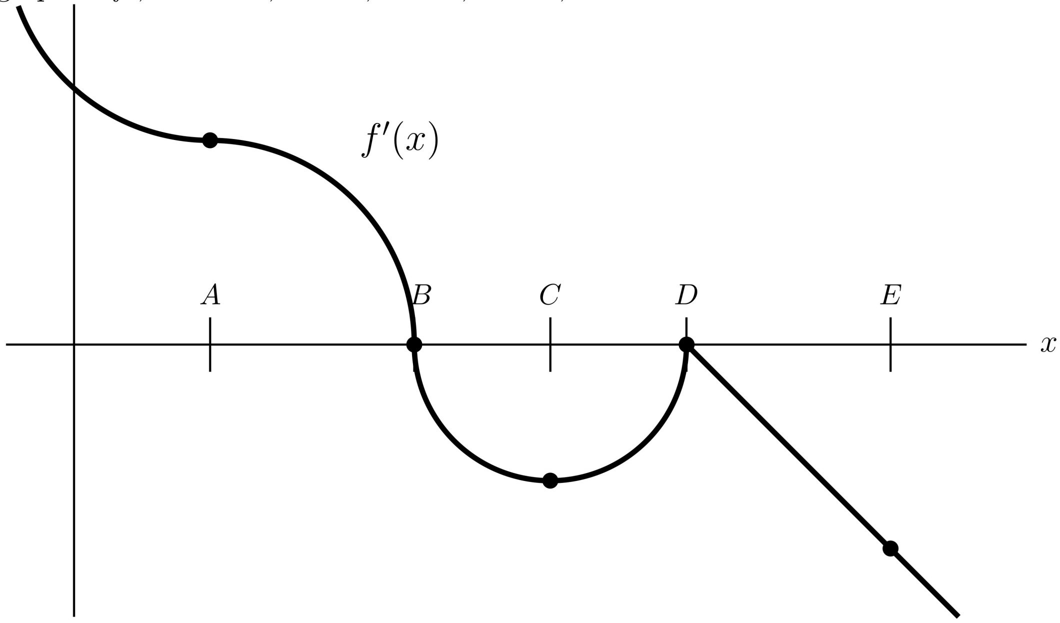
6. [12 points] The *derivative* of a function f is graphed below. Five points are marked on the graph of f', at $x = A$, $x = B$, $x = C$, $x = D$, and $x = E$.



For each of the following, circle ALL answers which are correct. Each part has at least one correct answer. Pay careful attention to whether each question is asking about f, f', or f''.

a. [2 points] The function f' has a local minimum when ______________.

$x = A$ $x = B$ $\boxed{x = C}$ $x = D$ $x = E$

b. [2 points] The function f is increasing when ______________.

$\boxed{x = A}$ $x = B$ $x = C$ $x = D$ $x = E$

c. [2 points] The function f has a critical point when ______________.

$x = A$ $\boxed{x = B}$ $x = C$ $\boxed{x = D}$ $x = E$

d. [2 points] The global maximum of f on the interval $A \leq x \leq E$ occurs when ______________.

$x = A$ $\boxed{x = B}$ $x = C$ $x = D$ $x = E$

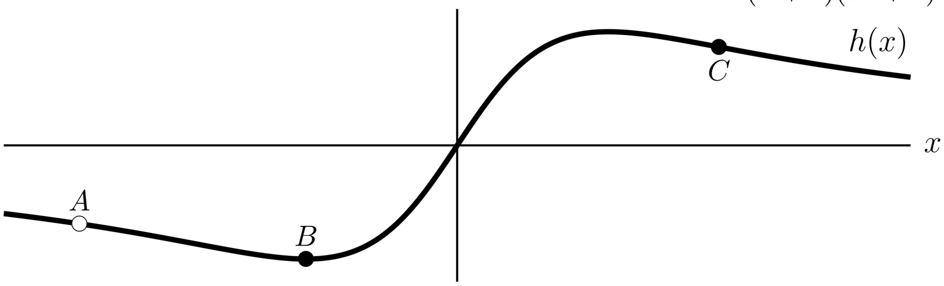
e. [2 points] The function f has an inflection point when ______________.

$x = A$ $x = B$ $\boxed{x = C}$ $\boxed{x = D}$ $x = E$

f. [2 points] The function f'' is undefined when ______________.

