Math 115 — Practice for Exam 3

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Name: SOLUTIONS

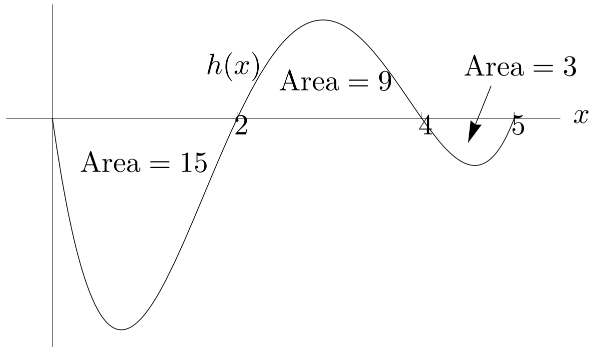
Instructor: ______________________ Section Number: ________

1. This exam has 5 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Winter 2012	3	2		15	
Winter 2014	3	1		10	
Winter 2015	3	6		6	
Fall 2016	3	5		10	
Fall 2015	3	1		12	
Total				53	

Recommended time (based on points): 64 minutes

2. [15 points] Using the graph of $h(x)$ shown below, compute each of the following quantities. If there is not enough information to compute the given quantity, write "not enough information". You do not need to explain your answers.



a. [3 points] $\int_2^0 \Big(h(x)+2\Big)dx$

Solution:
$$\int_2^0 \Big(h(x)+2\Big)dx = -\int_0^2 h(x)dx - \int_0^2 2dx = 15-4=11.$$

b. [2 points] $\int_0^5 3h(y)dy$

Solution:
$$\int_0^5 3h(y)dy = 3(-15+9-3) = -27.$$

c. [3 points] $\int_8^9 h(x-4)dx$

Solution:
$$\int_8^9 h(x-4)dx = \int_4^5 h(x)dx = -3.$$

d. [3 points] The average value of $h(x)$ on the interval $[-2,2]$, assuming that $h(x)$ is an even function.

Solution:
$$\frac{1}{4}\int_{-2}^2 h(x)dx = \frac{1}{4}\cdot 2\cdot \int_0^2 h(x)dx = \frac{-15}{2}.$$

e. [2 points] $H(2)$, where H is an antiderivative of h

Solution: Not enough information.

f. [2 points] $H(2)-H(0)$, where H is an antiderivative of h

Solution: By the fundamental theorem of calculus,
$$H(2)-H(0) = \int_0^2 h(x)dx = -15.$$

1. [11 points] The table below gives several values of a function $f(x)$ and its derivative. Assume that both $f(x)$ and $f'(x)$ are defined and differentiable for all x.

x	0	1	2	3	4	5	6
$f(x)$	0	3	4	2	-1	-3	5
$f'(x)$	4	2	-1	-5	-2	7	9
$f''(x)$	-1	-3	-5	0	4	3	1

Compute each of the following. Do not give approximations. If it is not possible to find the value exactly, write NOT POSSIBLE

a. [2 points] Find $\int_0^4 f''(x)\,dx$.

Solution: By the Fundamental Theorem of Calculus,

$$\int_0^4 f''(x)\,dx = f'(4) - f'(0) = -2 - 4 = -6.$$

Answer: $\int_0^4 f''(x)\,dx =$ -6

b. [2 points] Find $\int_2^5 (3f(x)+1)\,dx$.

Solution: In order to evaluate this exactly, we would need to know an antiderivative of $f(x)$. Since we don't know one, this is not possible to evaluate exactly.

Answer: $\int_2^5 (3f(x)+1)\,dx =$ NOT POSSIBLE

c. [3 points] Find the average value of $4f'(x)+x$ on the interval $[1,6]$.

Solution: The average value can be computed as an integral. Since an antiderivative of $4f'(x)+x$ is $4f(x)+\frac{1}{2}x^2$, we can compute the exact value of this integral with the Fundamental Theorem of Calculus:

$$\frac{1}{6-1}\int_1^6 \left(4f'(x)+x\right)\,dx = \frac{1}{5}\left(\left(4f(6)+\frac{1}{2}6^2\right) - \left(4f(1)+\frac{1}{2}1^2\right)\right) = 5.1$$

Answer: 5.1

d. [4 points] Assuming that $f(x)$ is an odd function, find $\int_{-3}^{3} f(x)\,dx$ and $\int_{-3}^{3} f'(x)\,dx$.

Solution: Note that

$$\int_{-3}^{3} f(x)\,dx = \int_{-3}^{0} f(x)\,dx + \int_0^3 f(x)\,dx,$$

and since $f(x)$ is an odd function, the two integrals on the right cancel out, leaving us with 0.

