when the ship's velocity is zero or negative, the t values are $[t_0, t_1]$, $[t_3, t_5]$ and t_7.

e. [2 points] For what intervals was the ship's velocity increasing?

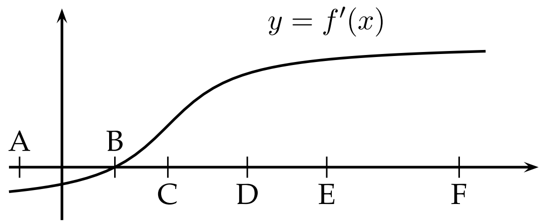
Solution: The ship's velocity is increasing when the acceleration is positive, and since $a(t) = v'(t) = w'(t) - c'(t)$, in order for $a(t) > 0$ we need $w'(t) > c'(t)$, *i.e.* that the slope of the tangent line to $w(t)$ is greater than the slope of the tangent line to $c(t)$. This happens on the intervals (t_0, t_2) and (t_4, t_6).

f. [4 points] Please circle each integral which is positive and underline each integral which is negative.

Solution:

$\int_{t_1}^{t_3} v(t)dt$ $\int_{t_5}^{t_7} v(t)dt$ $\int_{t_0}^{t_7} w(t)dt$ $\int_{t_3}^{t_5} c(t)dt$

1. (2 points each) Suppose f is a twice-differentiable function. Use the graph of the derivative f', shown below, to answer the following questions. No explanations are required.



(a) At which of the marked x-values does f attain a global minimum on the interval [A,F]?

B

(b) At which of the marked x-values does f attain a global maximum on the interval [A,F]?

F

(c) At which of the marked x-values does f' attain a global minimum on the interval [A,F]?

A

(d) At which of the marked x-values does f' attain a global maximum on the interval [A,F]?

F

(e) At which of the marked x-values does f'' attain a global maximum on the interval [A,F]?

C

(f) For which of the marked x-values does $\int_A^x f'(t)\,dt$ attain a global minimum on the interval [A,F]?

B

(g) For which of the marked x-values does $\int_A^x f'(t)\,dt$ attain a global maximum on the interval [A,F]?

F