$x = A$ $\boxed{x = B}$ $x = C$ $\boxed{x = D}$ $x = E$

8. [13 points] Below, there is a graph of the function $h(x) = \dfrac{2x^2+10x}{(x+5)(x^2+4)}$.



a. [3 points] The point A is a hole in the graph of h. Find the x- and y-coordinates of A.

Solution: Simplifying $h(x)$, we have $h(x) = \dfrac{2x(x+5)}{(x+5)(x^2+4)}$. Since the factor $(x+5)$ cancels, the hole occurs when $x = -5$. We look at the limit as x approaches -5 on the cancelled form to get the y-coordinate:

$$\lim_{x\to -5} h(x) = \lim_{x\to -5} \frac{2x}{x^2+4} = \frac{-10}{29},$$

Thus, $A = (-5, \frac{-10}{29})$.

b. [5 points] The point B is a local minimum of h. Find the x- and y-coordinates of B.

Solution: Using the quotient rule on the simplified form of h, we have $h'(x) = \dfrac{4-x^2}{(x^2+4)^2}$. This is never undefined, and it is equal to zero when $4 - x^2 = 0$ or $x = \pm 2$. From the graph, we can see that the local minimum occurs at $x = -2$. The y-coordinate here is $y = \frac{-4}{8} = -\frac{1}{2}$, so $B = (-2, -\frac{1}{2})$.

c. [5 points] The point C is an inflection point of h. Find the x- and y-coordinates of C.

Solution: We use the quotient rule again to find $h''(x) = \dfrac{2x^3 - 24x}{(x^2+4)^3} = \dfrac{2x(x^2-12)}{(x^2+4)^3}$. This is never undefined, and it is zero when $2x(x^2-12) = 0$, i.e. when $x = 0, \pm 2\sqrt{3}$. From the graph, we see that our x-coordinate must be $+2\sqrt{3}$, and then $y = \frac{4\sqrt{3}}{16} = \frac{\sqrt{3}}{4}$, so $C = (2\sqrt{3}, \frac{\sqrt{3}}{4})$.

2. [14 points] The table for the *derivative* of a function h with continuous first derivative is given below. Assume that between each consecutive value of x, the derivative h' is either increasing or decreasing. For each statement below, indicate whether the statement is true, false, or cannot be determined from the information given. No partial credit will be given.

x	-4	-3	-2	-1	0	1	2	3	4
$h'(x)$	2	3	1	-3	-4	-2	0	2	1

a.) The function h has a local maximum on the interval $-2 < x < -1$.

[True] False Not enough information

b.) The function h is negative on the interval $-1 < x < 1$.

True False **[Not enough information]**

c.) The function h is concave up on the interval $0 < x < 4$.

True **[False]** Not enough information

d.) The function h is decreasing on the interval $-3 < x < -2$.

True **[False]** Not enough information

e.) The function h has an inflection point on the interval $-1 < x < 1$.

[True] False Not enough information

f.) The derivative function, h', has a critical point at $x = 2$.

True False **[Not enough information]**

g.) The second derivative function, h'', is positive on the interval $0 < x < 3$.

True False **[Not enough information]**

7. [15 points] Suppose a is a positive constant and

$$f(x) = 2x^3 - 3ax^2.$$

a. [10 points] Find the absolute maximum and minimum values of $f(x)$ on the closed interval $[-a, \frac{3}{2}a]$. Specify all x values where the maximum and minimum value are achieved.

Solution: Seeking critical points, we take the derivative of f and set it equal to zero.

$$f'(x) = 6x^2 - 6ax = 6x(x - a) = 0.$$

Using this equation we find the critical points to be $x = 0, a$. Now we put the critical points and the endpoints of the interval back into the orginal function and compare the values. We compute $f(-a) = -5a^3,\ f(0) = 0,\ f(a) = -a^3,\ f(\frac{3}{2}a) = 0$.
This means the absolute max of f on this interval is 0 and this value is achieved at $x = 0, \frac{3}{2}a$. The absolute min is $-5a^3$ and this value is achieved at $x = -a$.

b. [5 points] Find all inflection points of $f(x)$.

Solution: Seeking inflection points, we find $f''(x) = 12x - 6a$. Setting this equal to zero we find $x = \frac{a}{2}$. To verify this is an inflection point we test $f''(0) = -6a < 0$ and $f''(a) = 6a > 0$. This means f'' changes sign at $x = \frac{a}{2}$, so it is an inflection point.