Also, since $f(x)$ is odd, we have $f(-3) = -f(3)$, and hence by the Fundamental Theorem of Calculus,

$$\int_{-3}^{3} f'(x)\,dx = f(3) - f(-3) = f(3) - (-f(3)) = 2f(3) = 4.$$

Answer: $\int_{-3}^{3} f(x)\,dx =$ 0 and $\int_{-3}^{3} f'(x)\,dx =$ 4

5. [5 points] After hearing of the *Illumisqati* activities from Erin and Elphaba, the Police storm the King's farmhouse and find ample evidence to convict him of kidnapping. However, since he is the King, charges can only be brought against him if the Police can show proficiency in mathematics. Help them by doing the following problem.

For c a constant, consider the function $B(u) = \arctan(u^c + 7)$.

Use the limit definition of the derivative to write an explicit expression for $B'(3)$.

Your answer should not involve the letter B. Do not attempt to evaluate or simplify the limit.

Answer: $B'(3) = \lim_{h\to 0} \dfrac{\arctan((3+h)^c+7) - \arctan(3^c+7)}{h}$

6. [6 points] Recall the following definitions:

- A function f is *even* if $f(-x) = f(x)$ for all x in the domain of f.
- A function f is *odd* if $f(-x) = -f(x)$ for all x in the domain of f.

Compute each of the integrals below. If not enough information is provided to answer the question, write NOT ENOUGH INFORMATION.

a. [2 points] Suppose g is a differentiable function on $(-\infty, \infty)$ and g' (the **derivative** of g) is a continuous odd function with $g(3) = 2$ and $g(7) = 9$. Find $\int_{-3}^{7} g'(x)\,dx$.

Solution: Since $g'(x)$ is odd, $\int_{-3}^{3} g'(x)\,dx = 0$ so we have

$$\int_{-3}^{7} g'(x)\,dx = \int_{3}^{7} g'(x)\,dx = g(7) - g(3) = 9 - 2 = 7.$$

Answer: $\int_{-3}^{7} g'(x)\,dx =$ 7

b. [2 points] Suppose that q is a continuous and even function on $(-\infty, \infty)$ and that $\int_{0}^{5} q(x)\,dx = -4$. Find $\int_{-5}^{5} (3q(x) + 7)\,dx$.

Solution: By the linearity properties of definite integrals,

$$\int_{-5}^{5} (3q(x)+7)\,dx = 3\left(\int_{-5}^{5} q(x)\,dx\right) + \int_{-5}^{5} 7\,dx = 3\left(\int_{-5}^{5} q(x)\,dx\right) + 70.$$

Since $q(x)$ is even, $\int_{-5}^{5} q(x)\,dx = 2\int_{0}^{5} q(x)\,dx = -8$.

Therefore, $\int_{-5}^{5} (3q(x)+7)\,dx = 3(-8) + 70 = 46$.

Answer: $\int_{-5}^{5} (3q(x)+7)\,dx =$ 46

c. [2 points] Let $h(x) = \ln x$ and suppose p is a differentiable function on $(-\infty, \infty)$ with $p(4) = 7$. Find $\int_{4}^{1} (h(x)p'(x) + h'(x)p(x))\,dx$.

Solution: Note that $h(x)p(x)$ is an antiderivative of $h(x)p'(x) + h'(x)p(x)$ so by the Fundamental Theorem of Calculus,

$$\int_{4}^{1} (h(x)p'(x) + h'(x)p(x))\,dx = h(1)p(1) - h(4)p(4) = \ln(1)p(1) - \ln(4)p(4) = -7\ln(4).$$

Answer: $\int_{4}^{1} (h(x)p'(x) + h'(x)p(x))\,dx =$ $-7\ln(4)$

5. [10 points] The table below gives several values of a function $q(u)$ and its first and second derivatives. Assume that all of $q(u)$, $q'(u)$, and $q''(u)$ are defined and continuous for all real numbers u.

u	0	1	2	3	4	5	6
$q(u)$	30	23	19	20	24	25	24
$q'(u)$	0	−6	−2	1	3	1	−2
$q''(u)$	−9	5	4	3	2	−5	0

Compute each of the following. Do not give approximations. If it is not possible to find the value exactly, write NOT POSSIBLE.

a. [2 points] Compute $\int_5^2 q''(t)\,dt$. $= q'(2) - q'(5) = (-2) - (1) = -3$

Answer: $\int_5^2 q''(t)\,dt =$ -3

b. [2 points] Compute $\int_1^5 (-2q''(u) + 2u)\,du$. $= -2q'(u) + u^2 \Big|_1^5 = \left[-2q'(5) + 5^2\right] - \left[-2q'(1) + 1^2\right]$

$= \left[-2(1) + 25\right] - \left[-2(-6) + 1\right] = 23 - 13$

Answer: $\int_1^5 (-2q''(u) + 2u)\,du =$ 10

c. [2 points] Suppose that $q(u)$ is an even function. Compute $\int_{-5}^5 q(u)\,du$.

