Math 115 — Practice for Exam 2

Generated February 22, 2017

Name: ______________________________

Instructor: ______________________ Section Number: __________

1. This exam has 11 questions. Note that the problems are not of equal difficulty, so you may want to skip over and return to a problem on which you are stuck.
2. Do not separate the pages of the exam. If any pages do become separated, write your name on them and point them out to your instructor when you hand in the exam.
3. Please read the instructions for each individual exercise carefully. One of the skills being tested on this exam is your ability to interpret questions, so instructors will not answer questions about exam problems during the exam.
4. Show an appropriate amount of work (including appropriate explanation) for each exercise so that the graders can see not only the answer but also how you obtained it. Include units in your answers where appropriate.
5. You may use any calculator except a TI-92 (or other calculator with a full alphanumeric keypad). However, you must show work for any calculation which we have learned how to do in this course. You are also allowed two sides of a $3'' \times 5''$ note card.
6. If you use graphs or tables to obtain an answer, be certain to include an explanation and sketch of the graph, and to write out the entries of the table that you use.
7. You must use the methods learned in this course to solve all problems.

Semester	Exam	Problem	Name	Points	Score
Fall 2016	2	11		6	
Fall 2010	2	6	cactus cube	10	
Winter 2011	2	6		15	
Winter 2016	2	8		14	
Winter 2012	2	5		12	
Fall 2012	2	4		13	
Winter 2013	2	4		11	
Winter 2014	2	10		10	
Fall 2014	2	1		5	
Winter 2015	2	3		8	
Winter 2016	2	10		9	
Total				113	

Recommended time (based on points): 102 minutes

10. [4 points] Let a and b be constants. Consider the curve $\mathcal{C}$ defined by the equation

$$\cos(ax) + by\ln(x) = 3 + y^3.$$

Find a formula for $\frac{dy}{dx}$ in terms of x and y. The constants a and b may appear in your answer. To earn credit for this problem, you must compute this by hand and show every step of your work clearly.

Answer: $\frac{dy}{dx} =$

11. [6 points] Let $h(x) = x^x$. For this problem, it may be helpful to know the following formulas:

$$h'(x) = x^x(\ln(x) + 1) \qquad \text{and} \qquad h''(x) = x^x\left(\frac{1}{x} + (\ln(x) + 1)^2\right).$$

a. [2 points] Write a formula for $p(x)$, the local linearization of $h(x)$ near $x = 1$.

Answer: $p(x) =$ ______________________________

b. [4 points] Write a formula for $u(x)$, the quadratic approximation of $h(x)$ at $x = 1$. (Recall that a formula for the quadratic approximation $Q(x)$ of a function $f(x)$ at $x = a$ is $Q(x) = f(a) + f'(a)(x - a) + \frac{f''(a)}{2}(x - a)^2$.)

Answer: $u(x) =$ ______________________________

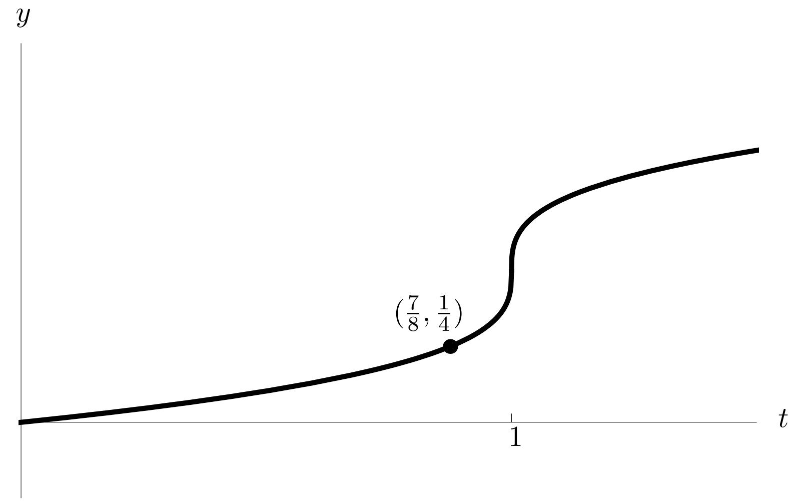
6. [10 points] Calvin is stuck in the desert, and he needs to build a cube out of cactus skins to hold various supplies. He wants his cube to have a volume of 8.1 cubic feet, but he needs to figure out the side length to cut the cactus skins the right size. He has forgotten his trusty calculator, so he decides to figure out the side length of his cube using calculus.

a. [5 points] Find a local linearization of the function $f(x) = (x + 8)^{1/3}$ at $x = 0$.

b. [3 points] Use your linearization to approximate $(8.1)^{1/3}$.

c. [2 points] Should your approximation from part **b.** be an over-estimate or an under-estimate? Why?