2. [13 points] The U-value of a wall of a building is a positive number related to the rate of energy transfer through the wall. Walls with a lower U-value keep more heat in during the winter than ones with a higher U-value. Consider a wall which consists of two materials, material A with U-value a and material B with U-value b. The U-value of the wall w is given by

$$w = \frac{ab}{b+a}.$$

Considering a as a constant, we can think of w as a function of b, $w = u(b)$.

a. [4 points] Write the limit definition of the derivative of $u(b)$.

Solution: The derivative of $u(b)$ is defined to be

$$u'(b) = \lim_{h \to 0} \frac{u(b+h) - u(b)}{h} = \lim_{h \to 0} \frac{a(b+h)/(b+h+a) - ab/(b+a)}{h}$$

b. [4 points] Calculate $u'(b)$. (You do **not** need to use the limit definition of the derivative for your calculation.)

Solution: By the quotient rule,

$$u'(b) = \frac{(b+a)(a) - ab}{(b+a)^2} = \frac{a^2}{(b+a)^2}.$$

c. [5 points] Find the x- and y-coordinates of the global minimum and maximum of $u(b)$ for b in the interval $[1, 2]$. Your answer may involve the parameter a. [Recall that $a, b > 0$.]

Solution: The derivative $u'(b)$ is strictly positive for all $b > 0$ by part **b.** This means there are not any critical points on $[1, 2]$ and u is strictly increasing so $b = 1$ is the global minimum while $b = 2$ is the global maximum. Now we compute $u(1) = \frac{a}{1+a}$ and $u(2) = \frac{2a}{2+a}$.

Global minimum on $[1, 2]$: $(1, \frac{a}{1+a})$

Global maximum on $[1, 2]$: $(2, \frac{a}{2+a})$

3. [12 points] Representative values of the derivative of a function $f(x)$ are shown in the table below. Assume $f'(x)$ is a continuous function and that the values in the table are representative of the behavior of $f'(x)$.

x	0	0.5	1	1.5	2	2.5	3
$f'(x)$	1	0.3	0	-0.1	-0.15	-0.12	-0.10

a. [6 points] Estimate the location of the global maximum and minimum of $f(x)$ on the closed interval $[0, 3]$. Justify your answers based on the data in the table.

Solution: We note that $f'(x) > 0$ for $x < 1$ and $f'(x) < 0$ for $x > 1$. Thus $f(x)$ has a local maximum at $x = 1$. Further, because there is only one change of sign in the derivative, we know that this is the global maximum. The global minimum will occur at one of the endpoints. It is not easy to tell at which endpoint this occurs, but because the negative slopes are of smaller magnitude (for $x > 1$) than the positive slopes (for $x < 1$), we expect that the global minimum occurs at $x = 0$.

b. [6 points] Can you tell from these data if $f(x)$ has any inflection points? If so, estimate the location of any inflection points and indicate how you know their locations. If not, explain why not.

Solution: We know that an inflection point occurs when $f'(x)$ goes from increasing to decreasing or vice versa. We can see from these data that $f'(x)$ is decreasing until sometime between $x = 2$ and $x = 2.5$, and increasing thereafter. Thus there is an inflection point at an x somewhere in $(2, 2.5)$.

9. [12 points] Suppose $w(x)$ is an everywhere differentiable function which satisfies the following conditions:

- $w'(0) = 0$.
- $w'(x) > 0$ for $x > 0$.
- $w'(x) < 0$ for $x < 0$.

Let $f(t) = t^2 + bt + c$ where b and c are positive constants with $b^2 > 4c$. Define $L(t) = w(f(t))$.

a. [2 points] Compute $L'(t)$. Your answer may involve w and/or w' and constants b and c.

Solution: $L'(t) = w'(t^2 + bt + c) \cdot (2t + b)$.

b. [4 points] Using your answer from (a), find the critical points of $L(t)$ in terms of the constants b and c.

Solution: $L(t)$ has critical points when $L'(t) = 0$. This happens only if $w'(t^2+bt+c) = 0$ or if $(2t + b) = 0$.
$w'(t^2 + bt + c) = 0$ means $t^2 + bt + c = 0$ by the first property of w' above. Solving using the quadratic formula, we have

$$t = -\frac{b}{2} \pm \frac{\sqrt{b^2 - 4c}}{2}$$

as critical points of $L(t)$. Both of these roots exist and are distinct since $b^2 > 4c$.
If $2t + b = 0$, we have $t = -\frac{b}{2}$ as a critical point. Altogether our critical points are $t = -\frac{b}{2}, -\frac{b}{2} + \frac{\sqrt{b^2-4c}}{2}, -\frac{b}{2} - \frac{\sqrt{b^2-4c}}{2}$.

c. [6 points] Classify each critical point you found in (b). Be sure to fully justify your answer.