Answer: $\int_{-5}^5 q(u)\,du =$ NOT POSSIBLE

d. [2 points] Suppose that $q(u)$ is an even function. Compute $\int_{-5}^5 (q'(u) + 7)\,du$.

$= q(u) + 7u \Big|_{-5}^{5} = \left[q(5) + 7(5)\right] - \left[q(-5) + 7(-5)\right] = \left[q(5) - q(-5)\right] + 70$ (equal)

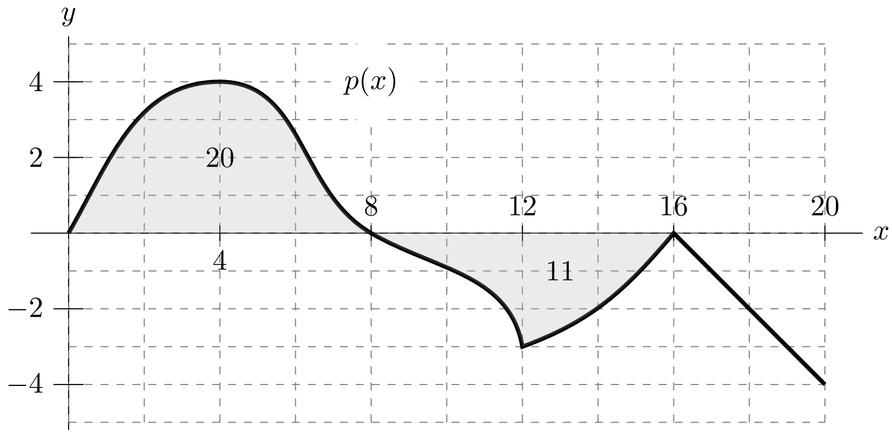
Answer: $\int_{-5}^5 (q'(u) + 7)\,du =$ 70

e. [2 points] Compute the average value of $-5q'(u)$ on the interval $[1, 4]$.

$\frac{1}{4-1}\int_1^4 -5q'(u)\,du = \frac{-5}{3} q(u)\Big|_1^4 = -\frac{5}{3}\left[q(4) - q(1)\right] = \frac{-5}{3}\left[24 - 23\right]$

Answer: $-5/3$

1. [12 points] Recall that a function h is odd if $h(-x) = -h(x)$ for all x. A portion of the graph of $p(x)$, an odd function, is shown below. Assume that the areas of the two shaded regions are 20 and 11, as indicated on the graph, and note that $p(x)$ is linear for $16 < x < 20$.



Remember to show your work throughout this problem.

a. [4 points] Compute the exact value of $\int_0^{20} (5 - 3p(x))\,dx$.

Solution: We have $$\int_0^{20} (5 - 3p(x))\,dx = \int_0^{20} 5\,dx - 3\int_0^{20} p(x)\,dx$$ $$= 100 - 3(20 - 11 - \frac{4 \cdot 4}{2}) = 97.$$

Answer: 97

b. [2 points] Compute the exact value of $\int_4^8 p'(x)\,dx$.

Solution: By the Fundamental Theorem, we have $\int_4^8 p'(x)\,dx = p(8) - p(4) = 0 - 4 = -4$.

Answer: -4

c. [3 points] Find the average value of $p(x)$ on the interval $-16 \le x \le 8$.

Solution: The average value is given by $\frac{1}{8-(-16)}\int_{-16}^{8} p(x)\,dx$. Since p is odd, we have $\int_{-8}^{8} p(x)\,dx = 0$, and $\int_{-16}^{-8} p(x)\,dx = -\int_8^{16} p(x)\,dx$. Thus, the average value is

$$\frac{1}{24}\left(\int_{-16}^{8} p(x)\,dx\right) = \frac{1}{24}\left(\int_{-16}^{-8} p(x)\,dx + \int_{-8}^{8} p(x)\,dx\right) = \frac{1}{24}\left(-\int_8^{16} p(x)\,dx + 0\right) = \frac{11}{24}.$$

Answer: $\frac{11}{24}$

d. [3 points] Use a right Riemann sum with 3 equal subintervals to estimate $\int_{12}^{18} p(x)\,dx$. Write out all terms of the sum.

Solution:

$$2\,(p(14) + p(16) + p(18)) = 2(-2 + 0 + (-2)) = -8.$$

Answer: $\int_{12}^{18} p(x)\,dx \approx$ -8