Group Work

2017/03/14

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1. (10 points) Suppose a paraboloid cup is inscribed in a hemisphere of radius R inches. The volume of the paraboloid is given by $\frac{1}{2}\pi r^2 h$. For what values of the parameter r and h is the volume of the cup maximized?

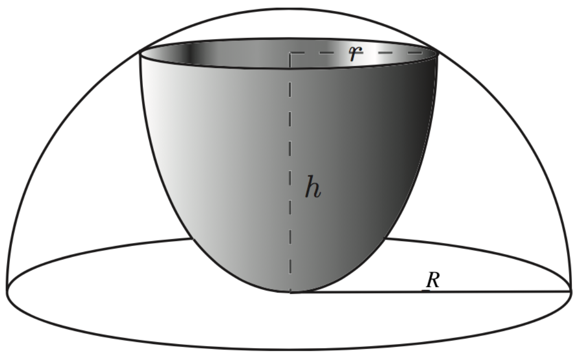

Figure 1: Paraboloid

Solution: Use Pythagorean Theorem we obtain a relation

$$r^2 + h^2 = R^2 \tag{1}$$

Solve (1) we get

$$r^2 = R^2 - h^2 \tag{2}$$

Now we want to maximize

$$V = \frac{1}{2}\pi r^2 h = \frac{1}{2}\pi(R^2 - h^2)h \tag{3}$$

Take the derivative of (3)

$$\frac{dV}{dh} = \frac{1}{2}\pi R^2 - \frac{3}{2}\pi h^2 \tag{4}$$

Solve $\frac{dV}{dh} = 0$ we get $h = \pm\frac{R}{\sqrt{3}}$. But h is a positive number, so $h = \frac{R}{\sqrt{3}}$.
To show that it's a local maximum, we use the second derivatice test:

$$\frac{d^2V}{dx^2} = -3\pi h < 0 \tag{5}$$

Therefore it's local maximum. Since it's the only critical point in the domain, it has to be a global maximum.

Once we get h, plug into (1) we get $r = \sqrt{R^2 - h^2} = \sqrt{\frac{2}{3}}R$. So the volume is maximized when $h = \frac{1}{\sqrt{3}}R$ inches and $r = \sqrt{\frac{2}{3}}R$ inches.