Solution: For simplicity, let's set $p = -\frac{b}{2} + \frac{\sqrt{b^2-4c}}{2}$ and $m = -\frac{b}{2} - \frac{\sqrt{b^2-4c}}{2}$.

We know that $f(t)$ is an upward opening parabola with roots at p and m. We also know $p > m$, so this means $f(t) > 0$ for $t < m$ and $t > p$. This also means $f(t) < 0$ for $m < t < p$. Thus by properties two and three of w' above we know $w'(f(t)) > 0$ for $t < m$ and $t > p$, and $w'(f(t)) < 0$ for $m < t < p$.

The expression $2t + b$ is positive for $t > -\frac{b}{2}$ and negative for $t < -\frac{b}{2}$.

Putting all of this information together gives us

$$L'(t) > 0$$

on the intervals $(m, -\frac{b}{2})$ and $(p, +\infty)$, and

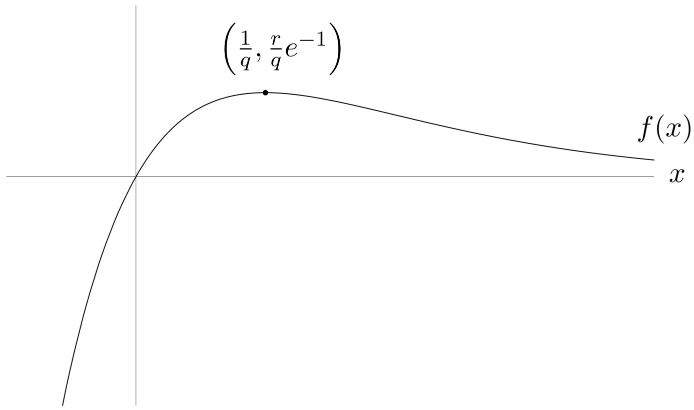
$$L'(t) < 0$$

on the intervals $(-\infty, m)$ and $(-\frac{b}{2}, p)$. Thus, by the first derivative test, the critical points $t = m = -\frac{b}{2} - \frac{\sqrt{b^2-4c}}{2}$ and $t = p = -\frac{b}{2} + \frac{\sqrt{b^2-4c}}{2}$ are local minima, and $t = -\frac{b}{2}$ is a local maximum.

8. [16 points] Below is the graph of the function

$$f(x) = rxe^{-qx},$$

where r and q are constants. Assume that both r and q are greater than 1. The function $f(x)$ passes through the origin and has a local maximum at the point $P = \left(\frac{1}{q}, \frac{r}{q}e^{-1}\right)$, as shown in the graph.



a. [4 points] Justify, using either the first-derivative test or second-derivative test, that the point P is a local maximum.

Solution: To apply the first-derivative test, first compute:

$$f'(x) = re^{-qx}(-qx+1).$$

Thus, there is indeed a critical point at $x = \frac{1}{q}$. Plugging in $x = 0$ (which is less than $\frac{1}{q}$), we find $f'(0) = r > 0$, while plugging in $x = \frac{2}{q}$ (which is greater than $\frac{1}{q}$), we find $f'(\frac{2}{q}) = -re^{-2} < 0$. Thus, f' changes from positive to negative at $x = \frac{1}{q}$, so it is a local maximum.
To apply the second-derivative test, compute $f''(x) = -rqe^{-qx}(-qx+2)$, so

$$f''(\tfrac{1}{q}) = -rqe^{-1} < 0.$$

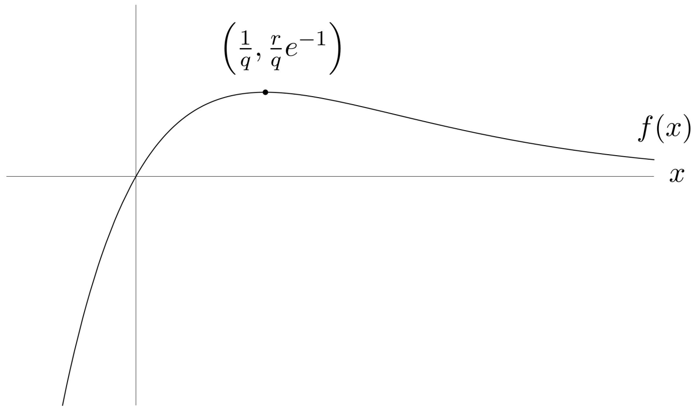
b. [2 points] What are the x-coordinates of the global maximum and minimum of $f(x)$ on the domain $[0, 1]$? (If $f(x)$ does not have a global maximum on this domain, say "no global maximum", and similarly if $f(x)$ does not have a global minimum.)