6. [15 points] Given below is the graph of a function $h(t)$. Suppose $j(t)$ is the local linearization of $h(t)$ at $t = \frac{7}{8}$.



a. [5 points] Given that $h'(\frac{7}{8}) = \frac{2}{3}$, find an expression for $j(t)$.

b. [4 points] Use your answer from (a) to approximate $h(1)$.

c. [3 points] Is the approximation from (b) an over- or under-estimate? Explain.

d. [3 points] Using $j(t)$ to estimate values of $h(t)$, will the estimate be more accurate at $t = 1$ or at $t = \frac{3}{4}$? Explain.

8. [14 points]
Suppose H is a differentiable function such that $H'(w)$ is also differentiable for $0 < w < 10$. Several values of $H(w)$ and of its first and second derivatives are given in the table on the right.

w	1	2	3	5	8
$H(w)$	6.3	5.4	5.2	4.8	0.7
$H'(w)$	−1.5	−0.4	−0.1	−0.6	−2.1
$H''(w)$	1.6	0.9	0	−0.8	−0.4

Assume that between each pair of consecutive values of w shown in the table, each of $H'(w)$ and $H''(w)$ is either always strictly decreasing or always strictly increasing. *Remember to show your work carefully.*

a. [3 points] Use an appropriate linear approximation to estimate $H(5.2)$.

Answer: $H(5.2) \approx$ ____________________

b. [5 points] Let $J(w)$ be the local linearization of H near $w = 2$, and let $K(w)$ be the local linearization of H near $w = 3$. Which of the following statements must be true? Circle all of the statements that must be true, or circle "NONE OF THESE".

$J(2) > H(2)$	$J(2.5) > H(2.5)$	$K(3.5) > H(3.5)$
$J(2) = H(2)$	$J(2.5) = H(2.5)$	$K(3.5) = H(3.5)$
$J(2) < H(2)$	$J(2.5) < H(2.5)$	$K(3.5) < H(3.5)$
$J'(2) > H'(2)$	$K(2.5) > H(2.5)$	$K'(3.5) > H'(3.5)$
$J'(2) = H'(2)$	$K(2.5) = H(2.5)$	$K'(3.5) = H'(3.5)$
$J'(2) < H'(2)$	$K(2.5) < H(2.5)$	$K'(3.5) < H'(3.5)$

NONE OF THESE

c. [3 points] Use the quadratic approximation of $H(w)$ at $w = 1$ to estimate $H(0.9)$. (Recall that a formula for the quadratic approximation $Q(x)$ of a function $f(x)$ at $x = a$ is $Q(x) = f(a) + f'(a)(x - a) + \frac{f''(a)}{2}(x - a)^2$.)

Answer: $H(0.9) \approx$ ____________________

d. [3 points] Consider the function N defined by $N(w) = H(2w^2 - 10)$, and let $L(w)$ be the local linearization of $N(w)$ at $w = 3$. Find a formula for $L(w)$. Your answer should not include the function names N or H.

Answer: $L(w) =$ ____________________

5. [12 points]

a. [3 points] Find the local linearization $L(x)$ of the function

$$f(x) = (1 + x)^k$$

near $x = 0$, where k is a positive constant.

b. [3 points] For which values of k does this local linearization give underestimates of the actual value of $f(x)$? (Show your work.)

c. [2 points] Suppose you want to use $L(x)$ to find an approximation of the number $\sqrt{1.1}$. What number should k be, and what number should x be?

d. [2 points] Approximate $\sqrt{1.1}$ using $L(x)$.

e. [2 points] What is the error in the approximation from part (d)?

4. [13 points] Let $f(x) = e^{\sin\sqrt{x}}$. Let P be the point on the graph of f at which $x = 4\pi^2 (\approx 39.4784)$.

a. [3 points] Calculate $f'(x)$.

b. [4 points] Find an **exact** formula for the tangent line $L(x)$ to $f(x)$ at P. **Exact** means your answer should not involve any decimal approximations.

c. [2 points] Use your formula for $L(x)$ to approximate $e^{\sin\sqrt{38}}$.

d. [4 points] Recall that the error, $E(x)$, is the actual value of the function minus the value approximated by the tangent line. Given the fact that in this case $E(39) \approx 0.000613$ and $E(40) \approx 0.000719$, would you expect $f''(4\pi^2)$ to be positive or negative? Explain, without doing any calculations.

