4.1 Using First and Second Derivatives & MVT

Math 115 Section 004

2/24/2017

1 Review

What do the derivatives tell us about the function?

- If $f' > 0$ on an interval, then f is **increasing** on that interval
- If $f' < 0$ on an interval, then f is **decreasing** on that interval
- If $f'' > 0$ on an interval, then the graph of f is **concave up** on that interval
- If $f'' < 0$ on an interval, then the graph of f is **concave down** on that interval

2 Definitions

Suppose p is a point in the domain of f, then
(Definitions related to the graph)

- f has a **local minimum** at p if $f(p)$ is less than or equal to the values of f for points near p
- f has a **local maximum** at p if $f(p)$ is greater than or equal to the values of f for points near p
- p is called an **inflection** point of f if the graph of f changes its concavity at p

(Definitions related to the derivative)

- p is called a **critical point** of f if $f'(p) = 0$ **or** $f'(p)$ is undefined. In either case, the point $(p, f(p))$ is also called a critial point; the function value $f(p)$ is called a critical value

3 Theorems

1. (Mean Value Theorem) If f is continuous on $a \leq x \leq b$ and differentiable on $a < x < b$, then there exists a number c with $a < c < b$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

   In other words, $f(b) - f(a) = f'(c)(b - a)$

2. Suppose p is a local maximum or minimum of the function f, p is not the endpoint of the interval and f is differentiablea at p. Then $f'(p) = 0$. Thus p is a critical point.

3. (Second-Derivative Test for Local Maxima and Minima)
   - If $f'(p) = 0$ and $f''(p) > 0$, then f has a local minimum at p
   - If $f'(p) = 0$ and $f''(p) < 0$, then f has a local maximum at p
   - If $f'(p) = 0$ and $f''(p) = 0$, the test tells us nothing.

4. Suppose p is an inflection point of the function f, p is not the endpoint of the interval and f'' is defined at p. Then $f''(p) = 0$.

5. If f has a continuous derivative, then an inflection point p of the function f is a local maximum or minimum of f'.

4 Discussion

Q1: How to detect a local maximum or minimum?

Answer: By *theorem 2* above, we know that a local maximum or minimum is necessarily a critical point. We just find all the critical points (**either** $f'(p) = 0$ **or** $f'(p)$ **is not defined**) and decide between them.

If p is a critical point such that $f'(p) = 0$, then we could either make a table like this

$x < p$	$x = p$	$x > p$
$f'(x) > 0$	$f'(x) = 0$	$f'(x) < 0$
f is increasing	f achieves local maximum	f is decreasing

or like this

$x < p$	$x = p$	$x > p$
$f'(x) < 0$	$f'(x) = 0$	$f'(x) > 0$
f is decreasing	f achieves local minimum	f is increasing

or we could use the second derivative test by *theorem 3* above.

If p is a critical point such that $f'(p)$ is not defined. Then we have to look at the graph.

Warning: A critital point p such that $f'(p) = 0$ doesn't have to be a local maximum or a local minimu. The reason is that it could fail the derivative test (i.e. $f''(p) = 0$). An example is that $f(x) = x^3$ at $x = 0$. Verify that this is not a local maximum or a local minimum but a critical point.

Q2: How to detect an inflection point?

Answer: By *theorem 4*, we could use the same ideal: first find all points such that $f''(p) = 0$. Then look at the sign of $f''(x)$ near p:

- If the sign is different for f'' on both sides of p, then p is an inflection point
- If the sign is the same, then p is not an inflection point.

5 Example

Find the x- and y- coordinates of all local minima, local maxmima and infelction points of the function

$$f(x) = e^{-18x^2+B}$$