Review for 4.1-4.3

Math 115 Section 004

3/10/2017

1 Vocabulary

What's the definition of each term?

- Local Extrema
- Global Extrema
- Critical Points
- Inflection Points

Check yourself:

- What's the relation between critical points and local extrema?
- What's the relation between local extrema and global extrema?
- What's the relation between inflection points and those points whose second derivative is zero?

2 Important Points

- The function doesn't necessarily have a derivative at critical points. For example, $f(x) = |x|$ at $x = 0$
- Critical points must be in the domain. For example, if $f(x)$ is defined on (a, b), then a and b are not critical points because they are not in the domain.
- Endpoints could be local extrema. For example, $f(x) = x$ is defined on $[1, 2]$, then $x = 1$ is a local (global) minimum and $x = 2$ is local (global) maximum.
- **Justification for a critical point to be a LOCAL extremum:** If p is a critical point and the derivative $f'(x)$ is defined near p, then we have to look at the sign of $f'(p - 0.1)$ and $f'(p + 0.1)$ (You need to explicitly calculate the value to show its sign, DON'T just write $f'(p-0.1) > 0$). If the sign **changes**, then p is a local extremum.

- **Justification for a critical point to be a LOCAL extremum:** If p is a critical point and the second derivative $f''(x)$ is defined at p, then if $f''(p) > 0$ (resp. $f''(p) < 0$), then p is a local minimum(resp. maximum). **If $f''(p) = 0$, then we cannot say anything about p.**
- **Justification for a critical point to be a GLOBAL extremum, One critical point case:** If p is a critical point other than the endpoint and it's the only critical point, then
  1. You need to justify that it's a local extremum
  2. Since it's the only critical point, a local extremum must be a global extremum.
- **Justification for a critical point to be a GLOBAL extremum, Several critical points case:** If we have several critical points, where endpoints are critical points, you have to compare the function value at these points, the largest one is the global maximum and the smallest one is the global minimum.
- **Justification for a critical point to be a GLOBAL extremum, Several critical points case:** If we have several critical points, where endpoints are **NOT** critical points, you have to compare the function value at these points and the limit value at endpoints. If the largest (resp. smallest) one occurs at some critical point, then it's a global maximum (resp. minimum). If the largest (resp. smallest) one occurs at the limit of endpoint, then the function doesn't have a global maximum (resp. minimum).
- You **CANNOT** use a graph of the original function to justify your answer if required to "**use calculus** to find and justify your answer".
- You **CAN** use a graph of the derivative function to justify your answer, you need to sketch the graph and label it to indicate that there is a sign change near the critical point.

3 Common Errors

1. Forget to check those points where f' doesn't exist. This happens a lot in a piecewise-defined function!
2. Forget to show that sign change of derivative.
3. Forget to justify that the only critical point is a local extremum.
4. Forget to provide justification of inflection points.
5. Write "critical points of f'" but actually mean "critical points of f"