4. [11 points]

a. [4 points] Find the tangent line approximation of the function

$$p(x) = 1 + x^k$$

near $x = 1$, where k is a positive constant.

b. [2 points] Suppose you want to use your tangent line from (a) to approximate the number $1 + \sqrt{0.95}$. What values of k and x would you plug in to your answer from (a)?

c. [2 points] Approximate $1 + \sqrt{0.95}$ using your tangent line from (a).

d. [3 points] Determine whether your approximation in (c) is an over- or underestimate. Be sure your reasoning is clear.

10. [10 points] Let $f(x)$ be a function with $f(1) = 5$, $f'(1) = -2$, and $f''(1) = 3$.

a. [2 points] Use the local linearization of $f(x)$ at $x = 1$ to estimate $f(0.9)$.

Answer: $f(0.9) \approx$ ____________________

b. [2 points] Do you expect your estimate from Part (a) to be an overestimate or underestimate? To receive any credit on this question, you must justify your answer.

c. [2 points] Use the tangent line approximation of $f'(x)$ near $x = 1$ to estimate $f'(1.1)$.

Answer: $f'(1.1) \approx$ ____________________

d. [4 points] Suppose that the tangent line approximation of $f(x)$ near $x = 8$ estimates $f(8.05)$ to be 3.75 and $f(8.1)$ to be 3.6. Find $f(8)$ and $f'(8)$.

Answer: $f(8) =$ ____________ and $f'(8) =$ ____________

1. [5 points] Let $h(x)$ be a differentiable function such that $h'(x)$ is also differentiable everywhere. Suppose that $h(3) = 9$, $h'(3) = 2$, and $h''(x) > 0$ for all real numbers x.

a. [2 points] Let $L(x)$ be the local linearization of $h(x)$ at $x = 3$. Find a formula for $L(x)$.

Answer: $L(x) =$ ______________________________

b. [3 points] Which of the following equalities could be true?
Circle all the statements that could be true or circle NONE OF THESE.
You do not need to explain your reasoning.

$h(-1) = -1$

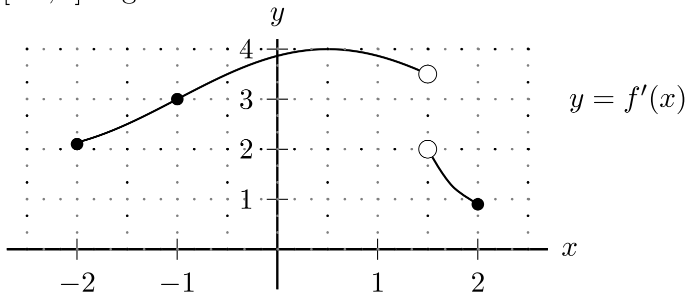
$h(-1) = 0$

$h(-1) = 1$

$h(-1) = 2$

NONE OF THESE

3. [8 points] Suppose $f(x)$ is a function that is continuous on the interval $[-2, 2]$. The graph of $f'(x)$ on the interval $[-2, 2]$ is given below.



a. [3 points] Let $L(x)$ be the local linearization of $f(x)$ at $x = -1$. Using the fact that $f(-1) = 4$, write a formula for $L(x)$.

Answer: $L(x) =$ ____________________

b. [2 points] Use your formula for $L(x)$ to approximate $f(-0.5)$.

Answer: $f(-0.5) \approx$ ____________________

c. [3 points] Is your answer from part (b) an overestimate or an underestimate of the actual value of $f(-0.5)$? Justify your answer.

Circle one: overestimate underestimate CANNOT BE DETERMINED

Justification:

10. [9 points] Consider the function h defined by $$h(x) = \begin{cases} Ax^4 & \text{if } x < 2 \\ Bx^3 + 80\ln\left(\frac{x}{2}\right) & \text{if } x \geq 2 \end{cases}$$

where A and B are constants.

a. [6 points] Find values of A and B so that h is differentiable.
Remember to show your work clearly.

Answer: $A =$ ____________________ and $B =$ ____________________

b. [3 points] Using the values of A and B you found in part **a.**, find the tangent line approximation for $h(x)$ near $x = 1$.

Answer: The tangent line approximation is given by $y =$ ____________________________