Solution: Since $q > 1$, the local maximum at $x = \frac{1}{q}$ is within this domain. Therefore, the global maximum occurs at $x = \frac{1}{q}$ and the global minimum occurs at $x = 0$.

c. [2 points] What are the x-coordinates of the global maximum and minimum of $f(x)$ on the domain $(-\infty, \infty)$? (If $f(x)$ does not have a global maximum on this domain, say "no global maximum", and similarly if $f(x)$ does not have a global minimum.)

Solution: The global maximum is at $x = \frac{1}{q}$ and there is no global minimum.

8. (continued) For your convenience, the graph of $f(x)$ is repeated below.



d. [4 points] Suppose that $g(x)$ is a function with $g'(x) = f(x)$. Find x-values of all local maxima and minima of $g(x)$. Justify that each maximum you find is a maximum and each minimum is a minimum.

Solution: The function $g(x)$ has critical points wherever $g'(x) = f(x) = 0$, which is only at $x = 0$. Since $f(x)$ changes from negative to positive at this point, the critical point at $x = 0$ is a local minimum by the first-derivative test.

e. [4 points] If $g(x)$ is as in part (d), for which x-values does $g(x)$ have inflection points? Show that these x-values are indeed inflection points.

Solution: The function $g(x)$ has inflection points when $g''(x) = f'(x)$ changes sign. This occurs precisely when $f(x)$ changes from increasing to decreasing (or vice versa), which is at $x = \frac{1}{q}$.

5. [14 points] The function f is has a continuous second derivative on the interval $10 \le x \le 19$. Some values of its derivative function f' are given in the table below.

x	10	11	12	13	14	15	16	17	18	19
$f'(x)$	-34	-3	-1	-2	-3	31	62	70	66	37

a. [4 points] f has exactly one inflection point on the interval $15 \le x \le 19$. Given the information provided, give the smallest x interval on which this inflection point is guaranteed to lie, making it clear whether your endpoints are included.

Solution: $16 < x < 18$ or $(16, 18)$.

b. [8 points] f has exactly four critical points, with x-values 11.2, 11.7, 12.6, and 14.2, respectively. Classify each point as a local minimum, a local maximum, or neither, given that f has either a local maximum or a local minimum at $x = 11.2$. For each point below, circle only one option.

At $x = 11.2$, f has **a local maximum** **[a local minimum]**

At $x = 11.7$, f has **[a local maximum]** **a local minimum** **neither**

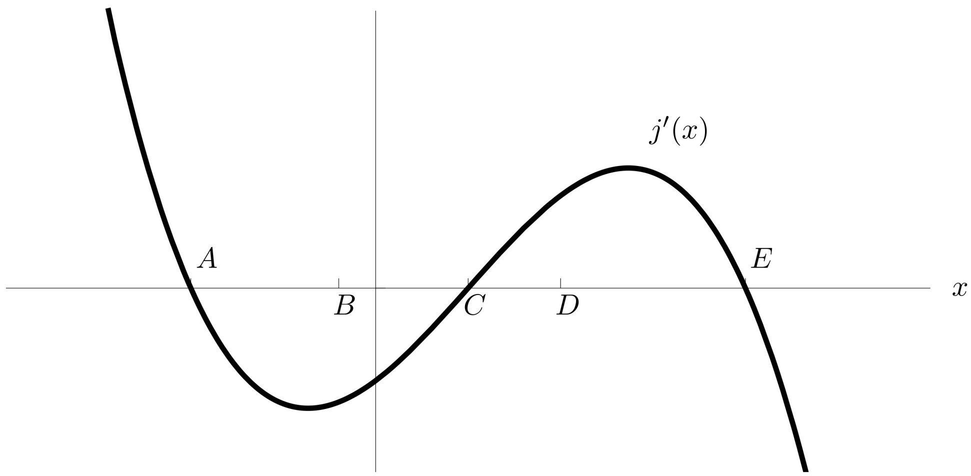
At $x = 12.6$, f has **a local maximum** **a local minimum** **[neither]**

At $x = 14.2$, f has **a local maximum** **[a local minimum]** **neither**

c. [2 points] Is there at least one inflection point on the interval $11 < x < 12$? (Circle one.)

[Yes] **No** **Not possible to determine**

1. [12 points] Consider the graph of $j'(x)$ given here. Note that this is not the graph of $j(x)$.



For each of (a)-(f) below, list **all** x-values labeled on the graph which satisfy the given statement in the blank provided. If the statement is not true at any of the labeled x-values, write "NP". You do not need to show your work. No partial credit will be given on each part of this problem.

(a) The function $j(x)$ has a local minimum at $x =$ **C**.

(b) The function $j(x)$ has a local maximum at $x =$ **A, E**.

(c) The function $j(x)$ is concave up at $x =$ **B, C, D**.

(d) The function $j(x)$ is concave down at $x =$ **A, E**.

(e) The function $j'(x)$ has a critical point at $x =$ **NP**.

(f) The function $j''(x)$ is greatest at $x =$ **C**.

9. [10 points] The function $f(x)$ is twice-differentiable. Some values of f and f' are given in the following table. In addition, it is known that $f''(x)$ is positive.

x	0	1	2	3	4
$f(x)$	7	6	7	9	12
$f'(x)$	-2	$\frac{1}{2}$	1	2	4

No partial credit will be given on any part of this problem.

a. [4 points] **Circle** any statement which is true, and **draw a line through** any statement which is false.

(i.) For some value of x with $0 < x < 1$, f has a critical point.

(ii.) ~~For some value of x with $1 < x < 2$, f has a critical point.~~

(iii.) ~~For some value of x with $2 < x < 3$, f has a critical point.~~

(iv.) ~~For some value of x with $3 < x < 4$, f has a critical point.~~

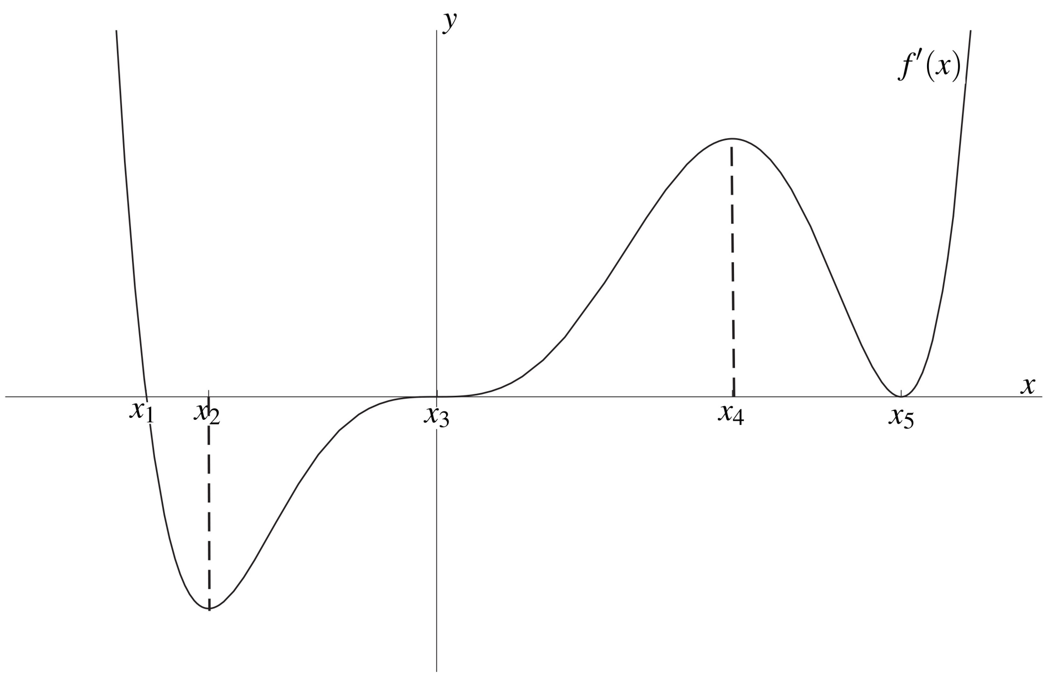
b. [3 points] If possible, find the global minimum value of $f(x)$ on the closed interval $[0, 4]$. (Give the y-coordinate, not the x-coordinate.) Do not give an approximation. If it is not possible to find it exactly, write "IT IS NOT POSSIBLE TO FIND IT EXACTLY."

Solution: We know that $f(x)$ is decreasing until some critical point p between 0 and 1 and is increasing after that (because we know that f' goes from negative to positive between 0 and 1 and never becomes negative again, since $f'' > 0$). The minimum occurs at some point that's not included in the table, so IT IS NOT POSSIBLE TO FIND IT EXACTLY.

c. [3 points] If possible, find the global maximum value of $f(x)$ on the closed interval $[0, 4]$. (Give the y-coordinate, not the x-coordinate.) Do not give an approximation. If it is not possible to find it exactly, write "IT IS NOT POSSIBLE TO FIND IT EXACTLY."

Solution: The only critical point is a local minimum, so the maximum value must be at one of the endpoints. Looking at the table, we see the maximum value is 12 (when $x = 4$).

1. (12 points) The graph below is a plot of $f'(x)$ (the *derivative* of f). Use the graph to answer questions about the function f.



(a) What are the critical points of f? x_1, x_3, x_5

(b) For what value(s) of x does f have a local maximum? x_1

(c) For what value(s) of x does f have a local minimum? x_3

(d) What are the inflection points of f? x_2, x_4, x_5

(e) On what interval(s) is f concave up? $[x_2, x_4], [x_5, \infty)$

(f) If f is a polynomial, what is the minimal degree of f? 7

2. (12 points) Suppose a is a positive (non-zero) constant, and consider the function

$$f(x) = \frac{1}{3}x^3 - 4a^2x.$$

Determine all maxima and minima of f in the interval $[-3a, 5a]$. For each, specify whether it is global or local.

We need to check values of f at the endpoints ($x = -3a$ and $x = 5a$) and wherever $f'(x) = 0$ or is undefined. Since $f'(x) = x^2 - 4a^2$, $f'(x)$ is defined for all x and $f'(x) = 0$ at $x = \pm 2a$. So, we have critical points $x = -2a, 2a$. We check all points individually to determine which are minima and which are maxima. We can use the second derivative, $f''(x) = 2x$ to help with the check.

- $\underline{x = -3a}$:
  $f'(-3a) = (-3a)^2 - 4a^2 = 5a^2 > 0$, so the function is increasing there, and this endpoint must be a (local) minimum. Since $f(-3a) = 3a^3$, we have the point $(-3a, 3a^3)$.
- $\underline{x = 5a}$:
  $f'(5a) = (5a)^2 - 4a^2 = 21a^2 > 0$, so this endpoint must be a (local) maximum. Since $f(5a) = \frac{65}{3}a^3$, we have the point $(5a, \frac{65}{3}a^3)$.
- $\underline{x = -2a}$:
  $f''(-2a) = -4a < 0$, so this must be a (local) maximum. Since $f(-2a) = \frac{16}{3}a^3$, we see that this occurs at the point $(-2a, \frac{16}{3}a^3)$.
- $\underline{x = 2a}$:
  $f''(2a) = 4a > 0$, so this must be a (local) minimum. Since $f(2a) = -\frac{16}{3}a^3$, we see that this minimum occurs at the point $(2a, -\frac{16}{3}a^3)$.

Comparing the y-values of the minima at $x = -3a, 2a$, we find that the global minimum occurs at $x = 2a$. Similarly, comparing the y-values of the maxima at $x = -2a, 5a$, we find the global maximum at the endpoint $x = 5a$.
Summing up:

- There's a local minimum at $(-3a, 3a^3)$.
- There's a local maximum at $(-2a, \frac{16}{3}a^3)$.
- There's a local and global minimum at $(2a, -\frac{16}{3}a^3)$.
- There's a local and global maximum at $(5a, \frac{65}{3}a^3)$.

4. [10 points] Let $h(x)$ be a twice differentiable function defined for all real numbers x. (So h is differentiable and its derivative h' is also differentiable.)
Some values of $h'(x)$, the derivative of h are given in the table below.

x	-8	-6	-4	-2	0	2	4	6	8
$h'(x)$	3	7	0	-3	-5	-4	0	-2	6

For each of the following, circle all the correct answers.
Circle "NONE OF THESE" if none of the provided choices are correct.

a. [2 points] Circle all the intervals below in which $h(x)$ must have a critical point.

$-8 < x < -6$ $\quad$ $\boxed{-6 < x < -2}$ $\quad$ $-2 < x < 2$ $\quad$ $\boxed{2 < x < 6}$ $\quad$ $\boxed{6 < x < 8}$

NONE OF THESE

b. [2 points] Circle all the intervals below in which $h(x)$ must have a local extremum (i.e. a local maximum or a local minimum).

$-8 < x < -6$ $\quad$ $\boxed{-6 < x < -2}$ $\quad$ $-2 < x < 2$ $\quad$ $2 < x < 6$ $\quad$ $\boxed{6 < x < 8}$

NONE OF THESE

c. [2 points] Circle all the intervals below in which $h(x)$ must have an inflection point.

$\boxed{-8 < x < -4}$ $\quad$ $-4 < x < 0$ $\quad$ $0 < x < 4$ $\quad$ $\boxed{2 < x < 6}$ $\quad$ $\boxed{4 < x < 8}$

NONE OF THESE

d. [2 points] Circle all the intervals below which must contain a number c such that $h''(c) = 2$.

$\boxed{-8 < x < -6}$ $\quad$ $-4 < x < -2$ $\quad$ $-2 < x < 0$ $\quad$ $\boxed{2 < x < 4}$ $\quad$ $6 < x < 8$

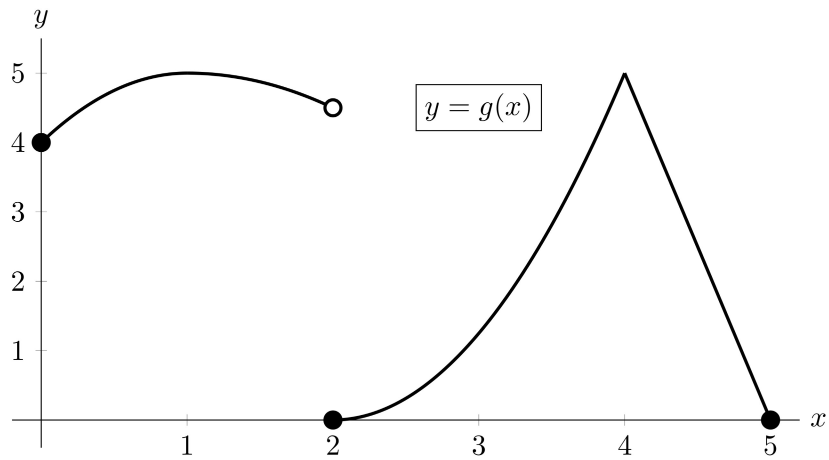
NONE OF THESE

e. [2 points] Suppose that $h''(x) < 0$ for $x < -8$, and $h(-8) = 7$. Circle all the numbers below which could equal the value of $h(-10)$.

$\boxed{-2}$ $\quad$ $\boxed{-1}$ $\quad$ $\boxed{0}$ $\quad$ 1 $\quad$ 2

NONE OF THESE

6. [14 points] The entire graph of a function $g(x)$ is shown below. Note that the graph of $g(x)$ has a horizontal tangent line at $x = 1$ and a sharp corner at $x = 4$.



For each of the questions below, circle all of the available correct answers.
(Circle NONE OF THESE if none of the available choices are correct.)

a. [2 points] At which of the following values of x does $g(x)$ appear to have a critical point?

[$x = 1$] [$x = 2$] $x = 3$ [$x = 4$] NONE OF THESE

b. [2 points] At which of the following values of x does $g(x)$ attain a local maximum?

[$x = 1$] $x = 2$ $x = 3$ [$x = 4$] NONE OF THESE

c. [6 points] Let $L(x)$ be the local linearization of $g(x)$ near $x = 3$. Circle all of the statements that are true.

$L(3) > g(3)$	$L(2.5) > g(2.5)$	$L(0) > g(0)$
[$L(3) = g(3)$]	$L(2.5) = g(2.5)$	$L(0) = g(0)$
$L(3) < g(3)$	[$L(2.5) < g(2.5)$]	[$L(0) < g(0)$]
$L'(3) > g'(3)$	[$L'(2.5) > g'(2.5)$]	[$L(5) > g(5)$]
[$L'(3) = g'(3)$]	$L'(2.5) = g'(2.5)$	$L(5) = g(5)$
$L'(3) < g'(3)$	$L'(2.5) < g'(2.5)$	$L(5) < g(5)$

NONE OF THESE

d. [2 points] On which of the following intervals does $g(x)$ satisfy the hypotheses of the Mean Value Theorem?

$[0, 2]$ $[0, 4]$ $[3, 5]$ [$[4, 5]$] NONE OF THESE

e. [2 points] On which of the following intervals does $g(x)$ satisfy the conclusion of the Mean Value Theorem?

$[0, 2]$ [$[0, 4]$] $[3, 5]$ [$[4, 5]$] NONE OF THESE